STANDARD EDUCATIONAL SERIES.

STANDARD

ELEMENTARY

ARITHMETIC

COMBINING

ORAL AND WRITTEN EXERCISES.

ST. LOUIS, MO.:
STANDARD SCHOOL BOOK COMPANY.

STANDARD EDUCATIONAL SERIES.

STANDARD FIRST READER.

STANDARD SECOND READER.

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STANDARD PRONOUNCING SPELLER.

STANDARD ELEMENTARY ARITHMETIC.

STANDARD COMPLETE ARITHMETIC.

STANDARD LANGUAGE LESSONS.

(*In preparation.*)

INTRODUCTION.

It is believed that there is a unity of plan and execution in the Elementary and the Complete Arithmetic of this course which is found in no other arithmetical series.

As the processes of oral and written arithmetic are precisely the same, the so-called Mental Arithmetic should not be made a distinct study. In fact, without ignoring other indispensable studies, there is no time for two daily lessons in arithmetic. The Progressive Oral and Written Drills, and the Inductive and Oral Exercises, provide so abundantly for developing and fixing all the introductory and fundamental facts, that a separate book would be superfluous.

In the Progressive Oral and Written Drills, advancement is made from the simpler to the more difficult combinations. The pupil almost unconsciously learns and retains the various facts or combinations, and recalls them without apparent thought. Such facility as this in addition, subtraction, multiplication and division, can be acquired only by the graded work of advancing from one digit to another.

The definitions, rules, analyses and manner of treatment, are the same in both books. The fundamental rules are developed separately and are afterward combined. It is believed that by treating them conjointly at the start, the child is brought too abruptly to the consideration of a union of principles, which, for the sake of simplicity and directness, should be treated apart. Mental growth and strength depend upon clear conceptions, and rest upon the fundamental principle, "Do one thing at a time and do it well." Hence, after treating addition and subtraction separately, they are combined, the same course being followed with multiplication and division; also with fractions.

In the Standard Arithmetics, the work, which is partly oral and partly written, has been prepared with great care, and will be found fresh, vigorous, and practical. The aim constantly has been to make the examples interesting problems of fact, and to give directness, simplicity, and clearness to every statement, so as to remove all needless difficulty from the task of the young student of numbers. Progress comes more from properly graded new matter than from wearisome repetition. Growth is the result not of repetitions but of additions: hence, space and time are not devoted to such repetitions. Pictures are discarded, because they divert attention and afford little aid in grasping arithmetical ideas.

No answers are given to the oral exercises, which are so constructed that the child can scarcely go astray in their solution. Should he do so, other pupils will be sure to detect the error. The answers to the written problems (except in a few instances) will be found at the end of the book.

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PART I.

PART II.

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HOW TO USE THIS BOOK.

I. MAKING NUMBERS.

AFTER counting with and without objects up to 10, show pupils how numbers are made, using the numeral frame or objects. Thus: 4 consists of four 1's; two 2's; a 3 and a 1; or one 4.

ILLUSTRATION: |||| ; || || ; ||| | ; ||||.

Remark.—*Require pupils to show and tell how* 5, 6, 7, 8, *and* 9 *are made.*

II. PROGRESSIVE DRILLS.

Take three combinations as suggested and drill upon them until the pupils know them thoroughly. Then take three more facts and drill upon them in the same manner, keeping up a continual review of the facts already learned. Since adding numbers is thinking them together, the pupil must be led to realize each combination so as to make it entirely his own.

ILLUSTRATION:

|| || 2 + 2
||||| || 5 + 2
|||||||| || 8 + 2

III. INDUCTIVE EXERCISES.

Facility in naming results of numbers applied to objects, is of prime importance. Such exercises conduce to correct and rapid thinking in arithmetic.

IV. ORAL EXERCISES.

Too great weight cannot be attached to the oral exercises. In every instance the problem should be stated clearly and but once. Then designate the boy or girl who is to solve it. The pupil should immediately repeat the example, and reason it out to a correct result. If he is unable to state it correctly, he should be charged with a failure. Attention will thus be fostered, memory strengthened, a rigidly logical method of solution acquired, and pupils will soon do creditable and even brilliant work.

V. WRITTEN EXERCISES.

In solving problems on the blackboard, the oral method should be insisted upon, until the analysis is thoroughly understood. He who formulated the first rule for the addition of numbers, must have done so from investigation, and such should be the course of every student of arithmetic.

ELEMENTARY ARITHMETIC.

PART I.

First Lessons in Numbers.

SUGGESTIONS.—First Lessons in numbers involve (1) counting, (2) making numbers, and (3) expressing numbers and indicating operations. A numeral frame and collections of objects such as pencils, marbles, toothpicks, etc., are needed for illustration. Counting is telling objects one by one in order to find out how many there are. Let children be employed in counting objects. Adding is joining things or numbers together so as to form a whole. Making numbers is putting smaller numbers together to form larger ones. Expressing numbers is using figures to represent the numbers and signs to show what is to be done with them.

I. Counting.

Give each child in the class a number of objects and ask him to hold up one of them. How many have you? *One.* Hold up another. Put them together. How many have you now? *Two.* Hold up two other objects. Hold them in your left hand and pick up another. How many have you in your left hand? *Two.* How many in your right hand? *One.* Put them together. How many have you now? *Three.* Pick up three other objects. Hold them in your left hand and pick up another. How many have you in your left hand? *Three.* How many in your right hand? *One.* Put them together. How many have you now? *Four.* What number is one less than three?—one more than three? Hold four objects in your left hand and pick up another. How many have you in your left hand? *Four.* How many in your

right hand? *One.* Put them together and tell how many you have. *Five.* What number is one less than four? What number is one more than four?

☞ Illustrate in every possible way, and continue these exercises until *six, seven, eight, nine* and *ten* are learned.

II. Making numbers and expressing them.

The children prepare their slates, and the teacher takes the numeral frame. Hold up two fingers. Show me two things. Make two marks on your slates. I will make them on the blackboard. Thus: | |. I will now show you another way to write two. Thus: 1 + 1 = 2, which is read ONE *and* ONE *is* TWO. This 1 is called one; this 2 is called two. See who can make them best on his slate. Let the pupils who make the best figures on their slates make them on the blackboard. Criticise and instruct children to make the figures plainly and to strive to get correct forms.

Hold up three objects. Make three marks on your slates. Thus: | | |. How many ones in three? I will now show you another way to make three. 1 + 1 + 1 = 3, which is read, ONE *and* ONE *and* ONE *is* THREE. Can you make three in some other way? || |. I will write it; 2 + 1 = 3. Please read what I have written. Two *and* ONE *is* THREE. Illustrate with the numeral frame, and show that by putting 1 + 1 + 1, or 2 + 1 together, three arises.

Hold up four objects. Make four marks. Thus: | | | |. I will show you another way to make four. 1 + 1 + 1 + 1 = 4. Please read what I have written. Can you make 4 in any other ways? Yes. || ||; ||| |. I will write this in figures. Thus: 2 + 2 = 4; 3 + 1 = 4. Please read these expressions.

In the same manner children should be taught to make the numbers 5, 6, 7, 8, 9 and 10. Thus:

| | | | | = 1 + 1 + 1 + 1 + 1 = 5.
|| || | = 2 + 2 + 1 = 5.
||| || = 3 + 2 = 5.
|||| | = 4 + 1 = 5.

REMARK.—The child, by easy steps, is led to make and combine small numbers, and learns to express them in arithmetical language. We thus go from objects to numbers, and by putting smaller numbers together to make larger ones, the child is employed in doing, and is developed by telling how to make the numbers up to 10.

To teach *naught*, make ten lines on the blackboard, rub them out and then ask, How many lines are left? None, 0. Make 0 on your slates.

You may now write quickly and neatly the figures as I name them: 1, 5, 3, 2, 4, 7, 6, 9, 8, 7, 4, 3, 1, 5, 8, 4, 6, 3, 2, 5, 4, 7, 8, 6, 8, 7, 4, 8, 1, 3, 0, 6, 3, 5, 4, 7, 4, 3, 2, 0, 8, 7, 9, 5, 4, 4, 7, 0, 9, 7, 6, 3, 8, 6, 4, 0, 8, 3, 7, 3, 8, 2, 8, 7, 0, 9, 6, 7, 8, 9, 5, 3, 2, 1, 4, 0, 6, 4, 5, 8.

III.—Using Signs and Writing Numbers greater than 9.

The child should now be taught how to use + and =, and how to write numbers by combining figures.

Place the proper figure on the right of the sign of equality in each of the following:

0 + 3 =	4 + 5 =	1 + 2 + 6 =	2 + 3 + 1 + 2 =
3 + 3 =	5 + 3 =	3 + 3 + 2 =	0 + 4 + 2 + 3 =
3 + 4 =	2 + 7 =	1 + 2 + 4 =	1 + 4 + 3 + 0 =
3 + 5 =	2 + 6 =	3 + 1 + 5 =	4 + 3 + 2 + 0 =
5 + 4 =	3 + 3 =	3 + 2 + 3 =	2 + 0 + 4 + 3 =
1 + 5 =	4 + 4 =	5 + 0 + 2 =	2 + 2 + 3 + 1 =
2 + 3 =	3 + 4 =	4 + 3 + 2 =	1 + 3 + 2 + 0 =
3 + 3 =	5 + 1 =	3 + 1 + 4 =	3 + 2 + 1 + 1 =
2 + 4 =	4 + 2 =	2 + 3 + 2 =	5 + 1 + 2 + 0 =
0 + 5 =	4 + 0 =	2 + 0 + 5 =	1 + 1 + 0 + 6 =

Count twenty. Now count backward from twenty to one. Beginning with 10, write all the numbers up to 20.

Write these numbers:

One, three, five; one, four, two; one, three, one; four, five, two; three, five, three; five, three, one.

Two, six, three; four, five, eight; six, four, two; five, four, three; one, six, five; four, five, six.

Three, five, seven; six, four, five; three, naught, five, two, seven, four; one, six, three; seven, six, two.

Four, eight, six; seven, three, naught; eight, five, four; seven, three, two; six, two, one; eight, naught, seven.

Five, nine, eight; six, seven, nine; seven, eight, three; six, five, naught; five, four, two; three, nine, four.

Six, ten, nine; eight, five, ten; three, naught, seven; five, six, four; three, nine, ten; eight, naught, five.

Seven, eleven, ten; seven, nine, eight; six, five, three; four, two, naught; five, ten, eleven; seven, naught, four.

Eight, six, seven; six, twelve, seven; four, eight, ten; eight, six, ten; four, three, eleven; nine, twelve, three.

Nine, eight, seven; seven, nine, ten; five, seven, eleven; seven, eleven, fourteen; eight, nine, two.

Ten, eight, nine; eleven, thirteen, eight; eight, six, twelve; nine, three, naught; six, eleven, ten.

Eleven, ten, six; seven, eight, four; thirteen, fourteen, ten; six, seven, nine; twelve, five, six; eight, six, twelve.

Twelve, eleven, eight; ten, six, seven; six, four, eight; eight, two, six; seven, four, nine; six, ten, thirteen.

Thirteen, seventeen, seven; eleven, nine, six; seven, ten, eleven; seven, three, twelve; eight, thirteen, eleven.

Fourteen, thirteen, nine; twelve, one, ten; nine, eight, fourteen; eleven, thirteen, ten; seven, thirteen, fifteen.

Fifteen, fourteen, twelve, naught; eleven, thirteen, seven, six; thirteen, ten, twelve, eight.

Sixteen, fifteen, eleven, fourteen; eight, five, thirteen, six; naught, nine, six, three; five, eight, ten, eleven.

Seventeen, sixteen, thirteen, fifteen; nine, four, fifteen, three; two, seven, nine, two.

Eighteen, fifteen, fourteen, twelve; ten, eleven, seventeen, eight; eighteen, naught, seventeen, sixteen.

Write the following with their answers:

$1 + 3 + 2 =$	$3 + 4 + 3 + 2 =$	$1 + 6 + 2 + 3 + 2 =$
$2 + 3 + 2 =$	$4 + 2 + 3 + 1 =$	$2 + 8 + 1 + 1 + 1 =$
$3 + 2 + 3 =$	$5 + 2 + 1 + 3 =$	$2 + 4 + 0 + 8 + 0 =$
$4 + 3 + 4 =$	$1 + 0 + 6 + 2 =$	$2 + 3 + 4 + 3 + 1 =$
$5 + 4 + 3 =$	$6 + 1 + 3 + 2 =$	$5 + 2 + 0 + 3 + 2 =$
$6 + 3 + 2 =$	$7 + 1 + 2 + 3 =$	$6 + 2 + 3 + 2 + 1 =$
$2 + 4 + 1 =$	$8 + 2 + 3 + 2 =$	$7 + 1 + 0 + 3 + 5 =$
$3 + 3 + 2 =$	$9 + 1 + 0 + 2 =$	$8 + 2 + 0 + 3 + 6 =$

To TEACHERS.—These exercises can be made very instructive and interesting. The teacher should call off the numbers in a clear, distinct voice, never repeating a number after once pronouncing it. The pupils should work rapidly. The pupils should never be allowed to pass any exercise until thoroughly familiar with it.

The expression $2 - 1 = 1$, is read, TWO *minus* ONE is ONE. The sign ($-$), is called *minus*.

Write the following:

Four plus two; five minus four; six plus three; three plus eight; five minus naught; four plus eighteen.

Six minus naught; four plus two; twelve minus ten; nine minus four; eleven plus seven; six minus two.

Five minus three; eight plus five; fifteen minus eleven; thirteen minus six; fifteen plus five; sixteen minus eight.

Three plus twelve; six minus one; eleven plus seven, nineteen minus seven; twelve plus four.

Eight plus ten; seven minus six; three plus eight; fourteen plus eight; twenty minus ten; nineteen plus one.

Ten minus nine; eleven plus three; sixteen minus fifteen; seventeen minus four; fifteen plus eighteen.

Nine plus seven; naught minus naught; seven plus six; thirteen minus six; eighteen plus two.

Seven minus five; twelve plus one; eleven minus three; eighteen plus eight; three plus two.

Six minus three; ten plus three; seven minus six; seventeen plus five; seven minus one.

Eight plus five; seven minus four; thirteen plus fifteen; three minus three.

Five plus thirteen; sixteen minus fourteen; twelve plus one; eleven minus seven.

Twenty minus eleven; eighteen plus three; eleven minus four; twelve plus eight.

Eighteen minus six; twenty plus ten; nineteen minus five; nine plus six; eight minus two.

Nineteen plus naught; nineteen minus nine; sixteen plus thirteen; seven minus naught.

Seventeen minus four; thirteen plus eight; fifteen plus one; eight minus six.

Write the following with their answers:

$2-1=$	$9-1=$	$3+2-1=$	$2+1+3-1=$
$3-1=$	$9-3=$	$4+1-2=$	$3+4+1-2=$
$4-1=$	$9-2=$	$5+2-4=$	$4+1+1-4=$
$5-2=$	$7-4=$	$6+1-3=$	$5+2+1-3=$
$6-3=$	$6-2=$	$7+2-2=$	$4+0+0-2=$
$8-3=$	$5-5=$	$8+1-3=$	$5+1+3-6=$
$8-4=$	$3-0=$	$3+5-4=$	$6+0+3-4=$
$6-4=$	$9-4=$	$3+3-0=$	$3+0+3-2=$
$7-0=$	$9-5=$	$4+2-2=$	$2+2+4-0=$
$8-6=$	$8-6=$	$5+2-4=$	$4+1+5-6=$

NOTE.—The design of the preceding lessons is to familiarize the pupil with the manner of making figures, and to acquaint him with the signs and the simplest operations in addition and subtraction. Having had a glimpse of the science of numbers, the study should now be taken up in a systematic manner.

Notation and Numeration.

DEFINITIONS.

ARTICLE 1.—A Unit is one, or a single thing; as one apple, one peach, one book, one.

ART. 2.—A Number is a unit, or a collection of units; as five apples, four peaches, nine books, five, four, nine.

Numbers are generally expressed by characters called *Figures.*

There are ten Figures. They are—

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Naught, one, two, three, four, five, six, seven, eight, nine.

The **Value of a Figure** is the number which it represents.

The figure naught, called also cipher or zero, expresses no value when used alone. The other figures are *Digits.*

Numbers are of two kinds, *Abstract* and *Concrete.*

An **Abstract Number** is a number not applied to any object; as 5, 4, 7.

A **Concrete Number** is a number applied to one or more objects; as 5 boys, 4 cows, 7 cherries, 1 horse.

ART. 3.—Notation is the art of writing numbers. When we do so by means of the figures given above, we use the *Arabic Notation.*

ART. 4.—Numeration is the art of reading numbers.

When a number is expressed by a single figure or digit, it means so many units; thus, 5, 8, 6, 7, mean 5 units, 8 units, 6 units, 7 units.

The right hand figure of any whole number always expresses units.

ART. 5.—Arithmetic treats of numbers and their uses.

To TEACHERS.—Definitions are means of explaining facts and terms. The teacher should analyze and illustrate each definition before the pupil memorizes it.

UNITS AND TENS.

ART. 6.—When a number is expressed by two figures, the second figure from the right expresses tens. Thus, 25 means 2 tens and 5 units; 63 means 6 tens and 3 units; 47 means 4 tens and 7 units.

Read 37, 84, 69, 94, 38, 49, 56, 27, 88, 11, 44, 79, 75, 66, 32, 99.

Ten is 10 times greater than 1; therefore, a figure in the tens' place is ten times greater than the same figure in the units' place.

1 ten, called ten = 10 units.
2 tens, called twenty = 20 units.
3 tens, called thirty = 30 units.
4 tens, called forty = 40 units.
5 tens, called fifty = 50 units.
6 tens, called sixty = 60 units.
7 tens, called seventy = 70 units.
8 tens, called eighty = 80 units.
9 tens, called ninety = 90 units.

ART. 7.—When we read units and tens, we do not always call the name *units* or *tens*. Thus, when I write 5 units, I say five; and 8 units would be eight. When I write 36, or 3 tens and 6 units, I say thirty-six; and 4 tens and 7 units are forty-seven.

Read the numbers I write on the board:

29, 56, 58, 67, 69, 73, 77, 84, 80, 90, 93, 96, 60, 48, 30, 97, 99.

Count from 1 to 15; 15 to 30; 30 to 45; 45 to 65; 65 to 85; 85 to 99.

Count backward from 99 to 85; 85 to 60; 60 to 45; 45 to 15; 15 to 1.

Write the following numbers:

13, 17, 19, 23, 38, 65, 73, 96, 98, 38, 83, 71, 65, 49, 55, 40, 89, 81, 11, 20, 55, 80, 97, 66, 40, 43, 71, 93.

UNITS, TENS, AND HUNDREDS.

ART. 8.—The first figure on the right expresses units, the second tens, and the third hundreds.

ART. 9.—A figure increases tenfold in value for every place it is moved to the left. Thus, when it is placed in the hundreds' place, its value is 10 times as great as when in the tens' place, and 100 times as great as when in the units' place.

One hundred, or ten tens is written	100.
Two hundred, or twenty tens is written	200.
Three hundred, or thirty tens is written	300.
Four hundred, or forty tens is written	400.
Five hundred, or fifty tens is written	500.
Six hundred, or sixty tens is written	600.
Seven hundred, or seventy tens is written	700.
Eight hundred, or eighty tens is written	800.
Nine hundred, or ninety tens is written	900.

Read the following:

126, 208, 345, 423, 545, 635, 723, 607, 831, 936, 646, 732, 803, 694, 930.

Write two hundred thirty-eight; five hundred five; six hundred twenty-three; seven hundred thirty-one; two hundred five; eight hundred; five hundred thirty-nine; nine hundred thirty-three; three hundred three; nine hundred ninety-nine; four hundred four; six hundred sixty-six; one hundred one; two hundred ten; five hundred fifty.

Write three hundred thirty-three; one hundred fifty;

eight hundred seventy-two; six hundred six; nine hundred forty-five; six hundred twenty-two; four hundred forty; six hundred one; three hundred; one hundred one; seven hundred nine; four hundred forty; two hundred twenty-five; six hundred two.

UNITS, TENS, HUNDREDS, AND THOUSANDS.

ART. 10.—The first figure on the right expresses units, the second tens, the third hundreds, the fourth thousands.

ART. 11.—Since a figure increases tenfold in value for every place it is moved to the left, when it stands in the thousands' place its value is 10 times as great as when in the hundreds' place, 100 times as great as when in the tens' place, and 1000 times as great as when in the units' place.

One thousand, or ten hundreds is written	1000.
Two thousand, or twenty hundreds is written	2000.
Three thousand, or thirty hundreds is written	3000.
Four thousand, or forty hundreds is written	4000.
Five thousand, or fifty hundreds is written	5000.
Six thousand, or sixty hundreds is written	6000.
Seven thousand, or seventy hundreds is written	7000.
Eight thousand, or eighty hundreds is written	8000.
Nine thousand, or ninety hundreds is written	9000.

Read the following:

2321, 2684, 3248, 5132, 4026, 3984, 4605, 7032, 8104, 7603, 7058, 4009, 7013, 9305, 9187, 6781, 4004, 3576, 7032, 9009.

Write the following numbers:

Three thousand six hundred twenty-four; four thousand three hundred twenty-one; five thousand four hun-

dred thirty-one; six thousand three hundred eighteen; two thousand seven hundred fifty-nine; seven thousand three hundred twenty-one; six thousand forty-eight; five thousand thirty-two; seven thousand one hundred one; eight thousand one hundred eight; seven thousand seven; nine thousand one.

HIGHER PERIODS THAN THOUSANDS.

ART. 12.—When we wish to read large numbers, we place a comma between the third and fourth figures from the right; another between the sixth and seventh; another between the ninth and tenth, and so on.

ART. 13.—Each group of three figures, thus pointed off, makes a *period*. The first period is the units' period; the second the thousands' period; the third the millions' period; the fourth the billions' period.

Billions' Period.		Millions' Period.		Thousands' Period.		Units' Period.	
Hundreds of Billions.		Hundreds of Millions.		Hundreds of Thousands.		Hundreds.	
Tens of Billions.		Tens of Millions.		Tens of Thousands.		Tens.	
Billions.		Millions.		Thousands.		Units.	
			2				5
						38	24
						452	382
				3,	618		731
				54,	539		694
				326,	586		743
			9,	915,	452		403
			43,	216,	538		543
		396,		405,	321		310
483,		275,		631,	802		241

Billions' Period.		Millions' Period.		Thousands' Period.		Units' Period.	
Hundreds of Billions.		Hundreds of Millions.		Hundreds of Thousands.		Hundreds.	
Tens of Billions.		Tens of Millions.		Tens of Thousands.		Tens.	
Billions.		Millions.		Thousands.		Units.	
							5
						38	24
						452	382
				3,	618		731
				54,	539		694
				326,	586		743
			9,	915,	452		403
			43,	216,	538		543
		396,		405,	321		310
483,		275,		631,	802		241

2

Write the following numbers and read them:

One hundred; one hundred five; one hundred eight; one hundred ten; one hundred twenty; one hundred twenty-eight; two hundred; two hundred five; two hundred seven; two hundred ten; two hundred fifty; three hundred; three hundred two; three hundred five; three hundred thirty; three hundred ninety-nine; four hundred; four hundred eight; four hundred eleven; four hundred seventy-four.

Five hundred; five hundred ten; five hundred eighteen; five hundred sixty-eight; six hundred; six hundred eighty; six hundred ninety; seven hundred; seven hundred two; seven hundred three; seven hundred seven; eight hundred; eight hundred four; eight hundred six; eight hundred eight; eight hundred twenty-three; nine hundred; nine hundred seven.

Write the following numbers, point them off into periods, and read them:

One thousand; one thousand twenty; one thousand fifty-five; one thousand one; one thousand three; one thousand eighty; one thousand eight; one thousand ninety; one thousand nine; one thousand nineteen; one thousand one hundred; one thousand one hundred ten; one thousand one hundred eleven; one thousand one hundred twenty-one; one thousand two hundred two.

One thousand seven hundred; one thousand seven hundred one; one thousand eight hundred one; one thousand eight hundred ten; one thousand nine hundred nine; two thousand eight; two thousand eighty; two thousand eight hundred; three thousand; three thousand three; three thousand thirty; five thousand.

Ten thousand ten; twenty thousand; two thousand five; twenty-five thousand fifty; twenty thousand ten; twenty thousand two hundred; twenty-four thousand twenty; twenty-three thousand three; twenty-four thousand nine; twenty-four thousand ninety.

Six thousand four; six thousand forty-four; six thousand eight hundred eight; seven thousand two hundred five; six thousand three hundred twenty-one; five thousand forty-one; six thousand one hundred three; seven thousand six; twenty-three thousand four hundred ninety; thirty-six thousand four hundred ninety-one; fifty-eight thousand three hundred eighty-five.

Forty-seven thousand three hundred five; forty-nine thousand six hundred six; fifty-six thousand three hundred; eighty-three thousand one; fifty-nine thousand nine; one hundred twenty-one thousand three hundred twenty-one; two hundred thirty-five thousand four hundred thirty-three; four hundred fifty-seven thousand three hundred twenty-eight; eight hundred twenty thousand ten; nine hundred thousand nine; one billion.

Five million sixteen thousand four hundred ninety-seven; sixteen million two thousand nine; ninety-seven million eight thousand forty; six hundred thousand fourteen; seven hundred million fifty thousand one hundred eight; five hundred million sixty thousand six.

REMARK.—When an order or period is missing, fill the vacant places with ciphers.

ROMAN NOTATION.

NOTE.—Omit until review.

ART. 14.—The **Roman Method of Notation** employs seven capital letters. They are: I, 1; V, 5; X, 10; L, 50; C, 100; D, 500; M, 1,000.

(1.) When a letter is repeated, its value is repeated the same number of times. Thus, II = 2; III = 3; XX = 20; XXX = 30.

(2.) When a letter is placed before one or more of less value, their values are added. Thus, XV = 15; LV = 55; CXX = 120; DCCC = 800.

(3.) When a letter is placed before one of greater value, the value of the less is taken from that of the greater. Thus, IV = 4; IX = 9; XC = 90.

(4.) A bar placed over a letter increases its value a thousand times. Thus, $\overline{X} = 10{,}000$; $\overline{L} = 50{,}000$; $\overline{C} = 100{,}000$.

Write the following numbers by the Roman notation: 5, 8, 6, 4, 10, 15, 18, 20, 35, 88, 121, 348, 610, 450, 832, 1,321, 2,453, 6,145, 3,853, 9,009.

Change the following numbers to the Arabic notation: III, VI, VIII, IX, XI, LV, LXII, LXIX, XC, CX, CXX, CCXX, CCXXV, CXXXI, CCCXXXVIII, DCCX, DCCXIX, DCCCXLIII, MDCXXXVII, $\overline{V}$, $\overline{M}$, MDCCCLXXXVI.

QUESTIONS FOR REVIEW.

What is a unit? What is a number? How are numbers generally expressed?

How many figures are there? Name them. What is the value of a figure? What is said of the figure naught? What are the other figures called?

What are the two kinds of numbers? What is an abstract number? Name three abstract numbers.

What is a concrete number? Name three concrete numbers.

What is notation? When do we use the Arabic notation?

What is numeration? When a number is expressed by a single figure, what does it mean? What does the right hand figure of any whole number always express? What is arithmetic?

Where is the tens' place? How is the value of a figure affected as it is moved to the left? How does the value of a figure in the thousands' place compare with the value of the same figure in the hundreds' place? In the tens' place? In the units' place?

How does the Roman method of notation differ from the Arabic? Name the letters used.

When a letter is repeated, how is its value affected? When a letter is placed before one of less value, what results? When it is placed before one of greater value, what results? How is the value of a letter affected by placing a bar over it?

Addition.

Progressive Oral and Written Drills.

SUGGESTION.—The pupil should be taught to count with and without objects. After counting, the first process in arithmetic is to add the digits to a given number. Let the given number be **2** and add to it **2, 5, 8; 1, 4, 7; 3, 6, 9** respectively.

SLATE AND BLACKBOARD EXERCISES.

2	2	2	2	2	2	2	2	2	2
2	5	8	1	4	7	3	6	9	0
—	—	—	—	—	—	—	—	—	—
2	2	2	2	2	2	2	2	2	2
12	15	18	11	14	17	13	16	19	10
—	—	—	—	—	—	—	—	—	—

☞ Count by 2's to 20. Begin with 1 and count by 2's to 20.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2	1	2	1	2	2	1	2	1	1
1	2	1	1	2	0	2	0	2	2
0	2	1	2	1	1	1	2	2	1
2	1	2	2	2	2	2	1	2	0
1	2	2	1	2	1	2	2	1	2
—	—	—	—	—	—	—	—	—	—

☞ Begin with 0, 7, 4, 11, 8, 12, 9, 6, or 13, and add those columns up and down.

SUGGESTION.—Take **3** and add to it 1, 4, 7; 3, 6, 9; 2, 5, 8 respectively. Three combinations are enough for a lesson. Illustrate each fact, so that the child may think the result clearly.

SLATE AND BLACKBOARD EXERCISES.

3	3	3	3	3	3	3	3	3	3
2	5	8	1	4	7	9	6	3	0
—	—	—	—	—	—	—	—	—	—
3	3	3	3	3	3	3	3	3	3
12	15	18	11	14	17	19	16	13	10
—	—	—	—	—	—	—	—	—	—

☞ Count by 3's to 30. Begin with 1 and count by 3's to 30. Begin with 2 and count by 3's to 30.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3	2	2	2	3	2	3	3	2	3
2	3	1	3	1	3	1	1	3	3
1	2	3	2	2	1	3	2	1	2
3	1	2	2	2	3	3	0	2	3
2	3	3	3	3	3	2	2	1	1
–	–	–	–	–	–	–	–	–	–

☞ Begin with 0, 8, 4, 11, 5, 13, 16, 9, 12, or 21, and add these columns up and down.

SUGGESTION.—After adding the digits to 4, add 12 and 4, 22 and 4, 32 and 4; 15 and 4, 25 and 4, 35 and 4; and so on.

SLATE AND BLACKBOARD EXERCISES.

4	4	4	4	4	4	4	4	4	4
2	5	8	1	4	7	3	6	9	0
–	–	–	–	–	–	–	–	–	–

4	4	4	4	4	4	4	4	4	4
12	15	18	11	14	17	13	16	19	10
—	—	—	—	—	—	—	—	—	—

☞ Count by 4's to 40. Begin with 1, 2, or 3, and count by 4's to 40.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3	4	4	1	3	4	4	4	4	2
4	1	3	4	2	3	2	3	3	4
2	4	4	2	4	2	3	2	4	3
3	3	3	4	1	4	4	4	2	4
4	4	2	3	4	3	3	2	1	2
–	–	–	–	–	–	–	–	–	–

☞ Add each of the above columns up and down, beginning with 0, 2, 13, 8, 17, 4, 16, 9, 15, 7, or 19.

SUGGESTION.—Abstract examples like the following, containing no figure greater than **4**, should be dictated by the teacher:

(1)	(2)	(3)	(4)
423244	342434	243143	344243
344432	434241	434432	411402
412343	340413	142244	324143
344234	414043	434422	443404
421443	342414	342244	342244
344324	424123	414422	224434
412432	341234	341144	443043
344343	412432	424422	344234

☞ For explanation, see **Article 22.**

SUGGESTION.—The process with the **5's** is the same as with **2, 3** and **4.** After adding the digits to 5, add 16 and 5, 36 and 5, 26 and 5, 46 and 5, and so on, until all combinations up to 49 and 5 are mastered.

BLACKBOARD DRILLS.

(1, 2, 3, 4) 5		(2, 3, 4) 5		(3, 4) 5		(4) 5		5	
3	·	13	·	23	·	33	·	43	·
5	·	15	·	25	·	35	·	45	·
9	·	19	·	29	·	39	·	49	·
2	·	12	·	22	·	32	·	42	·
7	·	17	·	27	·	37	·	47	·
1	·	11	·	21	·	31	·	41	·
6	·	16	·	26	·	36	·	46	·
4	·	14	·	24	·	34	·	44	·
8	·	18	·	28	·	38	·	48	·
0	·	10	·	20	·	30	·	40	·

Explanation.—The teacher or a pupil points to the dots on the right of the vertical line, while the pupils in turn name the results obtained by adding the number on the left of this line to the given number at the top. This exercise fixes the attention of each pupil, and may be introduced with the **2's, 3's,** and **4's** and also with the **6's, 7's, 8's** and **9's.** ☞ The numbers in parentheses should be used as review drills.

☞ Count by 5's to 50. Begin with 1, 2, 3, or 4 and count by 5's to 50.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
5	4	5	4	5	4	3	5	2	4
4	5	3	5	4	5	5	4	5	5
1	5	4	5	3	5	1	5	5	2
5	4	3	4	5	3	5	3	2	5
3	5	5	5	2	4	3	5	4	5
–	–	–	–	–	–	–	–	–	–

☞ Add each of the above columns up and down, beginning with 0, 7, 9, 18, 4, 22, 6, 17, 29, 8, or 14.

DICTATION.

Let the teacher dictate examples containing no figure greater than **5**. There should not be more than twelve numbers, nor should any number be greater than hundreds of thousands.

SUGGESTION.—The treatment of addition of **6's** is similar to the treatment of the other numbers, and involves three distinct drills. 1st. The addition of the digits to 6. 2d. The addition of all numbers up to 59 to 6. SEE BLACKBOARD DRILLS for the 5's. 3d. Counting by 6's, beginning with 0, 1, 2, 3, 4, or 5.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
6	6	5	4	6	6	6	6	6	4
3	5	4	5	6	1	4	6	6	4
6	4	6	6	4	6	6	4	6	5
2	6	3	3	6	6	6	6	1	6
5	1	6	4	5	2	3	3	6	6
–	–	–	–	–	–	–	–	–	–

☞ Add each column up and down, beginning with 0, 5, 19, 4, 27, 6, 15, 23, 8, 22, or 11.

DICTATION.

Let the teacher dictate examples containing no figure greater than **6.** There should not be more than twelve numbers, nor should any number be greater than hundreds of thousands. The numbers should be irregular, that is, not all contain the same number of figures.

SUGGESTION.—Treat the **7's** in the same way as the **5's** and **6's,** observing the three distinct kinds of drill up to 69. Another good written drill is this: Let the teacher arrange numbers and require pupils to add some given number to each of them. For instance, the teacher says: Add 7 to 8, 14, 6, 9, 24, 28, 16, 29, 44, 53. The pupils write 15, 21, 13, 16, 31, 35, 23, 36, 51, 60, if they add readily and accurately.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
7	6	7	6	7	5	7	6	7	7
6	7	6	7	5	6	7	7	4	1
7	7	7	5	4	4	4	7	6	5
5	2	4	7	7	3	5	5	7	7
2	4	1	2	1	2	1	7	2	3
–	–	–	–	–	–	–	–	–	–

☞ Add each of the above columns, beginning with 0, 5, 19, 4, 27, 6, 15, 23, 8, 22, or 11.

REMARK.—The units' figure of the sum of any number and a given number is always the same, whether the number is a digit, or simply fills the units' place. See DRILL on **5's.**

BLACKBOARD DRILLS.

1		**4**		**6**		**4**		**6**		**9**
	5				**5**				**5**	
3		**2**		**3**		**7**		**8**		**7**

REMARK.—The above arrangement of numbers in circles gives endless columns, and is an excellent device for utilizing the spare minutes in concert drills. The teacher points to the numbers and the pupils add silently until the teacher calls for results. Go slowly at first, but gradually increase in rapidity as pupils acquire facility in adding.

DICTATION.

Let the teacher dictate examples containing no figure greater than **7**. No number should exceed millions.

SUGGESTION.—The 8's and 9's are treated in the same manner as the other numbers.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	7	6	8	7	6	7	6	7	8
7	6	8	6	8	8	8	8	8	7
8	5	8	8	8	7	8	7	8	8
5	8	5	2	5	8	7	8	8	6
4	6	3	8	8	5	8	6	7	8
–	–	–	–	–	–	–	–	–	–

☞ Add each of the above columns up and down, beginning with 0, 17, 8, 35, 7, 29, 54, 39, 6, 15, or 26.

DICTATION.

Let the teacher dictate numbers containing no figure greater than **8**. The numbers may be as great as hundreds of millions.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	9	7	4	8	9	8	9	7	8
9	8	8	9	9	8	7	6	9	9
6	6	6	5	9	9	9	9	6	7
7	9	5	9	6	6	8	7	9	8
9	8	9	2	9	9	6	9	5	9
–	–	–	–	–	–	–	–	–	–

☞ Add each of the above columns up and down, beginning with 0, 11, 17, 13, 18, 14, 12, 19, 10, 16, or 21.

DICTATION.

Let the teacher dictate examples of numbers containing any figure and not more than four periods. Let the numbers be irregular and contain ciphers.

☞ These *inductive exercises* are reviews of the facts of addition in the concrete. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 15.—1. How many books are 2 books and 3 books?

2. Thomas spent 3 cents for a pen and 5 cents for a lead pencil: how much did he spend for both?

3. There are 4 birds on one limb and 4 on another: how many birds are there on both limbs?

4. William picked 4 apples from one tree and 5 from another: how many apples did he pick?

5. If you read 3 pages on Monday and 6 on Tuesday, how many pages will you read during the two days?

6. Mary paid 5 cents for some pears and 5 cents for some apples: how much did she pay for both?

7. A house has 6 windows on one side and 5 on another side: how many windows are there on the two sides?

8. James has two Bantam hens; one laid 7 eggs and the other 4: how many eggs were laid by both hens?

9. Susan has 5 books and Jennie 6: how many books have both?

10. In the yard of my house are 8 maple trees, and 5 elm trees: how many trees are there in my yard?

11. John found 7 chestnuts under one tree and 7 under another tree: how many chestnuts did he find?

12. Annie spelled 6 words and Helen spelled 5: how many words were spelled by both?

13. A man paid 7 dollars for a coat and 6 dollars for a hat: how much did he pay for both?

14. Richard had 5 marbles and bought 5 more: how many marbles had he then?

15. A boy has 6 cents in one pocket, and 8 in another: how many cents has he in both pockets?

16. Conrad found 4 plums in his pudding and his sister found 8 in her pudding: how many plums did both find?

17. Mr. Jones owns 10 horses and Mr. Smith owns 6: how many horses do both own?

18. A cow gave 6 quarts of milk one day and 7 quarts another day: how many quarts did she give in the two days?

19. Harry rode 6 miles and walked 9: how many miles did he go?

20. A farmer has 8 pigs in one pen and 5 in another: how many pigs are there in both pens?

21. There are 7 pupils in a geography class and 6 in an arithmetic class: how many pupils are in both classes?

22. Henry picked 9 quarts of cherries and John picked 7 quarts: how many quarts did both pick?

23. Miriam is 6 years old and Jennie is 8 years older: how old is Jennie?

24. Lewis received 7 demerit marks and Isaac 8: how many demerit marks did both receive?

25. Newton paid 8 dimes for a knife and 9 dimes for a bat: how many dimes did he pay for both?

26. Alexander shot 7 quails and Peter shot 16: how many quails did they shoot?

27. Caroline has 8 books and Edith has 13: how many books have they?

28. Orlando bought a kite for 10 cents and a string for 14 cents: how much did he pay for both?

29. Newton's father gave him 11 dollars on his birthday and his mother gave him 10 dollars: how many dollars did Newton receive?

30. Carrie gathered 14 roses from one bush and 13 from another: how many roses did she gather?

DEFINITIONS.

ART. 16.—Addition is the process of uniting two or more numbers into one.

ART. 17.—The **Sum** or **Amount** is the number obtained by addition. Thus, when 3 and 5 are added, the *sum* or *amount* is 8; when 4 and 5 are added, the sum or amount is 9.

ART. 18.—The **Sign of Addition** (+) is called *plus*, and means *more*. It is read *plus*. When placed between two numbers, it shows that they are to be added. Thus, 2 + 6 is read 2 *plus* 6, and means that 2 and 6 are to be added; 4 + 3 is read 4 *plus* 3, and means that 4 and 3 are to be added.

ART. 19.—The **Sign of Equality** is =. It is read *equals*. It shows that the expression on the left of the sign is equal to the expression on the right of that sign. Thus, 5 + 6 = 11, is read 5 plus 6 equals 11, and means that when 5 and 6 are added, the sum is 11; 7 + 9 = 16, is read 7 plus 9 equals 16, and means that when 7 and 9 are added, the sum is 16.

The character $ means *dollars* and is placed before the number to which it belongs. Thus, for convenience, 25 dollars is written $25; 50 dollars is written $50, and 175 dollars is written $175.

ART. 20.—Principle 1. *Only like numbers can be added.*

Thus: 4 sheep, 6 lambs, 7, 8 horses, and 4 boys cannot be added, because they do not have like units.

2. *Only units of the same order can be added.*

Thus: 6 units, 7 tens, 4 thousands, 8 hundreds, and 4 ten thousands cannot be added, unless they are reduced to units, which is done when we say 6 + 70 + 4,000 + 800 + 40,000 = 44,876.

EXERCISES IN ADDING THE DIGITS.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	
11.	2 +	1 +	3 +	3 +	4 +	3 +	5 +	4 +	6 +	5 =	?
12.	1 +	2 +	2 +	4 +	3 +	4 +	3 +	5 +	3 +	6 =	?
13.	0 +	2 +	3 +	2 +	4 +	2 +	5 +	3 +	6 +	3 =	?
14.	2 +	1 +	1 +	4 +	2 +	4 +	2 +	5 +	4 +	6 =	?
15.	1 +	2 +	2 +	3 +	4 +	3 +	5 +	2 +	6 +	5 =	?
16.	2 +	1 +	3 +	4 +	4 +	4 +	1 +	5 +	2 +	6 =	?
17.	2 +	2 +	2 +	4 +	3 +	4 +	5 +	2 +	6 +	4 =	?
18.	1 +	2 +	3 +	2 +	4 +	2 +	4 +	5 +	1 +	6 =	?
19.	2 +	1 +	2 +	3 +	4 +	3 +	5 +	3 +	6 +	5 =	?
20.	1 +	2 +	3 +	4 +	3 +	4 +	3 +	5 +	4 +	6 =	?

REMARK.—Adding in line across the page shows the use of the signs + and =.

	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	
31.	6 +	5 +	6 +	4 +	6 +	7 +	6 +	7 +	6 +	7 =	?
32.	3 +	6 +	5 +	6 +	4 +	5 +	7 +	5 +	7 +	6 =	?
33.	6 +	4 +	6 +	3 +	6 +	7 +	5 +	6 +	7 +	7 =	?
34.	5 +	5 +	5 +	6 +	2 +	6 +	7 +	4 +	5 +	7 =	?
35.	6 +	6 +	4 +	6 +	6 +	5 +	7 +	7 +	6 +	5 =	?
36.	1 +	3 +	6 +	5 +	5 +	7 +	6 +	7 +	7 +	6 =	?
37.	6 +	4 +	5 +	6 +	6 +	6 +	7 +	6 +	7 +	7 =	?
38.	2 +	6 +	4 +	6 +	4 +	7 +	5 +	7 +	6 +	4 =	?
39.	5 +	6 +	6 +	5 +	6 +	6 +	7 +	5 +	7 +	7 =	?
40.	6 +	5 +	6 +	4 +	6 +	7 +	4 +	7 +	7 +	6 =	?

REMARK.—The teacher will find these graded exercises valuable as review drills. By changing a figure here and there, an endless variety of slate and blackboard work may be provided.

	41.	42.	43.	44.	45.	46.	47.	48.	49.	50.	
51.	8 +	7 +	8 +	5 +	8 +	6 +	9 +	8 +	9 +	8 =	?
52.	7 +	8 +	6 +	8 +	5 +	8 +	7 +	9 +	8 +	9 =	?
53.	8 +	6 +	8 +	3 +	8 +	5 +	8 +	7 +	9 +	8 =	?
54.	6 +	8 +	3 +	7 +	4 +	8 +	9 +	8 +	7 +	9 =	?
55.	8 +	5 +	8 +	6 +	8 +	9 +	8 +	9 +	9 +	8 =	?
56.	5 +	8 +	4 +	8 +	6 +	8 +	9 +	6 +	8 +	7 =	?
57.	8 +	4 +	8 +	5 +	8 +	9 +	6 +	9 +	6 +	9 =	?

ORAL EXERCISES.

ART. 21.—1. John bought 4 apples and James bought 5 apples: how many apples did both buy?

Solution.—Both bought the sum of 4 apples and 5 apples, or 9 apples.

2. A family ate 6 slices of bread for breakfast and 7 slices for dinner: how many slices did they eat?

3. Mary solved 7 problems on Monday and 7 on Tuesday: how many problems did she solve?

4. Samuel found 9 eggs in one nest and 11 in another: how many eggs did he find?

5. Richard counted 12 cows in one field and 10 in another: how many cows did he count in the two fields?

6. Thomas had 7 cakes and Ann gave him 4: how many cakes did Thomas then have?

7. Susan received 8 cents from her mother and 9 from her father: how many cents did she receive?

8. During the Revolution, there were 13 States. Since then 25 new States have been added: how many States now compose the Union?

9. Jackson paid 10 cents for some pea-nuts, 6 cents for some candy, and 5 cents for a glass of lemonade: how much money did he spend?

10. After Jackson ate the pea-nuts and candy, and drank the lemonade, his father paid $2 for a carriage to take him home, $1 for medicine, and $3 for the doctor's visits: how much did Jackson's father pay?

11. A boy counted 7 ducks on a pond, 5 in a yard, and 6 in a road: how many ducks did he count?

12. John is 5 years old, Harry is 6 years older, and Richard is 8 years older than Harry: how old is Richard?

13. On the first bench in school sat 6 boys, on the

second 5 boys, and on the third 4 boys: how many boys sat on the three benches?

14. In a certain class there were 4 boys and 5 girls, and in another class 7 boys and 6 girls: how many pupils were in the two classes?

WRITTEN EXERCISES.

ART. 22.—1. Add 324, 432, 546, 318.

Process.

$$\begin{array}{r} 324 \\ 432 \\ 546 \\ 318 \\ \hline 1620 \end{array}$$

Analysis.—We write the numbers so that units of the same order stand in the same column, draw a line underneath, and begin at the right to add.

Beginning with the order of units, we add each column separately. Thus, 8 + 6 + 2 + 4 = 20 units, which = 2 tens and 0 units. We write 0 in the units' place and add the 2 with the column of tens.

Two tens from the units' column + 1 + 4 + 3+ 2 = 12 tens, which = 1 hundred and 2 tens. We write 2 in the tens' place and add the 1 with the column of hundreds.

One hundred from the tens' column + 3 + 5 + 4 + 3 = 16 hundreds, which = 1 thousand and 6 hundreds, which we write in their proper places.

NOTE.—In adding, name results only. Thus, instead of saying 8 and 6 are 14 and 2 are 16 and 4 are 20, say 8, 14, 16, 20.

ART. 23.—Rule for Addition.—*Write the numbers so that units of the same order stand in the same column, draw a line underneath and begin at the right to add.*

If the sum of any column is less than 10, *write it under the column added; and if* 10 *or more, write the units' figure under the column added, and add the ten or tens with the next column. Write the total sum of the last column.*

Proof.—*Begin at the top and add each column downward. If the sums agree, the work is probably correct.*

Add the following, explain the process, prove and deduce the rule:

2.	3.	4.	5.	6.
38240	42326	61263	31266	91238
29032	16343	52312	82013	86327
61284	59369	83263	10000	41237
23214	47073	28431	21936	56320
52631	62168	91203	42761	79101
45232	32542	26361	83429	28013
71623	63263	87204	56127	49087
56	84	37	32	323

7.	8.	9.	10.	11.
126932	412036	583882	943263	723828
423288	366492	328472	813928	436023
721127	931245	417069	830244	521328
332493	269758	531627	243586	214348
727932	426361	580021	500000	325627
388832	666555	555777	499896	232848
921326	720007	600409	200909	408084
287784	238724	319802	321211	343255
326548	568638	535809	680680	707077
262	816	2 6	3 7 7	39 2 1

12.	13.	14.	15.
184246	3827682	5463288	3129369
491029	8321	862746	628731
603622	682407	2135040	1000
78346	5324638	812684	982
543120	279324	3258	4826
8928	832421	3186297	72318
27482	678723	4238456	708426
487438	41242	325868	7216387
216130	923280	542928	4328421
50000	61826	5668632	52630289
3		3 1	07 9

16.	17.	18.
27658427	43225894	61231025
3105235	26305326	32454234
63452314	81299876	7123156
60154	4301812	85294
7315235	821327	71231505
617389	89	1492
2105614	1026	88
731420	72438	294
57624	283263	5312105
3107102	72305280	700500
6324251	122209326	92048
43142178	432028726	24651253
9 3	7 2 5 3 3	20 88 9 4

19. Illustrate by an original example the addition of five numbers.

20. Illustrate by an original example the addition of ten numbers.

21. Illustrate by an original example the addition of fifteen numbers.

ORAL EXERCISES.

ART. 24.—1. There are 5 blackbirds sitting on a tree, 4 cows in the shade under the tree, and 3 boys bathing in the pond: how many are there altogether? How many *what?*

To TEACHERS.—The pupil will probably answer "Twelve," thereby affording a good opportunity to make clear to him, by a number of illustrations, the fact that *only like numbers can be added.* These oral exercises should all be solved in rigid accordance with the model given on page 31.

2. If there are 4 cows under one tree, 3 under a second, and 5 under a third, how many cows are there altogether?

3. Frank knew only 12 answers to his geography questions, and he was obliged to remain after school to learn the 6 answers which he missed: how many questions were in his lesson?

4. In digging sand at the sea-shore, Robert threw out 11 shovelfuls and Lawrence 13: how many shovelfuls were thrown out?

5. Wilmot threw 9 stones into the water and Edward threw 10: how many did both throw?

6. Molly, Susan and Maria each spelled 7 words: how many words did they all spell?

7. Mr. Higgens paid $14 for a coat and $6 for a pair of trousers: how much did the garments cost?

8. Charles bought some eggs at a store, he let the basket fall and broke 7; he afterwards fell and broke 8: how many eggs were broken?

9. Albanus gathered 9 quarts of hickory nuts from one tree and 11 from another: how many quarts did he gather?

10. Mr. Hunt spent 12 cents for apples and 10 cents for oranges: how much money did he spend?

11. If Harry rides 13 miles and walks 11, how many miles does he travel?

12. Joseph paid 6 cents for a lead pencil, 4 cents for a piece of rubber, and 5 cents for a bottle of ink: how much did the articles cost him?

13. A pound of sugar cost 8 cents, a pound of beef 22 cents, and a pound of cheese 16 cents: how much did they all cost?

14. Arthur has 6 cents, Robert 19 cents, and Andrew 5 cents: how much have they all?

15. If a quart measure contains 11 peaches, how many peaches are contained in 3 quart measures?

16. John swam 3 times across a stream 20 yards wide: how far did he swim?

17. Woodbury was 15 minutes late to school on Monday; 10 minutes late on Wednesday, and 12 minutes late on Friday: how many minutes was he tardy during the week?

18. There are three brothers whose hights are respectively 5 feet, 4 feet, and 6 feet: what is their united hight?

19. Thomas caught 9 fishes one day, 7 fishes on another day, and 8 fishes on a third day: how many fishes did he catch in the three days?

20. The Declaration of Independence was passed July 4, 1776: when did its centennial take place?

21. A man gave $13 to one of his sons, $11 to another, and $10 to a third: how many dollars did he give to his three sons?

22. John answered 12 questions, Harry 5, and Charles 9: how many were answered by all?

WRITTEN EXERCISES.

ART. 25.—1. Washington was born in 1732 and lived 67 years: in what year did he die?

2. The Pacific Ocean was first seen by Balboa in 1513, and Jamestown, Virginia, was settled 94 years later: in what year was Jamestown settled?

3. Four stores in one block rent respectively for $1,820, $1,666, $1,743, $1,848: what is the total rental of the four stores?

4. The surrender of Yorktown took place in 1781 and gold was discovered 67 years later in California: in what year was gold discovered?

5. A man paid $12,450 for a farm, $5,500 for a house,

$2,700 for stock, and $236 for insurance: what was the total cost of his property?

6. A gentleman owned 8 farms whose areas were as follows: 339, 417, 601, 505, 388, 209, 708, and 299 acres respectively: what was their aggregate area?

7. The steamer *Etruria,* in the month of August, 1885, sailed from Queenstown to New York in 6 days, 5 hours, and 46 minutes. Reckoning from Liverpool, her run each day was: 240 miles, 424 miles, 464 miles, 450 miles, 465 miles, 464 miles, 71 miles: what was the number of miles run between Liverpool and New York?

8. The population of Great Britain and Ireland at the decennial census of 1881 was: England and Wales, 25,968,286; Scotland, 3,734,441; Ireland, 5,159,839; islands, 141,223; army, navy, and merchant seamen abroad, 242,844: what was the total population?

9. The population of the United States in 1880 was, native white, 36,843,291; native colored, 6,632,549; foreign born, 6,679,943: what was the total population of our country in 1880?

10. The foreign born population in the United States in 1880 was as follows: Born in Europe, 5,744,006; America, 807,157; Africa, 2,204; Asia, 107,703; other parts of the world, 18,873: what was the total foreign population?

11. The Chinese immigration from 1854 to 1868 was: 3,526; 4,733; 5,944; 5,128; 3,457; 5,467; 7,518; 3,633; 7,214; 2,795; 2,942; 2,385; 3,863: what was the total?

12. Inclusive of 1868, the Chinese immigration to June, 1884, was: 10,684; 14,902; 11,943; 6,039; 10,642; 18,154; 16,651; 19,033; 16,679; 10,379; 8,468; 9,189; 7,011; 20,727; 35,614; 381; 80: what was the total?

13. The immigration to the United States between and

including the years 1820 and 1879 was as follows: From England, 894,444; from Ireland, 3,065,761; from Scotland, 159,547; from Wales, 17,893; from Great Britain, not specified, 560,453: what was the total?

14. The total immigration for the period named above was: From Europe, 8,746,921; from Asia, 228,047; from Africa, 1,631; from British America, 568,941; from all other American countries, 97,007; from the Pacific islands, 10,474; from all the remaining parts of the world, 255,778: how much was the grand total?

15. Our specie imports in 1884 were: gold bars and bullion, $4,997,571; American coin, $3,824,962; foreign coin, $14,008,784; silver bars and bullion, $2,910,451; American coin, $686,182; foreign coin, $10,998,312: find the total of our specie imports for 1884.

16. A gentleman owns a house worth $12,500, a store worth $6,200, a farm worth $9,500, and his personal property amounts to $24,800: what is the amount of his estate?

17. A man willed $25,000 to each of his three sons, $12,400 to each of his two daughters, and $35,000 to his wife: what was the amount left to all?

18. Four persons entered into partnership. A put in $7,300; B, $8,400; C, $7,800, and D, $9,100: what sum did all put in?

19. A fruit-dealer shipped 4,592 bushels of apples in one week, 3,851 the next, and 4,101 the third week: what was the number of bushels shipped?

20. In a company numbering three partners, A owns $8,800 of the stock, B $200 more than A, and C $1,000 more than B: what is the amount of stock owned by all?

21. Illustrate by an original problem, addition of numbers.

Subtraction.

Progressive Oral and Written Drills.

SUGGESTION.—Subtraction is the process of finding the difference between two numbers. Take 2 from 5, 8, 11; 10, 4, 7; 3, 6, 9 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	11	4	10	7	3	6	9	2
Take	2	2	2	2	2	2	2	2	2	2
	–	–	—	–	—	–	–	–	–	–

From	4268	74684	6834246756
Take	2122	12212	2221222112

REMARK.—Dictate one figure at a time to be written in line.

SUGGESTION.—Take 3 from 5, 8, 11; 10, 4, 7; 12, 9, 6 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	11	10	4	7	12	9	6	3
Take	3	3	3	3	3	3	3	3	3	3
	–	–	—	—	–	–	—	–	–	–

From	7643	98675	9746543687
Take	3213	31323	3323133233

SUGGESTION.—Take 4 from 5, 8, 12; 11, 9, 6; 7, 10, 13 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	12	11	9	6	7	10	13	4
Take	4	4	4	4	4	4	4	4	4	4
	–	–	—	—	–	–	–	—	—	–

From	4874	68797	8745867498
Take	4234	42434	3243423434

SUGGESTION.—Take 5 from 7, 13, 6; 11, 14, 9; 8, 12, 10 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	7	10	6	11	14	10	18	13	9	5
Take	5	5	5	5	5	5	5	5	5	5
	–	—	–	—	—	—	—	—	–	–

From	7402434643	9164506474
Take	4435245345	3554155445

☞ For explanation, see **Article 35.**

SUGGESTION.—Take 6 from 12, 15, 8; 10, 14, 7; 13, 9, 11 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	8	10	14	7	13	9	11	6
Take	6	6	6	6	6	6	6	6	6	6
	—	—	–	—	—	–	—	–	—	–

From	9064874654	8402648735
Take	5646616556	5466366556

SUGGESTION.—Subtract 7 from 12, 15, 9; 11, 14, 10; 8, 13, 16 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	9	11	14	10	13	16	8	7
Take	7	7	7	7	7	7	7	7	7	7
	—	—	–	—	—	—	—	—	–	–

From	9740787465	8640346561
Take	7467477467	7463665667

SUGGESTION.—Subtract 8 from 12, 15, 10; 17, 11, 14; 9, 13, 16 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	10	17	11	14	9	13	16	8
Take	8	8	8	8	8	8	8	8	8	8
	—	—	—	—	—	—	–	—	—	–

From	9482716387	8741087647
Take	7878685878	7665867678

SUGGESTION.—Subtract 9 from 11, 14, 17; 10, 13, 16; 12, 15, 18 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	11	14	17	10	13	16	12	15	18	9
Take	9	9	9	9	9	9	9	9	9	9
	—	—	—	—	—	—	—	—	—	–

From	9876543210	1234567890
Take	9687938789	989489889

☞ These *inductive exercises* are a review of the facts of subtraction in the concrete, and are intermediate between examples and problems. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 26.—1. Wilson had 4 cents and lost 1 cent: how many cents had he left?

2. Lilian had 5 oranges and gave away 2: how many oranges had she left?

3. There were 6 passengers in a street car and 3 got out: how many passengers remained in the car?

4. George earned 5 cents and spent 4: how many cents had he left?

5. Mr. Brown had 6 cows and sold 3 of them: how many cows were left?

6. Alfred gave 8 cents for a top and sold it for 5 cents: how much did he lose?

7. A certain class consists of 7 girls and 4 boys: how many more girls are there than boys?

8. There were 9 cows in one pasture but 3 broke out: how many cows were left in the pasture?

9. A hunter fired into a flock of 7 birds and killed 4: how many birds did he miss?

10. Howard owed his brother 10 cents, and gave him 5: how much did he then owe his brother?

11. Kate put $9 in the savings bank and drew out $4: how much had she left in the savings bank?

12. Mary is 11 years old, and Dollie is 5: how much older is Mary than Dollie?

13. John paid 12 dimes for his history and 4 dimes for his reader: how much more did he pay for his history than for his reader?

14. Frank caught 8 perch and sold 3: how many perch had he left?

15. Mary had 9 cakes and gave 5 to a poor girl: how many cakes had Mary left?

16. Maria had 11 cents in her money-box and put 6 of them into the church collection: how much remained?

17. A grocer bought 10 barrels of potatoes and sold 7 of them: how many barrels had he left?

18. A lady bought 12 eggs, and broke 6 of them: how many eggs were unbroken?

19. Jacob earned $11 and paid $8 for a pair of boots: how many dollars had he left?

20. A passenger train of 10 cars contained 6 palace cars: how many were not palace cars?

21. Twelve boys tried to jump over a ditch and 8 fell in: how many jumped over the ditch?

22. In a class consisting of 11 girls, 8 recited their lessons correctly; how many girls missed their lessons?

23. Horace worked 12 days and James worked 9: how many more days did Horace work than James?

24. From a cask of cider containing 13 gallons, 8 leaked out: how many gallons were left in the cask?

25. A hen hatched 11 chicks, but a cat killed 7: how many chicks were left?

26. Eight boys were sitting on a fence and 5 got off: how many boys remained on the fence?

27. Simon had 9 apples and ate 4: how many apples were left?

28. Ten ducks were swimming on a pond and 3 flew away: how many were left?

29. Mr. Thomson raised 11 bushels of potatoes and sold 6: how many bushels did he keep?

30. Jack bought a pony for $12 and sold it for $7: how much did he lose?

DEFINITIONS.

ART. 27.—Subtraction is the process of finding the difference between two numbers.

ART. 28.—The **Minuend** is the number from which another number is to be subtracted.

ART. 29.—The **Subtrahend** is the number to be subtracted.

ART. 30.—The **Difference** is the number obtained by subtraction.

ART. 31.—The **Sign of Subtraction** is −. It is read *minus*. When placed between two numbers, it shows that the number on the right is to be subtracted from the number on the left. Thus, 8 − 3 is read 8 *minus* 3, and means that 3 is to be subtracted from 8.

Read: 11 − 3; 18 − 4; 16 − 4; 22 − 5; 41 − 17; 27 − 15; 33 − 11; 33 − 22.

What is the value of: 12 + 4 − 5? 11 + 2 − 5? 13 + 5 − 3? 15 + 4 + 2 − 8? 17 − 6 + 8? 35 + 8 − 6 + 10? 16 + 5 − 4 + 8?

Write 2 expressions containing the signs + and −.
Write 2 expressions containing the signs +, −, and =.
Write 3 expressions containing the signs − and +.
Write 3 expressions containing the signs +, −, and =.
Write 5 expressions containing the signs −, +, and =.
Write 5 expressions containing the signs +, −, and =.

Art. 32. In writing dollars and cents, the two denominations are separated by a period. Two figures on the right of the period represent cents and those on the left represent dollars. Thus, 12 dollars and 18 cents are written $12.18; 125 dollars and 5 cents are written $125.05.

ART. 33.—Principles. 1. *Only like numbers can be subtracted.*

2. *If the minuend and subtrahend are equally increased or diminished, the difference remains the same.*

ORAL EXERCISES.

ART. 34.—1. James had 5 apples and ate 2 : how many apples had he left ?

Solution.—He had left the difference between 5 apples and 2 apples, or 3 apples.

2. Mary had 6 cents and spent 2 : how many cents had she left ?

3. Robert put 7 cents in his pocket, but when he reached home he had only 3 cents : how many cents did he lose on the way ?

4. Susan washed 8 plates and broke 3 of them : how many were left whole ?

5. Samuel had 11 hickory nuts and gave 7 to his squirrel : how many nuts were left ?

6. Warner's suit of 5 yards was cut from a roll of cloth containing 12 yards : how many yards were left ?

7. Harry's father gave him $14 to buy a pony ; the pony cost $11 : how much money had Harry left ?

8. Nine boys were riding down hill on a sled, and 4 fell off : how many remained on the sled ?

9. George had saved 25 cents to spend on the Fourth of July ; at the end of half an hour he had 5 cents left : how much had he spent ?

10. Fifteen persons were in a street car and 10 got out : how many were left in the car ?

11. In a certain school there are 22 pupils; 12 are boys: how many are girls?

12. A farmer had 18 sheep; his neighbor's dog killed 10: how many escaped?

13. Of 19 men who went to war, 11 were killed: how many were not killed?

14. Charles copied 20 words, 12 of which he spelled correctly: how many words did he spell incorrectly?

15. John is 11 years old and his brother is 19: how much younger is John than his brother?

16. If Johnson earns $28 a month and spends $14, how much does he save?

17. Kate received a present of 20 cents and spent 14 cents: how many cents had she left?

18. Thomas was paid 25 cents for picking cherries; he spent 5 cents for pea-nuts and 10 cents for candy: how much money had he left?

19. Frank bought some walnuts for 10 cents and some almonds for 15 cents; he gave the clerk a silver half-dollar: how much change should Frank receive?

20. Thomas bought a bat for 25 cents and a ball for 50 cents; he gave the shop-keeper a dollar bill: how much change should Thomas receive?

21. The subtrahend is 25, and the minuend 40: what is the difference?

22. The minuend is 38, and the difference 24: what is the subtrahend?

23. The difference is 24, and the minuend 68: what is the subtrahend?

24. The minuend is 48, and the subtrahend 16: what is the difference?

25. The subtrahend is 32, and the minuend 97: what is the difference?

WRITTEN EXERCISES.

ART. 35.—1. From 7686 take 3792.

Process.

7686
3792
3894

Analysis.—We write the subtrahend under the minuend, units under units, tens under tens, hundreds under hundreds, and begin at the right to subtract.

Two units from 6 units leave 4 units, which we write in the units' place. Nine tens cannot be subtracted from 8 tens, so we add 10 tens to 8 tens in the minuend, making 18 tens. Nine tens taken from 18 tens leave 9 tens, which we write in the tens' place. Since we increased the minuend by 10 tens, we must increase the subtrahend by 1 hundred, equal to 10 tens. Eight hundreds cannot be taken from 6 hundreds, so we add 10 hundreds to 6 hundreds in the minuend, making 16 hundreds. Eight hundreds taken from 16 hundreds, leave 8 hundreds, which we write in the hundreds' place. Since we increased the minuend by 10 hundreds, we must increase the subtrahend by 1 thousand, equal to ten hundreds. Four thousands from 7 thousands leave 3 thousands, which we write in the thousands' place.

Therefore, 3792 subtracted from 7686 leaves 3894.

Proof.—3792 + 3894 = 7686.

ART. 36.—Rule for Subtraction.—*Write the subtrahend under the minuend, placing units of the same order in the same column, and begin at the right to subtract.*

Subtract each order in the subtrahend from the order above it, placing the difference beneath. If any order in the subtrahend is greater than the order above it, add 10 *to the latter and subtract. In such case increase the next subtrahend order by* 1 *before subtracting.*

Proof.—*Add the subtrahend to the difference, and if the sum equals the minuend the work is correct.*

2.	3.	4.	5.	6.	7.	8.
339	438	572	493	667	949	2138
242	254	381	364	378	849	1236
9	8	1	1	2	0	9

9.	10.	11.	12.	13.
$42.37	$56.32	5312	$621.31	74132
20.44	23.15	4410	413.12	66510
$21	$ 3.1	9	$ 08.1	762

14. From eighteen thousand three hundred six, take thirteen thousand four hundred nine.

15. The minuend being ten thousand four hundred forty-eight, and the subtrahend seven thousand three hundred twenty-five, what is the difference?

16. The minuend being twelve thousand three hundred thirty-three, and the subtrahend six thousand two hundred forty-nine, what is the difference?

17. The minuend being one hundred twenty-six thousand two hundred forty-seven, and the subtrahend one hundred nine thousand nineteen, what is the difference?

18. The minuend is four million two hundred forty-six thousand eleven; the subtrahend, three hundred twenty-eight thousand one hundred one: what is the difference?

19. The subtrahend is one million two hundred forty-eight thousand six hundred thirty; the minuend two million one hundred fifty-three thousand forty-eight: what is the difference?

20. Mr. Joslin's farm contained 494 acres, and from it he sold 296 acres: how many acres had he left?

21. A certain regiment went into battle with 684 men and 293 were killed: how many escaped?

22. Jamestown was settled in 1607 and the surrender of Yorktown took place in 1781: how many years apart were the two events?

23. Mr. Winkler bought a piano for $363 and sold it for $750: how much profit did he make?

24. America was discovered in 1492: how many years have elapsed since?

25. A farm was bought for $11,450 and sold for $10,360: what was the loss?

26. Washington was born in 1732 and died in 1799: how old was he when he died?

27. The greatest width of the Atlantic Ocean is 4,150 miles and the greatest width of the Pacific Ocean is 10,000 miles: how much wider is the Pacific than the Atlantic?

28. The combined area of Virginia, New York, Massachusetts, and Pennsylvania is 145,150 square miles: how much larger is California, the area of which is 158,360 square miles?

29. How much less is the area of California than that of Texas, which is 265,780 square miles?

30. How much less is the area of Texas than that of Alaska, which is 577,390 square miles?

31. The area of Asia is 17,137,780 square miles: how much greater is that than the area of Africa, the latter being 11,511,496 square miles?

32. How much less than the area of Asia is that of North America, which is 9,349,585 square miles?

33. The cost of a horse was $245 and the cost of a carriage $128: how much did the cost of the horse and carriage exceed $78, the cost of the harness?

34. The estimated population of Asia is 793,742,303: that of Africa is 205,412,267: how much does the former exceed the latter?

35. How much less than the population of Asia is that of Europe, which is 330,003,664?

36. The cost of three farms is $48,246: how much does that exceed the cost of a share in a silver mine for which $39,204 is paid?

37. Illustrate by an original problem, subtraction of numbers.

Review Problems.

ORAL EXERCISES.

ART. 37.—1. William has 9 cents in one pocket and 8 cents in another. If he spends 7 cents, how much money will he have left?

Solution.—In both pockets he has the sum of 9 cents and 8 cents, or 17 cents. If he spend 7 cents, he will have left the difference between 17 cents and 7 cents, or 10 cents.

2. If Robert spends 10 minutes in the study of geography, 10 minutes in the study of grammar, and then plays ball 50 minutes, how much more time does he spend in play than in study?

3. If Susan pays 10 cents for a doll and 5 cents for some candy, how much change should she receive from a twenty-five-cent piece?

4. There were 8 cows in one field, 7 in another, and 10 in a third. After 15 cows were driven away, how many cows were left in the fields?

5. John had 12 cents; he lost 8 cents, 6 of which Henry found and gave back to John: how many cents had John then?

6. Thomas climbed 20 feet up a telegraph pole; he slipped back 5 feet and then climbed up 8 feet: how high was he above ground?

7. Runyon bought a book for 15 cents and a hat for 20 cents. He handed the shopkeeper a half-dollar: how much change should he receive?

8. George bought a dog for $2, a ball for $1, and a bicycle suit for $6: how much change should he receive out of a ten-dollar bill?

9. Hugh paid $2 for a whip, $3 for a bridle, and $10 for a saddle: how much did the cost of the saddle exceed that of the bridle and whip?

10. Jennie's lesson contained 15 examples. She solved 5 in the morning and 4 in the afternoon: how many remained unsolved?

11. Thomas is 10 years old and his sister 8 years old: how much less is the sum of their ages than the age of their father, who is 36 years old?

12. Richard earned $11 in one week and $12 in another. His employer paid him $4: how much did he still owe Richard?

13. Maria having 25 cents, spent 8 cents one day and 7 cents another day: how many cents had she left?

14. Jane wrote 10 words on her slate and 14 on the blackboard. She misspelled 8 of them: how many did she spell correctly?

15. Ellis started to ride 100 miles on his bicycle. The first day he rode 20 miles, the second 30 miles, and the third 40 miles: how many miles still remained for him to ride?

WRITTEN EXERCISES.

ART. 38.—1. Mr. Thomas deposited $250.45 in the bank on Monday, $323.52 on Tuesday, $638 on Wednesday. On Thursday he drew out $721.21: how much still remained to his credit?

2. Mr. Langley paid \$8,324 for a farm, and expended \$275 for fences, \$1,823 for buildings, and \$360 for interest. He sold the farm for \$13,400: how much did he gain?

3. A speculator bought a tract of land for \$18,350. He divided it into 3 parts, and sold the first for \$8,580, the second for \$12,250, and the third for \$10,480: what was his profit?

4. Mrs. Chatten went shopping with 3 ten-dollar bills and 2 five-dollar bills. She paid \$16 for some silk, \$8 for a pair of shoes, and \$2 for a pair of gloves: how much had she left?

5. A laborer earned \$32 the first month, \$28 the second month, and \$31 the third month. His expenses during that time were \$47.50: how much did he save?

6. If I live 37 years longer, I shall be 82 years old: how old was I 29 years ago?

7. A gentleman left \$25,665 to his wife and two sons. One son received \$9,500, and the other \$9,800: how much did the widow receive?

8. Three ranchmen left Texas each with a herd of cattle. In the first there were 3,250, in the second 2,948, and in the third 3,120. On the way northward 638 cattle were lost: how many cattle were left?

9. A man whose store was insured for \$12,500, and stock for \$6,800, suffered a loss of \$23,250 on his building and stock: what was his total loss?

10. If Mr. Hagar's income is \$5,600 a year, and he spends \$850 for servants, \$450 for rent, and \$1,254 for other expenses, how much has he left at the end of the year?

11. The area of Alabama is 52,250 square miles and of Arkansas 53,850 square miles: how much more is the sum of these areas than that of Colorado, which is 103,925 square miles?

12. The area of the public land is 5,740 square miles; of the District of Columbia, 70 square miles; of Rhode Island, 1,250 square miles; of Delaware, 2,050 square miles. How much less is the area of the foregoing than that of Arizona, which is 113,020 square miles?

13. The population of New York City, according to the tenth census of the United States, was 1,206,299; of Philadelphia, 874,170; of Chicago, 503,185: how much less is the joint population of those three cities than that of London, which was 3,814,571?

14. The population of San Francisco, by the same census, was 233,959; of Sacramento, 21,420; of Baltimore, 332,313; of St. Louis, 350,518: compare the joint population of these four cities with that of Paris, which was 2,269,123.

15. The area of Idaho is 84,800 square miles; of the Indian Territory, 64,690 square miles; of Montana, 146,080 square miles; of Washington, 69,180 square miles: how much less is the sum of these areas than that of Brazil, which is 3,219,000 square miles?

16. The population of Yang Tchoo is 360,000; of Venice, 129,276; of San Luis Potosi, 343,000; and of Jerusalem, 28,000; how does their joint population compare with that of Pekin, which is 500,000?

17. The total population of the United States in 1880 was 50,155,783; the foreign born was 6,679,943; the native colored, 6,632,549: what was the native white population?

18. The number of miles of railroad in the United States in 1883 was 120,552. Of these, there were in New England 6,323 miles; in the Middle States 17,532 miles; in the Southern States 18,866 miles; in the Pacific States 7,486 miles; and the remainder in the Western States: how many miles were in the Western States?

Multiplication.

Progressive Oral and Written Drills.

SUGGESTION.—Multiplication grows out of addition, and is a short method of adding a number to itself. The numbers only that result from adding one to itself are called prime numbers, such as 1, 2, 3, 5, 7, 11, 13, 17, 19, etc. When a number larger than one is added to itself, the resulting number is called a multiple of that number. The multiples that arise from multiplying a digit by a digit, are the facts to be learned in multiplication, of which there are 36 only, as will be shown in the following drills.

DEVELOPMENT OF THE TABLE.

☞ Take columns of 2's and bring out the 8 facts of the multiplication of 2's. Thus :

									×2	
2	2	2	2	2	2	2	2		4	.
2	2	2	2	2	2	2	2		8	.
4	2	2	2	2	2	2	2	$2 \times 2 = 4$	2	.
	6	2	2	2	2	2	2	$3 \times 2 = 6$	6	.
		8	2	2	2	2	2	$4 \times 2 = 8$	3	.
			10	2	2	2	2	$5 \times 2 = 10$	7	.
				12	2	2	2	$6 \times 2 = 12$	1	.
					14	2	2	$7 \times 2 = 14$	5	.
						16	2	$8 \times 2 = 16$	9	.
							18	$9 \times 2 = 18$		

SLATE AND BLACKBOARD EXERCISES.

2122	12212	21220	21102	12221	22121
1	2	3	4	5	6

2122122122122	122121121212	2210212122222
7	8	9

REMARK.—The examples are graded, so that the multiplicand has no higher figure than the number treated, but the multiplier may be any of the nine digits. The pupil must be taught how to carry when the product is more than 9.

SUGGESTION.—Develop the table of 3's from columns of 3's. Thus:

								×3	
3	3	3	3	3	3	3		7	.
3	3	3	3	3	3	3		4	.
3	3	3	3	3	3	3		2	.
9	3	3	3	3	3	3	$3 \times 3 = 9$	8	.
	12	3	3	3	3	3	$4 \times 3 = 12$	2	.
		15	3	3	3	3	$5 \times 3 = 15$	9	.
			18	3	3	3	$6 \times 3 = 18$	3	.
				21	3	3	$7 \times 3 = 21$	6	.
					24	3	$8 \times 3 = 24$	5	.
						27	$9 \times 3 = 27$	1	.

REMARK.—As 3×2 is the same as 2×3, this combination educes no new fact, and it is therefore omitted. Only the seven new combinations with 3 are given.

SLATE AND BLACKBOARD EXERCISES.

2233	23213	231123	231233	233221	123332
1	2	3	4	5	6

231323231132	323103132332	321332323213
7	8	9

REMARK.—It is an excellent exercise to write a long line of 1's, 2's and 3's, and require pupils to multiply in class.

SUGGESTION.—Since 2×4 is the same as 4×2, and 3×4 as 4×3, there are only six new facts to be learned. Thus:

								×4	
4	4	4	4	4	4			7	.
4	4	4	4	4	4			2	.
4	4	4	4	4	4			9	.
16	4	4	4	4	4	$4 \times 4 = 16$		5	.
	20	4	4	4	4	$5 \times 4 = 20$		8	.
		24	4	4	4	$6 \times 4 = 24$		1	.
			28	4	4	$7 \times 4 = 28$		4	.
				32	4	$8 \times 4 = 32$		6	.
					36	$9 \times 4 = 36$		3	.

SLATE AND BLACKBOARD EXERCISES.

342442434241	424434343243	434144413424
1(2)	3(4)	5(6)
432432434424	243423344443	432143414424
7	8	9

REMARK.—Dictate a long line of figures containing no figure greater than 4, and multiply the number so written by all the digits. Such exercises fix the facts learned and assure certain and steady advancement in arithmetical knowledge.

SUGGESTION.—Since $5 \times 2 = 2 \times 5$; $5 \times 3 = 3 \times 5$, and $5 \times 4 = 4 \times 5$, these combinations need not be repeated. There are only five new facts to be learned. Thus:

						×5
5	5	5	5	5		3 .
5	5	5	5	5		6 .
5	5	5	5	5		4 .
5	5	5	5	5		1 .
25	5	5	5	5	$5 \times 5 = 25$	8 .
	30	5	5	5	$6 \times 5 = 30$	5 .
		35	5	5	$7 \times 5 = 35$	9 .
			40	5	$8 \times 5 = 40$	2 .
				45	$9 \times 5 = 45$	7 .

SLATE AND BLACKBOARD EXERCISES.

543515452453	454525543452	515454354521
1(2)	3(4)	5(6)
545325525355	453542253545	535454515455
7	8	9

SUGGESTION.—Count by 6 up to 54, and then ask, How many 6's make 42? 30? 48? 24? 18? and so on. Learn the four new combinations, 6×6; 7×6; 8×6; 9×6.

SLATE AND BLACKBOARD EXERCISES.

636266624653	563646465362	461645656245
1(2)	3(4)	5(6)
645653646366	646564566466	646345656636
7	8	9

SUGGESTION. — There are only three new combinations to be learned with the 7's, viz.: 7×7; 8×7; 9×7.

SLATE AND BLACKBOARD EXERCISES.

747647373746767 54	67674676764677675
2(3,4,5)	6(7,8,9)

REMARK.—It is a very profitable exercise to multiply a long example by each digit in turn.

SUGGESTION.—There are only two new combinations to be learned with the 8's, viz.: 8×8 and 9×8.

SLATE AND BLACKBOARD EXERCISES.

87887487868758678	87686878786868487
2(3,4,5)	6(7,8,9)

SUGGESTION.—There is but one combination to be learned with the 9's, viz.: $9 \times 9 = 81$.

SLATE AND BLACKBOARD EXERCISES.

98969497897896978	89879989698497987
2(3,4,5)	6(7,8,9)

REVIEW OF FACTS.

2×2, 3×2, 4×2, 5×2, 6×2, 7×2, 8×2, 9×2.
3×3, 4×3, 5×3, 6×3, 7×3, 8×3, 9×3.
4×4, 5×4, 6×4, 7×4, 8×4, 9×4.
5×5, 6×5, 7×5, 8×5, 9×5.
6×6, 7×6, 8×6, 9×6.
7×7, 8×7, 9×7.
8×8, 9×8.
9×9.

REMARK.—Keep up a continual oral review of the facts as learned.

The graded work in the fundamental rules, as presented in the "Progressive Oral and Written Drills," is the outcome of the best modern teaching, and, if carried out faithfully, will secure the great ends of (1) accuracy, (2) facility, and (3) rapidity in arithmetical operations. When the pupil has been taught to think all the primary combinations, advancement in applying numbers will be assured. Time spent in fixing the facts, will be a tenfold gain finally.

☞ These *inductive exercises* are a review of the facts of multiplication, and are intermediate between examples and problems. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 39.—1. John and James have each 2 cents: how many cents have both?

2. I saw 3 girls, each carrying 2 books: how many books had all?

3. Three boys each threw 3 stones at a dog: how many stones were thrown by all?

4. Mary received 4 credit marks in the forenoon and 4 in the afternoon: how many credit marks did she receive during the day?

5. There are 4 benches in a certain room, and 2 boys are sitting on each bench: how many boys are sitting on the 4 benches?

6. Julia put 3 cents in the collection-box on each of 3 Sundays: how many cents did she put in during that time?

7. Harmon set out 4 rows of trees, each row containing 4 trees: how many trees were in the 4 rows?

8. How much will 5 lemons cost at 4 cents each?

9. Sarah recited 5 lessons each day for 5 days: how many lessons did she recite?

10. Mr. Bonnel has 6 piles of wood, each containing 3 cords: how many cords has he?

11. A nickel is worth 5 cents: how many cents are 6 nickels worth?

12. John bought 4 tops and paid 6 cents apiece for them: how many cents did he pay for the tops?

13. Seven boys paid $3 each to the missionary cause: how many dollars did all pay?

14. Four men earned $8 each per week: how much did all earn in one week?

15. A dime is worth 10 cents: how many cents are 3 dimes worth?

16. How much will 4 bottles of ink cost at 9 cents a bottle?

17. In a certain Sunday-school are 8 classes, and each class contains 5 scholars: how many scholars are in the Sunday-school?

18. Ten girls each gave 5 cents to a poor woman: how much did they all give?

19. Warren wrote 6 lines a day in his copy-book for 11 days: how many lines did he write?

20. Joseph set 8 traps for rabbits, and during the winter caught 6 rabbits in each trap: how many rabbits did he catch?

21. A newsboy sold 10 papers each day for 9 days: how many papers did he sell?

22. Eleven men earned $7 apiece: how much did all earn?

23. A teacher gave 5 problems to each of 12 boys: how many problems did he give to all?

24. How many cents are 11 dimes worth?

25. How much will 9 Bibles cost at $6 each?

26. David walked 3 miles a day for 9 days: how far did he walk?

27. A man paid $4 for his hat, and 8 times as much for his overcoat: how much did he pay for his overcoat?

28. Frances weighs 10 times as much as her pet dog, whose weight is 7 pounds: what is the weight of Frances?

29. Julius gave 6 peaches to each of his 8 playmates: how many peaches did he give away?

30. How much will 12 pencils cost at 5 cents apiece?

31. Thomas is 9 years old and his father is 4 times as old: what is the age of his father?

32. Henry paid 4 cents apiece for 11 melons: how much did they cost?

33. There are 8 quarts in one peck: how many quarts are there in 8 pecks?

34. What do 7 loads of wood cost at $6 a load?

35. Belle gave 7 cents each for 12 spools of thread: how much did they all cost?

MULTIPLICATION TABLE.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5	6 × 1 = 6
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10	6 × 2 = 12
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15	6 × 3 = 18
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20	6 × 4 = 24
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25	6 × 5 = 30
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30	6 × 6 = 36
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35	6 × 7 = 42
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40	6 × 8 = 48
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45	6 × 9 = 54
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50	6 × 10 = 60
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44	5 × 11 = 55	6 × 11 = 66
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48	5 × 12 = 60	6 × 12 = 72

7 × 1 = 7	8 × 1 = 8	9 × 1 = 9	10 × 1 = 10	11 × 1 = 11
7 × 2 = 14	8 × 2 = 16	9 × 2 = 18	10 × 2 = 20	11 × 2 = 22
7 × 3 = 21	8 × 3 = 24	9 × 3 = 27	10 × 3 = 30	11 × 3 = 33
7 × 4 = 28	8 × 4 = 32	9 × 4 = 36	10 × 4 = 40	11 × 4 = 44
7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50	11 × 5 = 55
7 × 6 = 42	8 × 6 = 48	9 × 6 = 54	10 × 6 = 60	11 × 6 = 66
7 × 7 = 49	8 × 7 = 56	9 × 7 = 63	10 × 7 = 70	11 × 7 = 77
7 × 8 = 56	8 × 8 = 64	9 × 8 = 72	10 × 8 = 80	11 × 8 = 88
7 × 9 = 63	8 × 9 = 72	9 × 9 = 81	10 × 9 = 90	11 × 9 = 99
7 × 10 = 70	8 × 10 = 80	9 × 10 = 90	10 × 10 = 100	11 × 10 = 110
7 × 11 = 77	8 × 11 = 88	9 × 11 = 99	10 × 11 = 110	11 × 11 = 121
7 × 12 = 84	8 × 12 = 96	9 × 12 = 108	10 × 12 = 120	11 × 12 = 132

12 × 12 = 144

DEFINITIONS.

ART. 40.—Multiplication is the process of repeating one number as many times as there are units in another number. Thus: 27 + 27 + 27 = 3 times 27, or 81.

ART. 41.—The **Multiplicand** is the number repeated.

ART. 42.—The **Multiplier** is the number showing the number of times the multiplicand is to be repeated.

ART. 43.—The **Product** is the result of the multiplication.

The **Factors of a Number** are the numbers which, when multiplied together, produce that number.

The multiplicand and multiplier are factors of the product.

ART. 44.—The **Sign of Multiplication** (×) is read *times* or *multiplied by*, and when placed between two numbers, means that they are to be multiplied together. Thus, 2 × 5 is read 2 *times* 5, or 2 *multiplied by* 5.

ART. 45.—The Parenthesis ( ) signifies that the inclosed expression is to be considered as one number. Thus, 32 − (8 + 3) signifies that the sum of 8 and 3 is to be taken from 32.

ART. 46.—Principle. I. *The multiplicand and product are like numbers.*

II. *The multiplier is always abstract.*

ORAL EXERCISES.

ART. 47.—1. How much will 7 hats cost at $3 each?

Solution.—Seven hats will cost 7 times $3, or 21 dollars.

2. If 6 boys can lift as much as 1 man, how many boys will it take to lift as much as 8 men?

3. There are 12 window panes in 1 sash: how many window panes are there in 7 sashes?

4. At 6 cents apiece, how much do 7 lead pencils cost?

5. At 10 cents each, how much do 8 copy-books cost?

6. At 9 cents each, how much are 8 slates worth?

7. If 8 boys eat 12 almonds each, how many almonds are eaten by all?

8. Seven girls each recited 7 answers in geography: what was the whole number recited?

9. There are 4 pecks in 1 bushel: how many pecks in 9 bushels?

10. There are 8 quarts in 1 peck: how many quarts in 7 pecks?

11. Eight trains passed Eldon station one day, and each train consisted of 8 cars: how many cars passed Eldon that day?

12. A school-boy walked 6 miles to and from school each day: how many miles did he walk in 6 days?

13. Eight girls solved 11 problems each: how many did they all solve?

14. At Washington's Head-quarters, Newburg, there is a room with 8 doors: how many doors would there be in 7 similar rooms?

15. How much will 8 dozen eggs cost at 12 cents a dozen?

16. A farmer sold 9 calves at $7 apiece: how much did he receive for all?

17. Carrie is 6 years old and her grandfather is 12 times as old: what is the age of her grandfather?

18. If 8 bricks are placed in 1 pile, how many bricks are placed in 11 piles?

19. If a sheep is worth $4, what are 8 sheep worth?

20. There are 7 days in one week: how many days are there in 7 weeks?

21. If a bottle of ink is worth 10 cents, how much are 7 bottles worth?

22. Sarah reads 12 pages a day; how many pages does she read in 9 days?

23. If you ride 9 miles an hour, how far will you ride in 10 hours?

24. At $6 a barrel, what will 9 barrels of flour cost?

25. What will 11 pairs of shoes cost at $7 a pair?

26. Thomas owns 7 hens, which have 10 chicks each: how many chicks and hens does he own?

27. Caleb averaged 20 miles a day on his bicycle for 5 days: how many miles did he travel?

28. In a certain school are 12 rows of desks and 10 pupils sit in each row: how many pupils are there in the school?

29. What will 9 slates cost at 10 cents apiece?

30. How much will 9 yards of broadcloth cost at $6 per yard?

31. If 8 men can do a piece of work in 8 days, how long will it take one man to do it?

32. A boy saved 11 cents per week for 10 weeks: how much did he save?

33. What will 7 melons cost at 8 cents apiece?

34. There are 12 inches in one foot: how many inches are there in 7 feet?

35. How much must I pay for 9 pigs worth $7 each?

36. What will 9 pairs of shoes cost at $6 a pair?

37. When coal is worth $7 a ton, how much will 7 tons cost?

38. If I save $11 a month, how much can I save in 11 months?

ART. 48.—When the Multiplier is Not Greater than 12.

WRITTEN EXERCISES.

1. Multiply 634 by 6.

Process.

634
6
3804

Analysis.—We write the multiplier under the multiplicand, and begin at the right to multiply.

Six times 4 units are 24 units, equal to 2 tens and 4 units. We write 4 in the units' place and add the tens to the product of tens. Six times 3 tens are 18 tens, and 2 tens added are 20 tens, equal to 2 hundreds and 0 tens. We write 0 in the tens' place and add 2 to the product of hundreds. Six times 6 hundreds are 36 hundreds, and 2 hundreds added are 38 hundreds, equal to 3 thousands and 8 hundreds. We write the 8 in the hundreds' place and the 3 in the thousands' place.

1.	2.
42432314242143424	45435425145453432
8	9
3.	**4.**
56436456436246564	67464562067645675
5	4
5.	**6.**
67376473674657467	65767465767573747
7	9
7.	**8.**
86548678582468378	84689649698786498
6	8

REMARK.—Heretofore the pupil has multiplied by the digits only. He is now to learn to multiply by 10, 11 or 12, regarded as single numbers.

9.	10.
64654368784967567	87869789645786897
10	10

11.	12.
97524698704578987	4987468960978568 7
11	11
13.	**14.**
64058916849765268	58697123045987689
12	12

REMARK.—When the product is more than 100, be sure to carry the number expressed by the two left hand figures.

ART. 49.—When the Multiplier is Greater than 12.

WRITTEN EXERCISES.

1. What is the product of 384 multiplied by 32?

Process.

384
32
768
1152
12288

Analysis.—We write the multiplier under the multiplicand so that units of the same order stand in the same column, and begin at the right to multiply. Two times 4 units are 8 units, which we write in the units' place. Two times 8 tens are 16 tens, equal to 1 hundred and 6 tens. We write 6 in the tens' place and add the hundreds to the product of hundreds. Two times 3 hundreds are 6 hundreds, and 1 hundred added makes 7 hundreds, which we write in the place of hundreds.

Thirty (or 3 tens) times 4 units are 120 units, equal to 1 hundred and 2 tens. We write 2 in the tens' place and add 1 hundred to the product of hundreds. Thirty (or 3 tens) times 8 tens are 24 hundreds, and 1 hundred added makes 25 hundreds, equal to 2 thousands and 5 hundreds. We write 5 in the hundreds' place and add 2 thousands to the product of thousands.

Thirty (or 3 tens) times 3 hundreds are 9 thousands, and 2 thousands added make 11 thousands, equal to 1 ten thousand and 1 thousand, which we write in their proper places. Adding these partial products, we have 12288.

Therefore, $384 \times 32 = 12288$.

ART. 50.—Rule for Multiplying when the multiplier is greater than 12.—*Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

Multiply each order of the multiplicand by each order of the multiplier successively, beginning with the units. Write the first figure of each partial product under the multiplier that produced it, add the partial products together, and the sum will be the result required.

NOTE.—When either the multiplicand or multiplier or both end with one or more ciphers, omit the ciphers in forming the partial products and annex them to the final product.

To multiply any number by 10, 100, 1000, etc., annex to the number as many ciphers as there are in the multiplier.

Multiply

2.	544509	by	45.	**11.**	389125	by	508.
3.	623215	by	36.	**12.**	467142	by	128.
4.	734104	by	53.	**13.**	973250	by	163.
5.	856213	by	62.	**14.**	892388	by	327.
6.	794606	by	48.	**15.**	679640	by	421.
7.	875153	by	72.	**16.**	943375	by	529.
8.	672875	by	73.	**17.**	789428	by	348.
9.	894326	by	84.	**18.**	679640	by	521.
10.	763625	by	86.	**19.**	769724	by	388.

20. What will 323 pounds of butter cost at 27 cents a pound?

21. A drover sold 199 horses at $237 apiece: how much did he receive?

22. If a single box contains 393 lemons, how many lemons are there in 329 boxes?

23. If one book contains 564 pages, how many pages are there in 359 books?

24. If there are 4,693 bricks in one wall, how many bricks are there in 89 walls?

25. What will it cost to construct 1,284 miles of railroad at a cost of $5,948 a mile?

26. Allowing 365 days to a year, how many days has a man lived who is 63 years old?

27. If it takes 323 laborers 84 weeks to build a bridge, how long will it take one man to build it?

28. An agent sold 154 lots at $138 apiece: how much did he receive?

29. How much will 248 cows cost at $37 each?

30. Mr. Heisler paid $169 an acre for a farm containing 99 acres: how much did he pay?

31. A man bought 328 oxen and 28 times as many sheep: how many sheep did he buy?

32. A contractor paid $32 per month to each of 184 men: how much did he pay them all in a month?

33. How much will 2,525 tons of iron cost at $21 a ton?

34. If a man earns $784 a year, how much will he earn in 19 years?

35. How far will a railroad engineer travel in 229 days at the rate of 187 miles a day?

36. A certain army was composed of 111 regiments, each regiment containing 890 men: how many men were in the army?

37. In a mile are 5,280 feet: how many feet are in 783 miles?

38. In a cubic foot are 1,728 cubic inches: how many cubic inches are in 945 cubic feet?

39. A farmer sold 879 acres at $129 an acre: how much did he receive?

40. Illustrate by an original problem, the multiplication of numbers.

Review Problems.

ORAL EXERCISES.

ART. 51.—1. James bought 2 lead pencils at 5 cents each and 2 erasers at 3 cents each; he gave the clerk 20 cents: how much change should the clerk give James?

Solution.—2 lead pencils at 5 cents each cost 10 cents, and 2 erasers at 3 cents each, 6 cents; the pencils and erasers cost 10 cents plus 6 cents, or 16 cents. James should receive the difference between 20 cents and 16 cents, or 4 cents, change.

2. Stephen recited 17 verses and Marie 29 verses: how many more verses did Marie recite than Stephen?

3. Harmon counted 32 grasshoppers and Adolphus 18: how many more grasshoppers did Harmon count than Adolphus?

4. In a certain school there are 49 pupils and 21 of them are boys: how many are girls?

5. A farmer sold 10 dozen eggs at 20 cents a dozen: how much less than $3 did he receive?

6. Samuel bought 20 pigeons from John and 15 from Israel; 10 of them afterward escaped: how many pigeons had Samuel left?

7. There are 8 windows in the front of each of two houses and 12 on the eastern side of each: how many windows are on the front and eastern sides of the two houses?

8. Alfred borrowed 11 cents from his brother and 12 from his sister. He paid his brother 6 cents and his sister 5: how much money did he still owe?

9. Three boys went fishing; the first caught 8 fishes, the second 6, and the third 5: how many more fishes did the first two together catch than the third?

10. A cyclone destroyed three barns, worth respec-

tively $1,200, $1,400, and $1,000: what was the combined value of the barns destroyed?

11. In a spelling-match, Thomas spelled 20 words before missing, Jane 27, and Molly 16: how many words were correctly spelled by the three?

12. Two boys traveled in opposite directions from a certain place. One went 6 miles an hour, and the other 5 miles an hour: how far were they apart at the end of 5 hours?

13. Richard had $5, and his brother lacks $6 of having 4 times as much: how much has his brother?

14. A farmer sold 20 pumpkins at 15 cents each: how much less than $4 did he receive?

15. Henry bought 4 pounds of sugar at 7 cents a pound and 6 pounds of butter at 20 cents a pound: how much less than $2 did he pay?

16. A boy handed a clerk a dollar bill in payment for 2 pounds of cheese worth 16 cents a pound, and 1 quart of molasses worth 18 cents: how much change should the boy receive?

17. Edward earned $6 a week and spent $4: how much did he save in 10 weeks?

18. After buying 4 yards of muslin at 8 cents a yard and 4 spools of thread at 6 cents a spool, Mary had 9 cents left: how much had she at first?

19. James paid 84 cents for books and 10 cents for a slate: how much change should he receive if he gave in payment one dollar?

20. Harry sold 12 pigeons at 20 cents apiece and lost 60 cents: what did he pay for the pigeons?

WRITTEN EXERCISES.

ART. 52.—1. If the "Chicago Limited" averages 50 miles an hour, how far will it travel in 2 days of 24 hours each?

2. A grocer buys 65 barrels of pork for $360 and sells it at $6 a barrel: how much does he gain?

3. A book-keeper receives $80 per month and his monthly expenses are $35: how much does he save in a year?

4. A contractor hired 22 men at $2 a day. It took 20 days for him to complete his contract, for which he was paid $960: did he make or lose, and how much?

5. A farmer's daughters put up 19 tubs of butter, each weighing 49 pounds, for which they received 28 cents a pound: how much did they receive for the butter?

6. A merchant paid $5,600 for his store and $3,800 for his stock. He sold both for $11,500: how much did he gain?

7. A farmer having saved $468 a year for 12 years, went into Wall Street to deal in stocks. At the close of the second day he had lost $5,616: how much of his savings remained?

8. A dealer bought 74 horses at $250 apiece and sold them for $18,400: did he gain or lose, and how much?

9. There were 9 brothers, 5 of whom were each 6 feet tall and the others were each 5 feet tall: how much did their united height lack of 59 feet?

10. In a certain school there are 18 classes, each containing 48 pupils: if 11 classes were dismissed, how many pupils would be left in the building?

11. A merchant put all his money, $4,000, in business; the first year he gained $543, the second year he lost $245, the third year he lost $105, and the fourth year he gained $388: how much was he then worth?

12. Mr. McGovern bought 250 acres of land for $12,000; he sold 100 acres at $50 an acre, 100 more

acres at $47 per acre, and the remainder at $48 per acre: did he gain or lose, and how much?

13. How many more years is it since the discovery of America in 1492, than since the signing of the Declaration of Independence in 1776?

14. A regiment went into battle with 950 men; 27 deserted and 101 were killed and wounded: if the deserters rejoined the regiment after the battle, how many soldiers were then in the regiment?

15. A steamboat, chartered to carry 650 passengers, made 6 excursions, taking 745 persons each time: how many more persons were taken on the 6 excursions than was legal?

16. A grocer bought 54 barrels of sugar, each weighing 275 pounds, at 8 cents a pound: if he sells the sugar at 10 cents a pound, how much does he gain?

17. If sound moves 1,100 feet a second, how far will it move in one hour, or 3,600 seconds?

18. If I pay $6 a barrel for 275 barrels of flour and sell it at $7 a barrel, how much do I gain?

19. Three fields of corn each contain 80 rows, and each row has 300 hills, and each hill 3 stalks: how many stalks are in the three fields?

20. James sold 7 bushels of hickory nuts for 80 cents a bushel, and his brother sold 6 bushels of walnuts at 90 cents a bushel: which received the greater sum, and how much?

21. A sea-captain has crossed the Pacific Ocean 3 times, and another sea-captain has crossed the Atlantic Ocean 8 times. Estimating the width of the Pacific to be 10,000 miles, and that of the Atlantic 3,100 miles, how many miles farther did the first sea-captain sail than the second?

22. A railroad company's average monthly receipts from

passenger travel are $12,500, and from freight, $8,100. The total expenses for 12 months are $192,350: what is the amount of profit at the end of a year?

23. The yield of a silver mine in Colorado is: first quarter, $12,400; second quarter, $11,111; third quarter, $14,256; fourth quarter, $8,325. The quarterly expenses are $8,800: what is the profit at the end of the year?

24. A railroad engineer averages a run of 100 miles a day for 300 days in a year. At that rate for 8 years, how does the distance traveled by him compare with the distance between the earth and moon, which is 240,000 miles?

25. A drover paid $224 apiece for 38 horses. He sold 10 for $250 apiece; 15 for $220 apiece, and the remainder for $225 apiece: did he gain or lose, and how much?

26. A gentleman owning 6 buildings, insures one worth $9,000, for $8,000; 2 worth $8,500 apiece, for $7,700 each; and 3 worth $11,000 apiece, for $9,200 each. If all the buildings are burned, how much will he lose?

27. Two buildings worth $6,500 apiece are insured for $5,800 each, and two others worth $7,000 apiece are insured for $6,500 each. If all the buildings are burned, and one of the insurance companies fails to pay a policy of $6,500, what will be the total loss to the owner of the buildings?

28. A man bought a farm containing 323 acres, at $95 an acre. He invested $7,220 in improvements, and then sold the farm at $120 an acre: did he gain or lose, and how much?

29. Two vessels which were 825 miles apart sailed toward each other, one at the rate of 42 miles a day, and the other at the rate of 38 miles a day: how far were they apart at the end of 8 days?

30. One man is 133 miles in advance of another who

follows him at the rate of 83 miles a day: if the former travels 61 miles a day, how far will they be apart at the end of 6 days?

31. If a regiment of 985 soldiers consume 132 barrels of flour in 64 days, how long will the same amount of flour last one man?

32. There are 640 acres in one square mile: how many acres in the State of Missouri, which contains 69,415 square miles?

33. A grocer bought 42 barrels of flour at $6 a barrel. He sold 22 barrels of it at $7 and the remainder at $6 a barrel: how much did he gain?

34. A man bought 32 horses at $104 apiece, and sold 13 at $108 each, and the remainder at $111 each: how much did he gain?

35. I bought 2 farms, one containing 140 acres at $75 an acre, and the other 230 acres at $64 an acre: what did they cost?

36. The following is the Magic Square of Archimedes, in which no number is repeated:

22	21	13	5	46	38	30
31	23	15	14	6	47	39
40	32	24	16	8	7	48
49	41	33	25	17	9	1
2	43	42	34	26	18	10
11	3	44	36	35	27	19
20	12	4	45	37	29	28

Add each of these columns downward, and then add them across the page, and compare the different sums.

37. Illustrate by an original problem, addition, subtraction, and multiplication.

Division.

Progressive Oral and Written Drills.

SUGGESTION.—Division is the reverse of multiplication, and is the process of finding how many times one number is contained in another. Take the multiples of 2, and ask, How many 2's in 4? 12? 16? 6? 8? 14? 10? 18? Illustrate division by separating objects into groups of 2. If odd numbers are taken, there will always be one over, which may be written over the divisor, giving rise to the fractional form $\frac{1}{2}$.

SLATE AND BLACKBOARD EXERCISES.

2)468246 2)864691 2)604826 2)842697

☞ For explanation, see **Article 63.**

REMARK.—The dividend may contain any figure, but the divisor must not be larger than the digit which is the object of study.

What is $\frac{1}{2}$ of 4? 6? 8? 10? 12? 14? 16? 18?

SUGGESTION.—Count by 3 to 27. Ask, How many 3's in 9? 18? 12? 6? 21? 15? 24? 27?

Begin at 1 and count by 3's. The numbers thus obtained will contain a certain number of 3's with one over, which is $\frac{1}{3}$ of 3. How many 3's in 7? 13? 19? 25? 16? 28? 10? 22?

Begin at 2 and count by 3's. These numbers will all have 2 over, which is $\frac{2}{3}$ of 3. How many 3's in 8? 14? 20? 26? 17? 29? 11? 23?

SLATE AND BLACKBOARD EXERCISES.

3)345963 3)6481582 3)6378457 3)4579864

REMARK.—When there is 1 or 2 over with the last division, write what remains over the divisor.

What is $\frac{1}{3}$ of 6? 9? 12? 15? 18? 21? 24? 27?

What is $\frac{2}{3}$ of 6? 9? 12? 15? 18? 21? 24? 27?

NOTE.—The process of finding the fractional parts of multiples, is an excellent review exercise in division and multiplication.

SUGGESTION.—Count by 4's up to 36. Ask, How many 4's in 16? 24? 20? 32? 12? 36? 8? 28? Illustrate by groups of 4.

Begin with 1 and count by 4's up to 37. These numbers give a certain number of 4's and 1 over, which is ¼ of 4.

Begin with 2 and count by 4's up to 38. These numbers contain a certain number of 4's and 2 over, which is of 4.

Begin with 3 and count by 4's up to 39. These numbers contain a certain number of 4's, and 3 over, which is ¾ of 4.

SLATE AND BLACKBOARD EXERCISES.

4)4845286476 4)8976842676 4)6880462416

REMARK.—Dictate additional examples.

What is ¼ of 4? 8? 12? 16? 20? 24? 28? 32? 36?

☞ Find ²⁄₄ and ³⁄₄ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, and count by 5. Tell how many 5's and how many over there are in each number.

SLATE AND BLACKBOARD EXERCISES.

5)4056849686 5)8402868405 5)6847967245

What is ⅕ of 10? 30? 45? 20? 35? 15? 40? 25?

☞ Find ²⁄₅, ³⁄₅ and ⁴⁄₅ of the above numbers.

SUGGESTION.—After beginning with 0, 1, 2, 3, 4 or 5, and counting by 6's, let pupils take such columns as the following and tell how many 6's and how many over there are in each number. Thus: In 14 there are two 6's and 2 over, and so on.

14	34	54	4	24	44
39	59	9	29	49	19
50	0	20	40	10	30
8	28	48	18	38	58
21	41	11	31	51	1
47	17	38	57	7	27
12	32	52	2	22	42
36	56	6	26	46	16
53	63	23	43	13	33
5	25	45	15	35	55

SLATE AND BLACKBOARD EXERCISES.

6)7264816743 6)8463948765 6)4376798268

REMARK.—Dictate additional examples.

What is $\frac{1}{6}$ of 18? 36? 54? 42? 24? 12? 30? 48?

☞ Find $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$ and $\frac{5}{6}$ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, or 6, and count by 7's up to 69. After the pupil can do this with facility, let him tell how many 7's and how many over there are in each of the following numbers:

14	34	54	4	24	44	64
39	59	9	29	49	69	19
50	0	20	40	60	10	30
8	28	48	68	18	38	58
21	41	61	11	31	51	1
47	67	17	37	57	7	27
62	12	32	52	2	22	42
16	36	56	6	26	46	66
33	53	3	23	43	63	13
55	5	25	45	65	15	35

REMARK.—This table contains all numbers from 0 to 70.

What is $\frac{1}{7}$ of 14? 21? 28? 35? 42? 49?

☞ Find $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$ of the above numbers.

SLATE AND BLACKBOARD EXERCISES.

7)8964068976 7)8604964843 7)8774879543

REMARK.—Dictate additional examples.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, 6, or 7, and count by 8's up to 79. This exercise is not only a review of addition, but prepares the pupil for the analytic process of division.

☞ Tell how many 8's and how many over there are in each of the following numbers:

4	24	44	64	14	34	54	74
29	49	69	19	39	59	79	9
40	60	10	30	50	70	0	20
68	18	38	58	78	8	28	48
11	31	51	71	1	21	41	61
37	57	77	7	27	47	67	17
52	72	2	22	42	62	12	32
76	6	26	46	66	16	36	56
3	23	43	63	13	33	53	73
25	45	65	15	35	55	75	5

SLATE AND BLACKBOARD EXERCISES.

8)5487465346 8)9706843768 8)3807649876

What is $\frac{1}{8}$ of 16? 24? 32? 40? 48? 56? 64?

☞ Find $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$ and $\frac{7}{8}$ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, 6 or 7, and count by 8's up to 89. Arrange the numbers as in the table of 8's, and tell how many 9's and how many over there are in each number.

SLATE AND BLACKBOARD EXERCISES

9)6436794230 9)9835264984 9)6034687648

REMARK.—Dictate additional examples, and divide by each of the digits.

What is $\frac{1}{9}$ of 18? 27? 36? 45? 54? 63? 72? 81?

☞ Find $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$ and $\frac{8}{9}$ of the above numbers.

REMARK.—Finding the fractional parts of the multiples of the digits, fixes the tables of multiplication and division. No other drill will accomplish as much with as little effort. If the plans suggested heretofore are faithfully followed out, the pupil will have acquired such a mastery over pure numbers as to enable him to perform the required operations with accuracy, facility, and rapidity.

☞ These *inductive exercises* are a review of the facts of division in the concrete, and are intermediate between examples and problems. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 53.—1. If an orange is worth 2 cents, how many oranges can be bought for 4 cents?

2. Mr. Sutton divided six cents equally among his 3 sons: how many cents did each son receive?

3. Bertha practices 3 hours a day on her piano: how many days will it take her to practice 9 hours?

4. There are three feet in a yard: how many yards in 12 feet?

5. There are 4 quarts in a gallon: how many gallons in 12 quarts?

6. At 4 cents apiece, how many pears can be bought for 16 cents.

7. John expended 20 cents in buying nails at 5 cents a pound: how many pounds did he buy?

8. At $5 a ton, how many tons of coal can be bought for $30?

9. A farmer divided 36 apples equally among 6 boys: how many apples did each boy receive?

10. If a man works 7 hours a day, how many days will it take him to work 35 hours?

11. There are 8 quarts in a peck: how many pecks in 24 quarts?

12. An agent received $25 for books at $5 apiece: how many books did he sell?

13. There are 7 days in a week: how many weeks in 28 days?

14. How many hats at $5 each can be bought for $40?

15. A farmer who had 48 acres divided it into 8 fields of equal size: how many acres were in each field?

16. How many coats at $9 each can be bought for $27?

17. Thomson divided 36 apples equally among 9 of his playmates: how many apples did he give to each?

18. How long will it take a horse to trot 54 miles at the rate of 9 miles an hour?

19. There are 3 feet in a yard: how many yards are there in 75 feet?

20. If $63 is paid for the pasturage of 9 cows, what is paid for the pasturage of one cow?

21. How many tons of hay at $10 a ton can be bought for $60?

22. The traveling expenses of an agent were $66 for 11 days: how much were his expenses per day?

23. How long will it take a steamer to make a trip of 48 miles, if it runs 12 miles an hour?

24. A laborer earned $77 in 11 weeks: how much did he earn a week?

25. A certain school containing 72 pupils is divided into 12 equal classes: how many pupils are there in each class?

26. Hampden paid 40 cents for 4 quires of paper: how much did each quire cost?

27. How many quarts of milk at 8 cents a quart can be bought for 24 cents?

28. If it takes 10 yards of calico to make Mabel a dress, how many dresses can be made from 50 yards?

29. If wood is worth $5 a cord, how many cords can be bought for $45?

30. Robert received 88 cents for 11 quarts of blackberries: how much did he receive per quart?

31. There are 3 miles in one league: how many leagues in 27 miles?

DEFINITIONS.

ART. 54.—Division is the process of finding how many times one number is contained in another number.

ART. 55.—The **Dividend** is the number to be divided.

ART. 56.—The **Divisor** is the number by which we divide.

ART. 57.—The **Quotient** is the number showing how many times the divisor is contained in the dividend.

ART. 58.—The **Remainder** is what is left of the dividend when the division is not exact.

The divisor and quotient are factors of the dividend.

ART. 59.—The Sign of Division ($\div$) is read *divided by*. When placed between two numbers, it shows that the number on the left is to be divided by the number on the right. Thus, $12 \div 3$ is read 12 *divided by* 3. It may also be written $\frac{12}{3}$ and 3)12.

ART. 60.—Division is the reverse of multiplication.

ART. 61.—Principle.—*The dividend equals the product of the divisor and quotient, plus the remainder.*

ORAL EXERCISES.

ART. 62.—1. There are 6 lemons to be divided equally between 2 boys: how many lemons should each boy receive?

Solution.—Since 6 lemons are to be equally divided between 2 boys, each boy should receive $\frac{1}{2}$ of 6 lemons, or 3 lemons.

2. If 10 cents are divided equally between 2 boys, how many cents will each receive?

3. If 3 boys share 12 marbles equally, how many will each receive?

4. Four boys drive 16 cows to pasture, each driving the same number: how many does each drive?

5. Divide 24 cakes equally among 24 girls.

6. Jacob having 20 apples shared them equally with 3 of his companions: how many did each receive?

7. If 3 yards of ribbon cost 27 cents, what is the price per yard?

8. When oranges are 2 cents apiece, how many can be bought for 24 cents?

9. Mr. Linden divided 36 cents equally among his 6 children: how much did each receive?

10. Melton rode 60 miles on his bicycle in 3 days: how many miles did he travel each day?

11. Maggie solved 36 problems in 4 days: how many did she solve each day?

12. Mary practiced on a piano 30 hours in 5 days: how many hours did she practice a day?

13. The dividend is 64 and the divisor 8: what is the quotient?

14. The divisor is 4 and the quotient 9: what is the dividend?

15. The quotient is 7 and the divisor 6: what is the dividend?

16. The divisor is 6 and the dividend 66: what is the quotient?

17. The dividend is 72 and the quotient 9: what is the divisor?

18. The quotient is 9, the divisor 7 and the remainder 6: what is the dividend?

19. The divisor is 8, the quotient is 6 and the dividend is 53: what is the remainder?

20. The dividend is 79, the quotient is 8, and the remainder is 7: what is the divisor?

SLATE AND BLACKBOARD EXERCISES.

÷ 4	÷ 4	÷ 4	÷ 4	÷ 5	÷ 5	÷ 5	÷ 5	÷ 5
4 .	5 .	6 .	7 .	5 .	6 .	7 .	8 .	9 .
8 .	9 .	10 .	11 .	10 .	11 .	12 .	13 .	14 .
12 .	13 .	14 .	15 .	15 .	16 .	17 .	18 .	19 .
16 .	17 .	18 .	19 .	20 .	21 .	22 .	23 .	24 .
20 .	21 .	22 .	23 .	25 .	26 .	27 .	28 .	29 .
24 .	25 .	26 .	27 .	30 .	31 .	32 .	33 .	34 .
28 .	29 .	30 .	31 .	35 .	36 .	37 .	38 .	39 .
32 .	33 .	34 .	35 .	40 .	41 .	42 .	43 .	44 .
36 .	37 .	38 .	39 .	45 .	46 .	47 .	48 .	49 .

☞ The teacher or a pupil points to the dots on the right of the vertical lines, while the pupils in turn name results. Thus: If the dot on the right of 37 is pointed to, the pupil says 9 and 1 over; or, 7 and 2 over. This exercise is also well adapted to silent concert reviews. The teacher can extend it to the **6**'s, **7**'s, **8**'s and **9**'s.

ART. 63.—When the division is performed by writing results only, the process is called *short division.*

WRITTEN EXERCISES.

1. Divide 7389 by 9.

Process.

9)7389
821

Analysis.—We write the divisor on the left of the dividend with a line between them, and begin to divide at the left. Nine is not contained in the thousands' order of the dividend, hence we annex the 3 hundreds. Nine is contained in 73 hundreds 8 (hundreds) times with 1 hundred remainder. We write 8 underneath the dividend in the hundreds' place of the quotient. The one hundred remainder = 10 tens, and 8 tens added = 18 tens. Nine is contained in 18 tens 2 (tens) times. We write 2 in the tens' place of the quotient. Nine is contained in 9 units 1 time. We write 1 in the units' place of the quotient.

Therefore, 7389 ÷ 9 = 821.

ART. 64.—Rule for Short Division.—*Write the divisor at the left of the dividend with a line between them.*

Beginning at the left, divide each order of the dividend by the divisor, writing the proper quotient figure underneath. If there be a remainder, change it to the next lower order, add to it the number of that order in the dividend, and divide as before.

When there is a final remainder, write the divisor beneath it with a line between and annex it to the quotient.

Proof.—*Multiply the divisor by the quotient and add the remainder, if any. If the sum equals the dividend, the answer is correct.*

2. There are 12 eggs in a dozen: how many dozen are there in 48 eggs? In 60 eggs? In 72 eggs? In 96 eggs? In 144 eggs? In 288 eggs?

3. If a man earn $132 in 11 weeks, how much does he earn in 1 week? In 7 weeks?

4. If my horses eat 6 bushels of corn a day, in how many days will they eat 126 bushels?

5. A train of cars runs 560 miles in 10 hours: what is its average run an hour?

6. If a ton of coal costs $6, how many tons can you buy for $1,344? For $2,400?

7. If 9 imported Normandy horses cost $5,625, how much does one horse cost?

8. A peach orchard contains 3,114 trees in 9 equal rows: how many trees in a row?

9. If a steamer takes 11 days to force its way through 605 miles of ice, how many miles does it average a day?

10.
2)1472583698527419

11.
3)8527419638479026

12.
4)2583691478964587

13.
5)6842135704596842

14.
6)7951348627697043

15.
7)6987452314780694

16.
8)4689012684389659

17.
9)7846590181246531

REMARK.—Heretofore we have divided by the digits only. The pupil should now be trained to divide by 10, 11 and 12, regarded as single numbers.

18.
10)6846378197206431

19.
10)7684613469712078

20.
11)7643218754320187

21.
11)6284507931428967

22.
12)9487653812084726

23.
12)8432160847502689

24.
12)4689216846987203

25.
12)71648297342681654

ART. 65.—When the division is performed by writing the steps in the solution, the process is called *Long Division.*

WRITTEN EXERCISES.

ART. 66.—1. Divide 3181 by 24.

Process.

$$\begin{array}{r} 24)3181(132\tfrac{13}{24}. \\ 24 \\ \hline 78 \\ 72 \\ \hline 61 \\ 48 \\ \hline 13 \text{ rem.} \end{array}$$

Analysis.—We write the divisor at the left of the dividend with a line between them and a line on the right for the quotient. Twenty-four is contained in 31 hundreds, 1 (hundred) time. We write 1 in the hundreds' place of the quotient, multiply the divisor 24 by the quotient figure 1 (hundreds) and place the product 24 hundreds underneath the partial dividend 31 hundreds. Subtracting the product from the partial dividend gives a remainder of 7 hundreds. To this remainder we annex 8 tens of the dividend. Twenty-four is contained in 78 tens, 3 (tens) times. Writing 3 in the tens' place of the quotient, we multiply the divisor 24 by 3 (tens) and subtract the product 72 tens from the partial dividend, and to the remainder annex the units' figure of the dividend. Twenty-four is contained in 61 units 2 times with a remainder of 13. We express the remainder in the quotient by writing it over the divisor and annexing it to the quotient. Therefore, $3181 \div 24 = 132\frac{13}{24}$.

ART. 67.—Rule for Long Division.—*Write the divisor on the left of the dividend, with a line between them.*

Find how many times the divisor is contained in the fewest left hand orders of the dividend which will contain it, and write that number for the first figure of the quotient.

Multiply the divisor by this quotient figure, subtract the product from the orders divided, and annex to the remainder the next figure of the dividend. Divide as before until all the orders of the dividend are used.

If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.

Write the final remainder over the divisor and annex it to the quotient.

Proof.—*The proof is the same as in short division.*

NOTE.—Sometimes, when the divisor is large, it is difficult for the pupil to know at once the correct quotient figure. In such cases, he should place in the quotient the figure which seems to be the right one, and test it. If after multiplying the divisor by it, and subtracting from the dividend, he finds the remainder greater than the divisor, he will know that his quotient figure is too small. The most common trouble, however, comes from using too great a quotient figure. The moment the pupil has multiplied the divisor, he should see whether he has made such an error. In every case the remainder must be *less* than the divisor.

2. A yard is 36 inches in length: how many yards in 864 inches? In 936 inches? In 1,008 inches?

3. There are 24 hours in one day: how many days in 828 hours? In 852 hours? In 936 hours?

4. A barrel of beef contains 200 pounds: how many barrels in 164,000 pounds? In 200,000 pounds?

5. How many city lots may be bought for $248,000, when the price is $2,400 each? $3,100 each?

6. There are 60 seconds in one minute: how many minutes in 3,960 seconds? In 4,200 seconds?

7. How many pounds of beef at 26 cents a pound can be bought for 598 cents? For 1,300 cents?

8. At $125 apiece, how many horses can be bought for $24,375? For $25,000?

9. There are 480 sheets in one ream of paper: how many reams in 147,840 sheets?

10. If a man walk 29 miles a day, how long will it take him to walk 1,972 miles?

Solve the following problems and prove each:

11. 4466678 ÷ 16.
12. 2852892 ÷ 17.
13. 4568763 ÷ 18.
14. 6462487 ÷ 19.
15. 7286962 ÷ 22.
16. 5345873 ÷ 24.
17. 2988649 ÷ 26.
18. 6945872 ÷ 28.
19. 32618963 ÷ 32.
20. 48460497 ÷ 34.
21. 71863000 ÷ 4800.
22. 72054489 ÷ 52.

23. 80968462 ÷ 64.	**32.** 832706644 ÷ 234.
24. 91019875 ÷ 62.	**33.** 513010892 ÷ 472.
25. 101011563 ÷ 74.	**34.** 731814743 ÷ 416.
26. 606914972 ÷ 83.	**35.** 593419687 ÷ 292.
27. 713824879 ÷ 72.	**36.** 381013891 ÷ 463.
28. 923104487 ÷ 88.	**37.** 410305976 ÷ 624.
29. 856493689 ÷ 90.	**38.** 934016784 ÷ 712.
30. 610518742 ÷ 124.	**39.** 599890000 ÷ 3100.
31. 419362847 ÷ 136.	**40.** 471616470 ÷ 198.

Review Problems.

ART. 68.—1. A man earns $30 a week, spends $18, and saves the remainder: in how many weeks can he save $288?

2. I sold 5 pounds of butter at 30 cents a pound and 4 dozen eggs at 31 cents a dozen. I took my pay in muslin at 16 cents a yard: how many yards did I receive?

3. A woman bought a sewing machine for $64 and agreed to pay for it in installments of $4 a month: how long did it take her to pay for the machine?

4. A farmer sold 20 pigs and 13 cows for $490: if the pigs were worth $5 apiece, how much was each cow worth?

5. Mr. Brown sold 5 horses at $175 apiece and Mr. Jones sold 7 horses at $210 apiece. Mr. Brown had paid $900 for his horses and Mr. Jones $1,680 for his: how many dollars did each gain or lose?

6. Mr. Jenkins has 3 sons at labor; the eldest receives $28 each month and spends $14; the second receives $22 a month and spends $11, and the youngest receives $19 and spends $9: what are the total savings of the 3 each month?

7. If a train of cars averages 550 passengers a day for

one week and 580 passengers a day for 2 weeks, what is the average number of passengers carried a day for the 3 weeks?

8. A man bought a farm containing 120 acres at $80 an acre and sold it for $11,040: what did he gain per acre?

9. A farmer sold 260 dozen eggs at the rate of 6 dozen for 108 cents: what amount did he receive?

10. The distance from San Francisco to Chicago is 2,448 miles: how long would it take to travel the distance on a bicycle, at 48 miles a day?

11. What is the value of $(48,800 + 2,400) \div 128$?

12. What is the value of $(36,632 + 21,380 - 16,000) \div 384$? Of $(25,245 + 13,240 - 11,015) \div 148$?

13. What is the value of $(38,824 + 16,456 - 12,382 + 124,000) \div 482$?

14. What is the value of $(16,400 + 12,385 - 10,000) \times 5 \div 648$?

15. If 63 men lay 24,680 bricks in one day, how many bricks should 189 men lay in the same time?

16. If 12 yards of cloth are worth $72, how many yards must be given for 18 cords of wood worth $3 a cord?

17. A customer bought 5 pounds of sugar worth 7 cents a pound and 8 pounds worth 8 cents a pound, and paid for it with raspberries worth 11 cents a quart: how many quarts did it take to pay for the sugar?

18. How many days' work at $3 a day will pay for 6 weeks' board at $4 per week, and a barrel of flour worth $9?

19. A farmer sold 24 head of cattle at $48 a head, and 3 horses at $120 each, and then bought 28 sheep at $6 a head: how much money had he left?

20. A man bought a farm containing 160 acres at $45 an acre, and another farm containing 130 acres at $60 an acre, and then sold both farms for $17,500: how much did he gain?

21. A man receives a salary of $1800 a year; he pays $15 a month rent, $12 a month for help, $40 a month for provisions, and his other expenses amount to $650 each year: how much has he left at the close of the year?

22. A grocer bought 175 bushels of potatoes at 75 cents a bushel and 325 bushels of corn at 50 cents a bushel. He paid 5 cents a bushel for freight on the potatoes and 6 cents a bushel for freight on the corn: how much did the corn and potatoes cost him?

23. A farmer bought 15 pounds of sugar at 6 cents a pound, 11 pounds at 7 cents a pound, and gave in part payment 13 pumpkins at 8 cents apiece: how much did he still owe?

24. If a man lay 48 feet of wall in 6 days, how many feet can he lay in 16 days?

25. If 72 yards of cloth will make 8 suits, how many yards will make 9 suits?

26. How many barrels of flour worth $9 a barrel will it take to pay for 36 yards of broadcloth worth $4 a yard?

27. If 11 men earn $154 in one week, how much can 7 men earn in the same time?

28. A man divided 63 cents equally among 9 boys: how much did 7 of the boys together receive?

29. Illustrate by an original problem addition and subtraction of numbers.

30. Illustrate by an original problem multiplication and division of numbers.

31. Illustrate by an original problem addition, subtraction, multiplication, and division of numbers.

Properties of Numbers.

DEFINITIONS.

ART. 69.—An **Integer** is a whole number; as 1, 5, 6, 11.

Integers are either *Even* or *Odd.*

ART. 70.—An **Even Number** is a number that is exactly divisible by 2; as 4, 6, 8, 10, 24.

ART. 71.—An **Odd Number** is a number that is not exactly divisible by 2; as 3, 5, 7, 9, 17.

Numbers are either *Composite* or *Prime.*

ART. 72.—A **Composite Number** is one that is produced by multiplying together two or more numbers each greater than 1; as 6, 9, 10, 12, 15, 18, 20.

ART. 73.—A **Prime Number** is one that cannot be produced by multiplying together two numbers each greater than 1; as 5, 7, 11, 13, 17, 19, 23.

Numbers are prime to each other when no number greater than 1 exactly divides them; as 5, 7; 4 and 9.

ART. 74.—A **Prime Factor** is a factor that is a prime number; thus, 2, 3, and 5 are the prime factors of 30.

ART. 75.—A **Common Divisor** of two or more numbers is any number greater than 1 that exactly divides each of them; thus, 6 is a common divisor of 12, 18, 24; and 3 is a common divisor of 9, 15, 24, 36.

ART. 76.—The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them; thus, 8 is the greatest common divisor of 16, 24, and 32; and 12 is the greatest common divisor of 36, 48, 60, 72.

ART. 77.—A **Multiple** of a number is any number of

times that number; thus, 6, 9 and 12 are multiples of 3.

A **Common Multiple** of two or more given numbers is any number exactly divisible by each of them; thus, 24 is a common multiple of 4, 6, 8 and 12.

ART. 78.—The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each of them; thus, 20 is the least common multiple of 2, 4, 5, and 10.

Factors.

ORAL EXERCISES.

ART. 79.—1. What are the factors of 6?

Solution.—The factors of 6 are 2 and 3, because 2 times 3 is 6.

2. What are the factors of 8? 9? 10? 12?
3. What are the factors of 14? 18? 21? 24?
4. What are the factors of 22? 28? 30? 32?
5. What are the factors of 38? 40? 42? 44?
6. What are the factors of 49? 50? 55? 60?
7. What are the prime factors of 20?

Solution.—The prime factors of 20 are 2, 2, and 5, because 2, 2, and 5 are the only prime factors whose product is 20.

8. What are the prime factors of 16? 18? 25?
9. What are the prime factors of 21? 24? 28?
10. What are the prime factors of 44? 46? 49?
11. What are the prime factors of 72? 75? 78?
12. Of what number are 3, 3, and 3 the prime factors? 2, 2, 2, and 2? 3, 5, and 2? 5, 3, and 7?
13. What are the prime factors of 30? 32? 34?
14. What are the prime factors of 36? 40? 45?
15. What are the prime factors of 54? 64? 66?
16. What are the prime factors of 80? 84? 96?

WRITTEN EXERCISES.

ART. 80.—1. What are the prime factors of 420?

Process.

$$\begin{array}{r|l} 2 & 420 \\ 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ & 7 \end{array}$$

Analysis.—Dividing 420 by the prime number 2 gives 210 for a quotient; dividing again by 2, gives 105; dividing this by the prime number 3, gives 35; dividing this by the prime number 5, gives the prime number 7 as the quotient. Since we have divided only by prime numbers, and since our last quotient is a prime number, it is evident that the several divisors and the last quotient must form the prime factors of 420.

Therefore, 2, 2, 3, 5 and 7 are the prime factors of 420.

ART. 81.—Rule for finding the Prime Factors of a Number. *Divide the number by the least prime number that will divide it; divide the quotient thus obtained by the least prime number that will divide it, and continue the process until the quotient is a prime number; the several divisors and the last quotient are the prime factors required.*

2. What are the prime factors of 50? 60? 98? 99?

3. What are the prime factors of 33? 288? 144?

4. What are the prime factors of 360? 384? 424?

5. What are the prime factors of 250? 280? 300?

6. Name four numbers and find their prime factors.

Divisors.

ORAL EXERCISES.

ART. 82.—1. Find the common divisor of 8 and 10.

Solution.—Two is a common divisor of 8 and 10, because it is exactly contained in each of them.

2. Find the common divisors of 8 and 12; 9 and 18.

3. Find the common divisors of 18 and 24; 20 and 30.

4. Find the common divisors of 32 and 48; 30 and 50.

5. Find the greatest common divisor of 9 and 12; 8 and 14; 15 and 20; 16 and 24.

6. Find the greatest common divisor of 7 and 14; 10 and 15; 14 and 21; 12 and 20; 18 and 30; 36 and 48.

WRITTEN EXERCISES.

ART. 83.—1. Find the greatest common divisor of 12, 16, and 24.

Process.

$12 = 2 \times 2 \times 3$
$16 = 2 \times 2 \times 2 \times 2$
$24 = 2 \times 2 \times 2 \times 3$

Analysis.—By inspecting the prime factors of which 12, 16, and 24 are composed, we observe that 2 is the only prime factor common to all the numbers, and that it occurs twice as a factor of each number.

Therefore, 2×2, or 4, must be the greatest common divisor of 12, 16, and 24.

ART. 84.—Rule for finding the Greatest Common Divisor. *Resolve each number into its prime factors; the product of the prime factors common to all the numbers is the greatest common divisor.*

2. Find the greatest common divisor of 10 and 14; 8 and 30; 26 and 39; 25 and 45.

3. Find the greatest common divisor of 6, 8, and 10; 5, 10, and 15; 4, 8, and 12.

4. Find the greatest common divisor of 6, 16, and 36; 14, 21, and 42; 9, 12, and 27.

5. Find the greatest common divisor of 2, 6, and 8; 3, 9, and 12; 16, 20, and 24.

6. Give four numbers whose greatest common divisor is 8.

Multiples.

ORAL EXERCISES.

ART. 85.—1. Of what numbers is 12 a multiple? 15? 18? 20? 25? 28? 30? 45? 50? 60?

2. Give 3 multiples of 5; of 6; of 7; of 9; of 10; of 11; of 4; of 3; of 8; of 12.

3. Name a common multiple of 2 and 3; 3 and 4; 5 and 6; 5 and 8: 6 and 7; 4 and 10.

4. Name a common multiple of 3, 4, and 5; 2, 3, and 4; 4, 5, and 6; 4, 6, and 8.

5. Name the least common multiple of 3, 4, and 5; 2, 3, and 5; 3, 5, and 6; 3, 7, and 8.

WRITTEN EXERCISES.

ART. 86.—1. Find the least common multiple of 12, 16, and 24.

Process.

2)12, 16, 24
2) 6, 8, 12
2) 3, 4, 6
3) 3, 2, 3
1, 2, 1

$2\times2\times2\times3\times2=48$

Analysis.—Since 2 is a prime factor of each of the numbers, it is also a factor of the least common multiple. After dividing, there remain as the other factors of the numbers 6, 8 and 12. 2 is a prime factor of these numbers and hence is a factor of the least common multiple. Dividing by 2, there remain 3, 4 and 6. 2 is a prime factor of 4 and 6, and is therefore a prime factor of the least common multiple. Dividing by 2, there remain 3, 2 and 3. 3 is a prime factor of 3 and 3, and hence is another factor of the least common multiple. Dividing by 3, there remain 1, 2 and 1, which are prime to each other. Hence, the product of the factors 2, 2, 2, 3 and 2, is the least common multiple.

ART. 87.—Rule for finding the Least Common Multiple.—*Write the numbers in a horizontal line. Divide the given numbers by any prime number that is an exact divisor of two or more of them, and write the quotients and undivided numbers in a line beneath.*

Continue the division until the quotients and undivided numbers are prime to each other. The product of the divisors and the numbers in the last horizontal line is the least common multiple required.

NOTE.—When one of the numbers is an exact divisor of one of the others, it may be disregarded in finding the least common multiple.

2. Find the least common multiple of 12, 24, 42.

3. Find the least common multiple of 27, 54, 81.

4. Find the least common multiple of 25, 50, 75.

5. Find the least common multiple or 20, 30, 40.

6. Find the least common multiple of 36, 48, 60.

7. Find the least common multiple of 36, 40, 48.

8. Find the least common multiple of $50, $60, $75.

9. What is the least common multiple of 5, 10, 15, 25?

10. What is the least common multiple of 3, 4, 5, 8?

11. What is the least common multiple of 6, 8, 10, 12?

12. Find the least common multiple of 3, 6, 8, 10.

13. Find the least common multiple of 15, 20, 25, 30.

14. Find the least common multiple of 20, 24, 40, 50.

15. Find the least common multiple of 25, 30, 35, 40.

16. Find the least common multiple of 30, 35, 45, 50.

17. Find the least common multiple of 40, 48, 60, 64.

18. Find the least common multiple of 10, 24, 36, 44.

19. Find the least common multiple of 1, 2, 3, 4, 5, 6.

20. Name five numbers and find their least common multiple.

Short Methods.

CANCELLATION.

ART. 88.—1. How many times is 2 times 3 contained in 4 times 3? 3 times 5 in 6 times 5? 3 times any number in 6 times that number?

2. How often is 3 times 8 contained in 9 times 8? 5 times 10 in 10 times 10? 5 times any number in 10 times that number?

3. How often is 6 times 7 contained in 12 times 7? 6 times 9 in 18 times 9? 6 times any number in 18 times that number?

4. What is the quotient of $(12 \times 15) \div (4 \times 15)$? Of $(72 \times 8) \div (12 \times 8)$?

5. What is the quotient of $(18 \times 23) \div (9 \times 23)$? Of $(49 \times 27) \div (7 \times 27)$?

6. What is the quotient of $(10 \times 12) \div (5 \times 12)$? Of $(12 \times 3) \div (4 \times 3)$?

7. What is the quotient of $(120 \times 19) \div (12 \times 19)$? Of $(84 \times 7) \div (12 \times 7)$?

8. What is the quotient of $(121 \times 37) \div (11 \times 37)$? Of $(144 \times 40) \div (12 \times 40)$?

9. In finding the quotient what numbers may be omitted from the dividend and divisor?

10. Why does such omission not affect the value of the quotient?

ART. 89.—Cancellation is the process of shortening arithmetical operations by striking out equal factors from the dividend and the divisor.

ART. 90.—Principle.—*Dividing both dividend and divisor by the same number does not affect the quotient.*

WRITTEN EXERCISES.

ART. 91.—1. Divide $3 \times 5 \times 6 \times 10$ by $3 \times 8 \times 4 \times 6$.

Process.

$$\frac{\cancel{3} \times 5 \times \cancel{6} \times \cancel{10}^{5}}{\cancel{3} \times 8 \times \cancel{4} \times \cancel{6}_{2}} = \frac{25}{16} = 1\frac{9}{16}$$

Analysis.—The divisor is written under the dividend with a line between.

Since the factors 3 and 6 are common to both dividend and divisor, we strike them from both. Since the factor 2 is a factor of 10 in the dividend and of 4 in the divisor, we strike it from both, leaving 5 in the dividend and 2 in the divisor. The product of the remaining factors in the dividend is 25, and of those in the divisor 16. Hence, the quotient is $\frac{25}{16} = 1\frac{9}{16}$.

ART. 92.—Rule for Cancellation.—*Cancel from the dividend and the divisor all common factors, and divide the product of the remaining factors in the dividend by the product of the remaining factors in the divisor.*

NOTE.—Should all the factors in the dividend and divisor be canceled, the quotient is 1. If the 1 appears in the dividend it must be retained; if in the divisor, it may be disregarded.

What is the quotient of:

2. $\frac{24 \times 6 \times 12 \times 14}{18 \times 10 \times 8 \times 6}$?

3. $\frac{32 \times 8 \times 18 \times 20}{4 \times 16 \times 15 \times 18}$?

4. $\frac{20 \times 4 \times 16 \times 15}{10 \times 5 \times 8 \times 40}$?

5. $\frac{28 \times 9 \times 14 \times 30}{18 \times 20 \times 12 \times 16}$?

6. $\frac{25 \times 20 \times 10 \times 36}{15 \times 30 \times 35 \times 40}$?

7. $\frac{35 \times 15 \times 25 \times 20}{5 \times 25 \times 24 \times 30}$?

8. $\frac{40 \times 14 \times 18 \times 35}{10 \times 21 \times 36 \times 70}$?

9. $\frac{45 \times 18 \times 16 \times 15}{9 \times 54 \times 48 \times 90}$?

10. $\frac{68 \times 16 \times 19 \times 17}{17 \times 16 \times 57 \times 85}$?

11. $\frac{66 \times 13 \times 28 \times 63}{11 \times 39 \times 21 \times 126}$?

12. $\frac{88 \times 22 \times 44 \times 60}{8 \times 66 \times 55 \times 120}$?

13. A dealer bought 10 barrels of potatoes at $3 a barrel and paid for them with shoes at $3 a pair: how many pairs of shoes did it require?

14. How many tons of coal worth $6 a ton must be given for 12 barrels of flour worth $7 a barrel?

15. A man exchanged 24 sheep worth $4 each for pigs at $6 each: how many pigs did he get?

16. Illustrate Cancellation by an original problem.

ART. 93.—1. Multiply 24 by 10; by 100; by 1000.

It is evident that when any number is multiplied by 10, it is moved one order to the left; when multiplied by 100, it is moved two orders to the left, and when multiplied by 1000, it is moved three orders to the left.

$24 \times 10 = 240$; $24 \times 100 = 2400$; $24 \times 1000 = 24000$.

Therefore, *To multiply any number by* 10, 100, 1000, *etc., annex as many ciphers to the number as there are ciphers in the multiplier.*

Such being the law, it follows that to *divide* any number by 10, 100, 1000, etc., we cut off from the right of the number as many figures as there are ciphers in the divisor. Thus, $24000 \div 10 = 2400$; $24000 \div 100 = 240$; $24000 \div 1000 = 24$, and $17864 \div 1000 = 17$ with 864 remaining; $17864 \div 100 = 178$ with 64 remaining; $17864 \div 10 = 1786$ with 4 remaining.

2. Multiply 396 by 10; by 100; by 1000.

3. Divide 47,232 by 1000; by 100; by 10.

4. Multiply 78 by 1000; by 100; by 10.

5. Divide 15,600 by 10; by 1000; by 100.

6. A man divided 1000 cents among 10 poor children: how much did each receive?

7. How much will 94 quarts of blackberries cost at 10 cents a quart?

8. Illustrate by an original problem the process of multiplying a number by 10, 100, and 1000.

9. Illustrate by an original problem the process of dividing a number by 10, 100, and 1000.

REVIEW QUESTIONS.

What is addition? What is the sum or amount? What is the sign of addition? What does it show? What is the sign of equality, and what does it show? What is the principle of addition?

Give the rule for addition, and tell how the work is proved.

What is subtraction? What is the minuend? The subtrahend? What is the difference? What is the sign of subtraction? What does it show?

Give the rule for subtraction, and tell how the work is proved. What is the first principle of subtraction? The second principle?

What is multiplication? What is the multiplicand? The multiplier? The product? What are the factors of a number? What are the factors of the product? What is the sign of multiplication? What does it mean? What is the principle of multiplication?

Give the rule for multiplication. How do we multiply when either or both the terms end with ciphers?

What is division? What is the dividend? The divisor? The quotient? The remainder? What are factors of the dividend? What is the sign of division? How is it read, and what does it mean? Of what is division the reverse? What is the principle of division?

When is the process called long division? Give the rule. What is the method of proving the work?

What is an integer? What is an even number? An odd number? A composite number? A prime number? The factors of a number? A prime factor?

What is a common divisor of two or more numbers? What is the greatest common divisor? A multiple of a number? A common multiple of two or more numbers? The least common multiple of two or more numbers?

Give the rule for finding the prime factors of a number; for finding the greatest common divisor; for finding the least common multiple.

What is cancellation? What is the principle of cancellation? Give the rule. How do you multiply any number by 10, 100, 1000, etc.? How do you divide any number by 10, 100, 1000, etc.?

PART II.

Common Fractions.

INDUCTIVE EXERCISES.

To Teachers.—The teacher cannot fail to see the importance of carefully explaining the nature of fractions. Before giving the pupils a definition or term to learn, and before requiring them to perform an operation on slate or blackboard, the fundamental facts concerning fractions should be made clear to them.

An apple, a block of wood, or some simple object should be prepared for use in illustration. It should be cut in halves, or thirds, or fourths, and the teacher will find it a pleasing occupation to manipulate these before his class in such a manner that every one will speedily understand the subject.

ART. 94.—A melon is cut exactly through the middle, thus making two equal parts; what is each part called?

If each of these parts be divided in like manner into two equal parts, how many parts will there be? What is each part called?

What are two of the parts called? What are three of the parts called? What figure shows the number of parts into which the melon was first divided?

If a pear is sliced into three equal divisions, what is each one called? Suppose that each division is cut into two equal parts, how many are there? What is each called?

How many halves are there in a melon? In a banana? In a dollar? In a foot? In a mile? In anything? How many thirds? How many fifths? How many fourths? How many sevenths? How many tenths? How many fifteenths? How many twentieths?

You will observe that in writing the fraction one half, I place the figure 1 above a short, horizontal line, and the figure 2 below it. The 2 below the line shows into

how many parts the apple, banana, dollar, or whatever we are talking about, is divided. It therefore gives the name or *denomination* to the fraction, and is called the *denominator*, as $\frac{1}{2}$.

The figure 1 above the line shows the number of the parts that we have taken, and is therefore called the *numerator*, as $\frac{1}{2}$.

In the following fractions, name the numerator and the denominator, and tell into how many parts the unit, or object we are considering, is divided, and how many of those parts are meant:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{7}$, $\frac{4}{5}$, $\frac{4}{7}$, $\frac{2}{9}$, $\frac{5}{6}$, $\frac{4}{8}$, $\frac{5}{11}$,

$\frac{6}{7}$, $\frac{6}{10}$, $\frac{7}{8}$, $\frac{7}{11}$, $\frac{8}{13}$, $\frac{9}{15}$, $\frac{10}{18}$, $\frac{11}{20}$, $\frac{13}{24}$, $\frac{13}{26}$, $\frac{15}{21}$,

$\frac{17}{22}$, $\frac{18}{23}$, $\frac{7}{26}$, $\frac{17}{20}$, $\frac{11}{11}$, $\frac{19}{13}$, $\frac{14}{23}$, $\frac{15}{60}$, $\frac{17}{63}$, $\frac{19}{29}$, $\frac{24}{31}$.

Write in figures the following fractions:

1. Five eighths; six sevenths; eight ninths.
2. Nine tenths; eleven thirteenths; one nineteenth.
3. Four thirds; eight eighths; twelve thirteenths.
4. Five fortieths; ten thirtieths; fifty hundredths.
5. Eighteen twenty-firsts; seven elevenths.
6. Ten fifteenths; six sixths; seven tenths.
7. Three sevenths; fourteen fourteenths.
8. Seventeen twenty-fourths; one thirteenth.
9. Ten thirteenths; twelve seventeenths.
10. Eight twenty-firsts; eleven twenty-sevenths.
11. Nineteen twenty-fourths; twenty-three thirtieths; twenty-seven seventy-fifths; one hundred and three hundredths.
12. Write three fractions and name the numerator and denominator and tell what they signify.

INDUCTIVE EXERCISES.

ART. 95.—Mrs. Perkins has 3 boys to each of whom she gives $\frac{1}{6}$ of a melon. How many sixths does she give to all? How many times $\frac{1}{6}$ of a melon in $\frac{3}{6}$ of a melon?

If I write $\frac{1}{6}$ on the board, and wish to multiply it by 3, how can it be done? Then, if I multiply the numerator of a fraction, what effect has it on the value of the fraction?

How many sixths are in anything? Three sixths are what part of anything?

We have seen that if the numerator of $\frac{1}{6}$ is multiplied by 3, the result is $\frac{3}{6}$ or $\frac{1}{2}$. Now, instead of multiplying the numerator of the fraction $\frac{1}{6}$ by 3, let us divide the denominator by 3. The result is $\frac{1}{2}$, which you perceive is the same as we obtained by multiplying the numerator by 3. We conclude, therefore, that *if the numerator of a fraction is multiplied by a number, the fraction itself is multiplied by that number; and also, if the denominator of a fraction is divided by a number, the fraction itself is multiplied by that number.*

How many sixths of a melon did Mrs. Perkins give to her 3 boys? Since she divided the $\frac{3}{6}$ of a melon among the 3 boys, how much did each boy receive? You observe, therefore, *that when the numerator of a fraction is divided by a number, the fraction itself is divided by that number.* Thus, $\frac{3}{6}^{\div 3} = \frac{1}{6}$.

Suppose that instead of dividing the numerator, as we have just done, we multiply the denominator of the fraction $\frac{3}{6}$ by 3. What is the result? How many eighteenths are there in anything? How many eighteenths in $\frac{1}{6}$ of anything? You perceive that $\frac{1}{6}$ and $\frac{3}{18}$ have the same value. It follows, therefore, that *if the denominator of a fraction be multiplied by any number, the fraction itself is divided by that number.* Thus, $\frac{1}{6_{\times 3}} = \frac{1}{18}$.

If I multiply the numerator and denominator of $\frac{1}{6}$ by

3, what is the result? But we have just learned that $\frac{1}{6}$ and $\frac{3}{18}$ have the same value. We conclude, therefore, that *it does not change the value of a fraction to multiply the numerator and denominator by the same number.* Thus, $\frac{1\times3}{6\times3}=\frac{3}{18}$.

We will now divide the numerator and denominator of $\frac{3}{18}$ by 3. What is the result? You will notice that the fractions $\frac{3}{18}$ and $\frac{1}{6}$ have the same value. We conclude, therefore, that *it does not change the value of a fraction to divide its numerator and denominator by the same number.* Thus, $\frac{3\div3}{18\div3}=\frac{1}{6}$.

ANALYSIS OF FRACTIONS.

ART. 96.—All fractions, as well as all numbers, are derived from the unit 1: hence, all numbers, whether integral or fractional, are referred in analysis to the number 1.

ART. 97.—The unit 1 is the measure of all numbers, hence, 1 is the unit of a fraction; one of the equal parts into which the unit is divided is a fractional unit; and more than one of the equal parts is a collection of fractional units.

ART. 98.—To analyze a fraction is to name the unit of the fraction; the fractional unit; the number of fractional units taken; the factors of the fraction; and the value of the fraction.

Thus, in analyzing the fraction $\frac{3}{4}$ we say: The unit of the fraction is 1; the fractional unit, $\frac{1}{4}$; the number of fractional units taken, 3; the factors of the fraction, $\frac{1}{4}$ and 3; the value of the fraction, $\frac{3}{4}$.

In analyzing the fraction $\frac{4}{5}$ we say: The unit of the fraction is 1; the fractional unit, $\frac{1}{5}$; the number of fractional units taken, 4; the factors of the fraction, $\frac{1}{5}$ and 4; the value of the fraction, $\frac{4}{5}$.

Analysis of the fraction $\frac{5}{7}$: The unit of the fraction is 1; fractional unit, $\frac{1}{7}$; number of fractional units, 5; factors of the fraction, $\frac{1}{7}$ and 5; value of the fraction, $\frac{5}{7}$.

Analyze, according to the models given, the following fractions:

1.	$\frac{3}{8}$,	$\frac{5}{7}$,	$\frac{4}{9}$.	6.	$\frac{1}{9}$,	$\frac{4}{11}$,	$\frac{1}{12}$.
2.	$\frac{5}{8}$,	$\frac{17}{24}$,	$\frac{16}{19}$.	7.	$\frac{7}{17}$,	$\frac{9}{19}$,	$\frac{13}{14}$.
3.	$\frac{3}{5}$,	$\frac{5}{6}$,	$\frac{9}{10}$.	8.	$\frac{3}{10}$,	$\frac{3}{7}$,	$\frac{4}{9}$.
4.	$\frac{7}{8}$,	$\frac{8}{9}$,	$\frac{2}{7}$.	9.	$\frac{2}{13}$,	$\frac{1}{2}$,	$\frac{3}{8}$.
5.	$\frac{5}{9}$,	$\frac{8}{10}$,	$\frac{7}{12}$.	10.	$\frac{5}{16}$,	$\frac{3}{19}$,	$\frac{5}{21}$.

PRINCIPLES AND DEFINITIONS.

ART. 99.—*Multiplying the numerator of a fraction by any number multiplies its value by that number,* because it increases the number of fractional units. Thus, $\frac{1}{2}^{\times 3} = \frac{3}{2}$; $\frac{2}{3}^{\times 4} = \frac{8}{3}$; $\frac{3}{4}^{\times 5} = \frac{15}{4}$.

ART. 100.—*Dividing the denominator of a fraction by any number multiplies its value by that number,* because it increases the size of the fractional unit. Thus, $\frac{3}{4_{+2}} = \frac{3}{2}$; $\frac{5}{6_{+2}} = \frac{5}{3}$; $\frac{5}{8_{+4}} = \frac{5}{2}$.

ART. 101.—*Multiplying the denominator of a fraction by any number divides the value of the fraction by that number,* because it decreases the size of the fractional unit. Thus, $\frac{2}{3_{\times 3}} = \frac{2}{9}$; $\frac{5}{7_{\times 2}} = \frac{5}{14}$; $\frac{7}{8_{\times 3}} = \frac{7}{24}$.

ART. 102.—*Dividing the numerator of a fraction by any number divides the value of the fraction by that number,* because it decreases the number of fractional units. Thus, $\frac{2}{3}^{+2} = \frac{1}{3}$; $\frac{3}{4}^{+3} = \frac{1}{4}$; $\frac{5}{6}^{+5} = \frac{1}{6}$.

ART. 103.—*Multiplying both numerator and denomina-*

tor of a fraction by the same number does not change its value, because the increase in the number of fractional units is equaled by the decrease in their size. Thus, $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$; $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$; $\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$.

ART. 104.—*Dividing both numerator and denominator of a fraction by the same number does not change its value,* because the decrease in the number of fractional units is equaled by the increase in their size. Thus, $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$; $\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$; $\frac{25}{30} \div \frac{5}{5} = \frac{5}{6}$.

ART. 105.—A **Fraction** is one or more of the equal parts of a unit. It is an expression of division.

ART. 106.—A fraction is expressed by two numbers, written one above a short horizontal line, and the other below it. It is a quotient.

ART. 107.—The **Denominator** of a fraction shows into how many equal parts the unit is divided, and is written below the line. It is the divisor.

ART. 108.—The **Numerator** of a fraction shows how many of the equal parts of the unit or number are taken, and is written above the line. It is the dividend.

ART. 109.—The **Terms of a Fraction** are the numerator and denominator.

ART. 110.—A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{13}{16}$.

ART. 111.—An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as, $\frac{5}{5}$, $\frac{7}{7}$, $\frac{8}{5}$, $\frac{11}{7}$.

ART. 112.—A **Simple Fraction** is one having a single integral numerator and denominator; as, $\frac{3}{7}$, $\frac{12}{17}$, $\frac{49}{63}$.

ART. 113.—A **Compound Fraction** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{4}{5}$; $\frac{3}{4}$ of $\frac{6}{7}$.

ART. 114.—A **Complex Fraction** is one whose nu-

merator or denominator or both are fractional; as, $\frac{\frac{3}{4}}{5}$, $\frac{7}{\frac{2}{3}}$, $\frac{\frac{2}{3}}{\frac{5}{6}\text{ of }\frac{7}{8}}$, $\frac{3\frac{1}{2}+2\frac{1}{4}}{6\div\frac{3}{4}}$.

ART. 115.—A Mixed Number is a whole number and fraction united; as, $2\frac{1}{2}$, $3\frac{1}{4}$, $7\frac{4}{5}$.

ART. 116.—To change Fractions to their Lowest Terms.

ORAL EXERCISES.

ART. 117.—A fraction is in its lowest terms or simplest form when no number greater than 1 will exactly divide its terms.

1. Change $\frac{6}{10}$ to its lowest terms.

Solution.—It is readily seen that 2 exactly divides both terms of the fraction. Dividing the terms by 2 we obtain $\frac{3}{5}$. We cannot divide the terms of $\frac{3}{5}$ by any number greater than 1; hence, $\frac{6}{10}$ changed to its lowest terms is $\frac{3}{5}$. Thus, $\frac{6}{10}\div\frac{2}{2}=\frac{3}{5}$.

Change to their lowest terms:

2. $\frac{3}{6}$, $\frac{4}{8}$, $\frac{4}{12}$.
3. $\frac{5}{15}$, $\frac{5}{20}$, $\frac{6}{18}$.
4. $\frac{8}{12}$, $\frac{3}{9}$, $\frac{7}{14}$.
5. $\frac{8}{20}$, $\frac{8}{40}$, $\frac{9}{27}$.
6. $\frac{7}{42}$, $\frac{8}{10}$, $\frac{10}{12}$.
7. $\frac{14}{16}$, $\frac{13}{39}$, $\frac{20}{30}$.
8. $\frac{26}{39}$, $\frac{28}{40}$, $\frac{30}{45}$.
9. $\frac{22}{24}$, $\frac{24}{48}$, $\frac{34}{40}$.
10. $\frac{12}{16}$, $\frac{20}{35}$, $\frac{50}{75}$.

ART. 118.—Rule for changing Fractions to their Lowest Terms.—1. *Find the greatest common divisor of the terms of the fraction, and* (2) *Divide both terms of the fraction by their greatest common divisor.*

WRITTEN EXERCISES.

1. $\frac{3}{81}$, $\frac{32}{40}$, $\frac{21}{72}$.
2. $\frac{38}{40}$, $\frac{54}{60}$, $\frac{60}{70}$.
3. $\frac{15}{48}$, $\frac{33}{88}$, $\frac{69}{81}$.
4. $\frac{160}{380}$, $\frac{164}{382}$, $\frac{185}{185}$.
5. $\frac{120}{200}$, $\frac{117}{351}$, $\frac{192}{224}$.
6. $\frac{42}{63}$, $\frac{46}{88}$, $\frac{36}{72}$.
7. $\frac{187}{289}$, $\frac{480}{732}$, $\frac{144}{1728}$.
8. $\frac{125}{275}$, $\frac{130}{300}$, $\frac{156}{240}$.
9. $\frac{320}{340}$, $\frac{235}{500}$, $\frac{275}{400}$.
10. $\frac{380}{570}$, $\frac{540}{624}$, $\frac{756}{846}$.

ART. 119.—To change Mixed Numbers to Improper Fractions.

ORAL EXERCISES.

1. How many thirds are there in $11\frac{2}{3}$?

Solution.—Since there are $\frac{1}{3}$ in 1 unit, in 11 units there are 11 times $\frac{1}{3}$ or $\frac{33}{3}$, and $\frac{2}{3}$ added make $\frac{35}{3}$. **Proof:** $35 \div 3 = 11\frac{2}{3}$.

Change the following mixed numbers to improper fractions:

2.	$3\frac{1}{2}$,	$5\frac{1}{3}$.	**7.**	$6\frac{3}{4}$,	$7\frac{2}{3}$.	**12.**	$10\frac{1}{2}$,	$9\frac{1}{3}$.
3.	$5\frac{2}{3}$,	$3\frac{4}{7}$.	**8.**	$1\frac{10}{11}$,	$4\frac{3}{7}$.	**13.**	$5\frac{7}{8}$,	$2\frac{8}{9}$.
4.	$6\frac{1}{7}$,	$2\frac{3}{4}$.	**9.**	$2\frac{5}{9}$,	$5\frac{6}{8}$.	**14.**	$3\frac{3}{7}$,	$8\frac{5}{8}$.
5.	$7\frac{1}{6}$,	$5\frac{2}{7}$.	**10.**	$6\frac{3}{5}$,	$3\frac{9}{10}$.	**15.**	$6\frac{7}{8}$,	$4\frac{9}{10}$.
6.	$4\frac{2}{5}$,	$6\frac{5}{6}$.	**11.**	$7\frac{2}{3}$,	$5\frac{7}{10}$.	**16.**	$7\frac{4}{7}$,	$9\frac{3}{8}$.

ART. 120.—Rule for changing Mixed Numbers to Improper Fractions.—*Multiply the integer by the denominator of the fraction, add the numerator to the product and write the sum over the given denominator.*

WRITTEN EXERCISES.

ART. 121.—Change the following mixed numbers to improper fractions:

1.	$11\frac{3}{7}$,	$9\frac{5}{12}$.	**7.**	$6\frac{7}{15}$,	$11\frac{14}{25}$.
2.	$10\frac{9}{11}$,	$12\frac{4}{7}$.	**8.**	$7\frac{3}{13}$,	$18\frac{8}{15}$.
3.	$9\frac{3}{15}$,	$11\frac{10}{11}$.	**9.**	$9\frac{16}{25}$,	$13\frac{4}{13}$.
4.	$7\frac{10}{12}$,	$10\frac{4}{15}$.	**10.**	$11\frac{17}{25}$,	$15\frac{11}{20}$.
5.	$12\frac{6}{20}$,	$9\frac{11}{12}$.	**11.**	$9\frac{1}{8}$,	$18\frac{18}{25}$.
6.	$8\frac{13}{25}$,	$6\frac{7}{20}$.	**12.**	$10\frac{1}{9}$,	$19\frac{2}{8}$.

13. $12\frac{5}{11}$, $20\frac{2}{3}$. **17.** $16\frac{13}{20}$, $24\frac{21}{25}$.

14. $15\frac{6}{13}$, $31\frac{6}{11}$. **18.** $19\frac{10}{13}$, $35\frac{24}{25}$.

15. $13\frac{11}{15}$, $22\frac{7}{13}$. **19.** $18\frac{17}{30}$, $26\frac{11}{13}$.

16. $17\frac{12}{20}$, $33\frac{12}{15}$. **20.** $41\frac{24}{25}$, $37\frac{5}{20}$.

ART. 122.—To change Improper Fractions to Whole or Mixed Numbers.

ORAL EXERCISES.

1. How many units in $\frac{11}{4}$?

Solution.—Since there are $\frac{1}{4}$ in 1 unit, $\frac{11}{4}$ is equal to as many units as $\frac{1}{4}$ is contained times in $\frac{11}{4}$, which is $2\frac{3}{4}$ times. **Proof:** $11 \div 4 = 2\frac{3}{4}$.

Change the following to whole or mixed numbers:

2. $\frac{7}{5}$, $\frac{6}{4}$, $\frac{9}{3}$. **6.** $\frac{10}{8}$, $\frac{13}{13}$, $\frac{15}{6}$. **10.** $\frac{18}{9}$, $\frac{17}{6}$, $\frac{12}{4}$.

3. $\frac{8}{4}$, $\frac{6}{3}$, $\frac{7}{4}$. **7.** $\frac{12}{5}$, $\frac{11}{11}$, $\frac{8}{4}$. **11.** $\frac{15}{4}$, $\frac{13}{10}$, $\frac{14}{7}$.

4. $\frac{9}{8}$, $\frac{7}{7}$, $\frac{8}{3}$. **8.** $\frac{20}{10}$, $\frac{18}{9}$, $\frac{19}{9}$. **12.** $\frac{14}{4}$, $\frac{13}{6}$, $\frac{17}{4}$.

5. $\frac{10}{8}$, $\frac{7}{6}$, $\frac{8}{7}$. **9.** $\frac{16}{5}$, $\frac{14}{3}$, $\frac{13}{5}$. **13.** $\frac{25}{6}$, $\frac{20}{4}$, $\frac{30}{6}$.

ART. 123.—Rule for changing an Improper Fraction to a Whole or Mixed Number.—*Divide the numerator of the fraction by the denominator. If there be a remainder, write the denominator under it and annex the resulting fraction in its simplest form to the quotient.*

WRITTEN EXERCISES.

1. $\frac{120}{19}$, $\frac{118}{17}$. **6.** $\frac{94}{47}$, $\frac{93}{18}$.

2. $\frac{113}{27}$, $\frac{110}{29}$. **7.** $\frac{48}{17}$, $\frac{125}{25}$.

3. $\frac{99}{41}$, $\frac{90}{45}$. **8.** $\frac{120}{33}$, $\frac{322}{108}$.

4. $\frac{51}{60}$, $\frac{30}{22}$. **9.** $\frac{647}{336}$, $\frac{303}{101}$.

5. $\frac{37}{30}$, $\frac{109}{71}$. **10.** $\frac{320}{2}$, $\frac{1728}{12}$.

ART. 124.—To change a Fraction or an Integer to an Equivalent Fraction having a Given Denominator.

ORAL EXERCISES.

1. How many eighths are there in $\frac{1}{2}$?

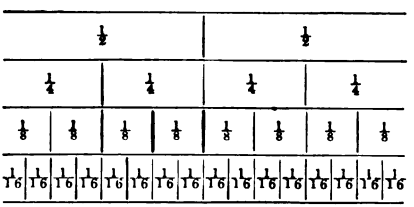
Solution.—Since there are $\frac{8}{8}$ in 1, in $\frac{1}{2}$ of 1 there are $\frac{1}{2}$ of $\frac{8}{8}$, or $\frac{4}{8}$.

2. How many halves are there in 1?

3. How many fourths in 1?

4. How many eighths in 1?

5. How many sixteenths in 1?



6. How many fourths in one half? How many eighths in one half? How many eighths in one fourth?

7. How many sixteenths in one eighth? In one fourth? In one half?

8. How many eighths in $\frac{3}{4}$? In $\frac{2}{4}$? In $\frac{4}{4}$?

WRITTEN EXERCISES.

ART. 125.—1. Change $\frac{1}{3}$ and $\frac{3}{5}$ to 15ths.

Process.

$\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$

$\frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$

Analysis.—It is evident that to change $\frac{1}{3}$ and $\frac{3}{5}$ to 15ths, their terms must be multiplied by the numbers which will change each denominator to 15. $\frac{1}{3}$ must be multiplied by $\frac{5}{5}$ and $\frac{3}{5}$ must be multiplied by $\frac{3}{3}$.

Rule for changing a Fraction to an Equivalent Fraction having a Given Denominator.—*Divide the given denominator by the denominator of the fraction and multiply both its terms by the quotient thus obtained.*

NOTE.—An integer is changed to an equivalent fraction having a given denominator by multiplying it by the denominator required and writing the product over the given denominator.

Change:

2.	$\frac{1}{2}$,	$\frac{2}{3}$,	to 12ths;	$\frac{1}{4}$,	$\frac{1}{5}$,	to	60ths.
3.	$\frac{1}{3}$,	$\frac{3}{4}$,	to 60ths;	$\frac{1}{6}$,	$\frac{1}{3}$,	to	36ths.
4.	$\frac{3}{5}$,	$\frac{3}{4}$,	to 20ths;	$\frac{3}{4}$,	$\frac{3}{5}$,	to	80ths.
5.	$\frac{5}{7}$,	$\frac{3}{14}$,	to 42ds;	$\frac{1}{5}$,	$\frac{3}{4}$,	to	120ths.
6.	$\frac{1}{6}$,	$\frac{2}{3}$,	to 12ths;	$\frac{3}{8}$,	$\frac{1}{3}$,	to	96ths.
7.	$\frac{2}{5}$,	$\frac{4}{7}$,	to 35ths;	$\frac{1}{4}$,	$\frac{1}{7}$,	to	28ths.

ART. 126.—To change Fractions to Equivalent Fractions having a Common or a Least Common Denominator.

WRITTEN EXERCISES.

1. Change $\frac{3}{4}$ and $\frac{5}{6}$ to equivalent fractions having a common denominator.

Process.

$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$

$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

Analysis.—The common denominator must be a common multiple of the denominators given. A multiple of 4 and 6 is their product, 24. Changing $\frac{3}{4}$ to 24ths by multiplying both terms by 6, we obtain the equivalent fraction $\frac{18}{24}$; changing $\frac{5}{6}$ to 24ths by multiplying both terms by 4, we obtain the equivalent fraction $\frac{20}{24}$.

Rule for Changing Fractions to Equivalent Fractions having a Common Denominator.—*Multiply both terms of each fraction by all the denominators except its own.*

Change to equivalent fractions having a common denominator:

2.	$\frac{2}{3}$,	$\frac{3}{4}$;	$\frac{2}{3}$,	$\frac{1}{5}$;	$\frac{2}{3}$,	$\frac{3}{4}$,	$\frac{1}{6}$.
3.	$\frac{1}{5}$,	$\frac{1}{6}$;	$\frac{1}{3}$,	$\frac{1}{9}$;	$\frac{3}{5}$,	$\frac{4}{7}$,	$\frac{1}{3}$.
4.	$\frac{1}{3}$,	$\frac{1}{4}$;	$\frac{2}{5}$,	$\frac{1}{6}$;	$\frac{5}{6}$,	$\frac{3}{4}$,	$\frac{1}{7}$.
5.	$\frac{3}{4}$,	$\frac{1}{3}$;	$\frac{2}{5}$,	$\frac{3}{7}$;	$\frac{5}{6}$,	$\frac{3}{7}$,	$\frac{1}{4}$.
6.	$\frac{1}{2}$,	$\frac{1}{4}$;	$\frac{2}{5}$,	$\frac{3}{7}$;	$\frac{8}{9}$,	$\frac{9}{10}$,	$\frac{10}{12}$.

ART. 127.—Since it is necessary to change fractions to equivalent fractions having a common denominator in order to add or subtract them, it is more convenient to have the common denominator as small as possible. Hence, we change the fractions to equivalent fractions with the *least common denominator.*

WRITTEN EXERCISES.

1. Change $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{10}$, to equivalent fractions with the least common denominator.

Process.

$\frac{2}{3} \times \frac{10}{10} = \frac{20}{30}$

$\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

$\frac{7}{10} \times \frac{3}{3} = \frac{21}{30}$

Analysis.—The least common denominator of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{10}$, is the least common multiple of 3, 5, and 10, or 30.

To change each fraction to 30ths, we divide 30 by each denominator in turn, and multiply both terms of each fraction by the quotient thus obtained.

Rule for finding the Least Common Denominator.—*Find the least common multiple of the denominators, divide this by each denominator in turn, and multiply both terms of each fraction by this quotient.*

Change the following to equivalent fractions having the least common denominator:

2. $\frac{3}{5}$, $\frac{4}{7}$; $\frac{5}{6}$, $\frac{3}{4}$.

3. $\frac{2}{3}$, $\frac{3}{6}$; $\frac{2}{3}$, $\frac{3}{8}$.

4. $\frac{3}{4}$, $\frac{5}{8}$; $\frac{5}{6}$, $\frac{5}{9}$.

5. $\frac{5}{6}$, $\frac{1}{9}$; $\frac{2}{7}$, $\frac{7}{8}$.

6. $\frac{6}{6}$, $\frac{5}{14}$; $\frac{2}{9}$, $\frac{4}{27}$.

7. $\frac{4}{5}$, $\frac{9}{15}$; $\frac{2}{3}$, $\frac{11}{21}$.

8. $\frac{7}{8}$, $\frac{5}{12}$; $\frac{4}{9}$, $\frac{1}{21}$.

9. $\frac{1}{3}$, $\frac{3}{8}$; $\frac{3}{5}$, $\frac{5}{12}$.

10. $\frac{1}{6}$, $\frac{5}{8}$; $\frac{11}{12}$, $\frac{1}{18}$.

11. $\frac{4}{5}$, $\frac{1}{12}$; $\frac{3}{4}$, $\frac{3}{20}$.

12. $\frac{3}{7}$, $\frac{2}{21}$; $\frac{5}{12}$, $\frac{4}{15}$.

13. $\frac{4}{5}$, $\frac{14}{15}$; $\frac{2}{3}$, $\frac{3}{7}$.

14. $\frac{3}{8}$, $\frac{5}{9}$; $\frac{5}{8}$, $\frac{3}{20}$.

15. $\frac{5}{6}$, $\frac{11}{40}$; $\frac{1}{10}$, $\frac{1}{12}$, $\frac{5}{16}$.

Addition of Fractions.

ORAL EXERCISES.

ART. 128.—1. George had $\frac{1}{4}$ of a dollar and his father gave him $\frac{2}{4}$ of a dollar: how much then had George?

2. Mary paid $\frac{1}{8}$ of a dollar for a doll and $\frac{2}{8}$ of a dollar for a ribbon: how much did she pay for both?

3. John spent $\frac{1}{3}$ of a dollar and James $\frac{2}{3}$ of a dollar: how much did both spend?

4. I paid $\frac{1}{2}$ of a dollar for a book and $\frac{1}{2}$ of a dollar for some postage stamps: how much did I expend?

5. Samuel gave $\frac{1}{8}$ of a bushel of oats to his horse and $\frac{3}{8}$ to his donkey: how much did he give to both?

6. Jennie bought two kinds of ribbon; for one kind she paid $\frac{1}{4}$ of a dollar, and for another $\frac{3}{4}$ of a dollar: how much did she pay for both?

7. If a chicken costs $\frac{1}{6}$ of a dollar and a duck $\frac{2}{3}$ of a dollar, how much do both cost?

8. If I sold $\frac{3}{8}$ of an acre of land to one man and $\frac{3}{10}$ of an acre to another, how much did I sell to both?

9. A farmer sold $\frac{1}{8}$ of his grain to one man and $\frac{2}{8}$ to another: what part of his grain did he sell?

10. Mr. Brown spent $\frac{1}{3}$ of his money at one time and $\frac{1}{4}$ at another time: what part did he spend?

11. A grocer sold $\frac{1}{3}$ of a dozen eggs to one man and $\frac{2}{6}$ of a dozen to another: what part of a dozen did he sell to both?

ART. 129.—Principle.—*Only like fractions can be added.*

12. A boy paid $\frac{1}{8}$ of a dollar for a reader and $\frac{1}{2}$ of a dollar for an arithmetic: what did he pay for both books?

Solution.—He paid the sum of $\frac{1}{5}$ of a dollar and $\frac{1}{2}$ of a dollar. $\frac{1}{5} = \frac{2}{10}$ and $\frac{1}{2} = \frac{5}{10}$; $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$: he paid $\frac{7}{10}$ of a dollar.

13. A slate cost $\frac{2}{5}$ of a dollar and a geography $\frac{3}{4}$ of a dollar: what was the cost of both?

14. William had $\frac{1}{4}$ of a dollar and his mother gave him $\frac{3}{10}$ of a dollar: how much had he then?

15. Susan had $\frac{1}{6}$ of a dozen of pins and Caroline had $\frac{3}{4}$ of a dozen: how many pins had both?

Find the sum of:

16. $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{2}$ and $\frac{1}{6}$; $\frac{1}{4}$ and $\frac{1}{6}$.

17. $\frac{2}{3}$ and $\frac{1}{4}$; $\frac{1}{6}$ and $\frac{1}{6}$; $\frac{2}{3}$ and $\frac{2}{4}$; $\frac{1}{5}$ and $\frac{3}{6}$.

18. $\frac{3}{4}$ and $\frac{1}{8}$; $\frac{1}{5}$ and $\frac{1}{10}$; $\frac{2}{5}$ and $\frac{2}{3}$; $\frac{1}{6}$ and $\frac{1}{2}$.

19. $\frac{1}{4}$ and $\frac{1}{6}$; $\frac{2}{5}$ and $\frac{1}{3}$; $\frac{2}{5}$ and $\frac{1}{6}$; $\frac{3}{4}$ and $\frac{1}{6}$.

20. $\frac{5}{6}$ and $\frac{1}{2}$; $\frac{1}{8}$ and $\frac{1}{6}$; $\frac{3}{5}$ and $\frac{1}{8}$; $\frac{4}{6}$ and $\frac{9}{10}$.

WRITTEN EXERCISES.

1. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$.

Process.

$\frac{2}{3} = \frac{20}{30}$

$\frac{3}{5} = \frac{18}{30}$

$\frac{5}{6} = \frac{25}{30}$

30

3 | $10 \times 2 = 20$

5 | $6 \times 3 = 18$

6 | $5 \times 5 = 25$

63

$\frac{20}{30} + \frac{18}{30} + \frac{25}{30} = \frac{63}{30} = 2\frac{3}{30} = 2\frac{1}{10}$

Analysis.—Since only like fractions can be added, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$ must be changed to equivalent fractions having a common denominator. $\frac{2}{3} = \frac{20}{30}$, $\frac{3}{5} = \frac{18}{30}$, $\frac{5}{6} = \frac{25}{30}$, and the sum of these $= 2\frac{1}{10}$.

Rule for Addition of Fractions.—*Change the fractions to equivalent fractions having a common denominator, add their numerators and write their sum over the common denominator. When there are mixed numbers or integers, add the whole numbers and fractions separately, and then add their sums.*

NOTE.—If the sum be an improper fraction, change it to a whole or mixed number, and fractional results to their lowest terms.

Find the sum of:

2. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

3. $\frac{5}{6} + \frac{3}{8} + \frac{3}{4}$.

4. $\frac{1}{5} + \frac{5}{6} + \frac{7}{10}$.

5. $\frac{3}{8} + \frac{1}{5} + \frac{3}{10}$.

6. $\frac{1}{4} + \frac{1}{2} + \frac{7}{15}$.

7. $\frac{2}{3} + \frac{1}{20} + \frac{9}{10}$.

8. $\frac{3}{4} + \frac{5}{6} + \frac{11}{12}$.

9. $\frac{4}{5} + \frac{5}{6} + \frac{7}{10}$.

10. $3\frac{2}{3} + 5\frac{1}{8} + 6\frac{3}{4}$; $\frac{7}{8} + \frac{6}{7} + \frac{3}{4} + 3\frac{4}{8}$.

11. $7\frac{2}{5} + 8\frac{1}{2} + 9\frac{3}{4}$; $10 + 5\frac{1}{4} + 3 + 2\frac{5}{6}$.

12. A farmer received $\$48\frac{3}{5}$ for a cow, and $\$184\frac{3}{4}$ for a horse: how much did he receive for both?

13. A dealer placed $158\frac{2}{3}$ tons of coal in a bin which already contained $119\frac{4}{5}$ tons: how many tons were in the bin?

14. A farmer sold $40\frac{2}{3}$ acres to one man, $109\frac{1}{5}$ to another, and $97\frac{3}{4}$ to a third: how many acres did he sell?

15. A horseman rode $48\frac{5}{6}$ miles the first day, $56\frac{2}{3}$ miles the second day, and $47\frac{1}{4}$ miles the third day: how far did he ride?

16. A man earned $\$63\frac{4}{5}$ in one month, $\$64\frac{5}{8}$ in another, and $\$59\frac{1}{10}$ in a third: how much did he earn in three months?

17. I own three piles of wood; the first contains $17\frac{2}{3}$ cords, the second $25\frac{5}{6}$ cords, and the third $37\frac{5}{8}$ cords: how many cords are there in all?

18. A tailor bought a roll of cloth containing $26\frac{3}{4}$ yards, another containing $57\frac{1}{8}$ yards, and another containing $101\frac{7}{10}$ yards: how many yards did he buy?

19. A steamer sailed $240\frac{5}{8}$ miles the first day, $321\frac{3}{8}$ miles the second day, $303\frac{3}{4}$ miles the third day, and

411$\frac{5}{11}$ miles the fourth day: how many miles did she sail in the 4 days?

20. A hat cost \$3$\frac{1}{2}$, a vest \$5$\frac{1}{4}$, a coat \$15$\frac{4}{5}$, and a pair of trousers \$7$\frac{1}{3}$: how much did they all cost?

21. A contractor delivered 15$\frac{5}{6}$ tons of coal the first day, 26$\frac{3}{4}$ tons the second day, 33$\frac{1}{3}$ tons the third day, 41$\frac{1}{5}$ tons the fourth day, and 21$\frac{7}{10}$ tons the fifth: how many tons did he deliver?

22. A clothier bought three pieces of cloth containing 132$\frac{3}{4}$ yards, 145$\frac{5}{8}$ yards, and 204$\frac{11}{16}$ yards: how many yards did he buy?

23. A railroad train went 215$\frac{3}{4}$ miles in one day, 310$\frac{3}{5}$ miles the second day, 300$\frac{4}{5}$ miles the third day: how far did it go?

24. One farm contains 170$\frac{3}{4}$ acres, another 203$\frac{4}{5}$ acres, and another 217$\frac{1}{3}$ acres: how many acres in the three farms?

25. A nurseryman's expenses were: wages paid to men, \$163$\frac{3}{4}$; wages paid to a boy, \$27$\frac{3}{5}$; cost of trees and plants, \$39$\frac{7}{10}$; amount paid for a horse, \$131$\frac{1}{2}$: what were the total expenses?

26. Mr. Brown paid \$1,324$\frac{1}{4}$ for the mason work on his barn; \$1,821$\frac{1}{5}$ for the carpenter work; and the hay and grain were worth \$320$\frac{7}{8}$. If the building and its contents are burned, what will be his loss?

27. A grocer has in his cellar 170$\frac{3}{4}$ gallons of molasses, 46$\frac{1}{8}$ gallons of syrup, 98$\frac{1}{2}$ gallons of vinegar, and 111$\frac{1}{16}$ gallons of kerosene: what is the total number of gallons?

28. Arthur's pedometer recorded as follows: First day, 23$\frac{3}{4}$ miles; second day, 24$\frac{3}{5}$ miles; third day, 27$\frac{1}{2}$ miles; fourth day, 26 miles; fifth day, 30$\frac{1}{3}$ miles: how many miles did Arthur walk?

29. Illustrate by an original problem addition of fractions.

Subtraction of Fractions.

ORAL EXERCISES.

ART. 130.—1. Peter had $\frac{3}{4}$ of a dollar and spent $\frac{1}{4}$ of a dollar: how much had he left?

2. Mary had $\frac{3}{8}$ of a quart of cherries and gave away $\frac{2}{8}$ of a quart: what part of a quart had she left?

3. A man owned $\frac{5}{8}$ of an acre of land and gave away $\frac{1}{8}$ of an acre: what part of an acre had he left?

4. Samuel walked $\frac{7}{8}$ of a mile, while Jacob walked $\frac{5}{8}$ of a mile: how much farther did Samuel walk than Jacob?

5. John having $\frac{4}{5}$ of a dollar, gave $\frac{1}{5}$ of a dollar for some sugar: how much had he left?

6. A man who owned $\frac{3}{3}$ of a ship, sold $\frac{2}{3}$ of the ship: what part did he then own?

7. Thomas gathered $\frac{1}{2}$ of a bushel of walnuts and sold $\frac{1}{4}$ of a bushel: what part of a bushel did he keep?

8. A girl paid $\frac{3}{10}$ of a dollar for a reader and $\frac{1}{5}$ of a dollar for a copy book: how much more did she pay for the reader than for the copy book?

9. If I have $\frac{11}{12}$ of a dozen of eggs, and break $\frac{1}{6}$ of a dozen, what part of a dozen will I have left?

10. A boy paid $\frac{1}{2}$ of a dollar for a bat and sold it for $\frac{1}{5}$ of a dollar: how much did he lose?

11. Jacob earns $\frac{5}{8}$ of a dollar a day and Joseph earns $\frac{1}{4}$ of a dollar: how much more does Jacob earn than Joseph?

ART. 131.—Principle.—*Only like fractions can be subtracted.*

12. Newton paid $\frac{1}{4}$ of a dollar for a reader and sold it for $\frac{1}{5}$ of a dollar, how much did he lose?

Solution.—He lost the difference between $\frac{1}{4}$ of a dollar and $\frac{1}{5}$ of a dollar. $\frac{1}{4} = \frac{5}{20}$ and $\frac{1}{5} = \frac{4}{20}$; $\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$. He lost $\frac{1}{20}$ of a dollar.

13. A man owning $\frac{2}{3}$ of a farm sold $\frac{1}{4}$ of the farm: what part of the farm did he still own?

14. A farmer sowed $\frac{9}{10}$ of an acre with wheat and $\frac{1}{3}$ of an acre with barley: how much more did he sow with wheat than with barley?

15. From $\frac{5}{6}$ of a barrel of sugar, a grocer sold $\frac{1}{4}$ of a barrel: what part of a barrel remained?

WRITTEN EXERCISES.

ART. 182.—1. From $\frac{7}{12}$ take $\frac{3}{16}$; $4\frac{1}{3}$ take $2\frac{1}{2}$.

1st Process.

$\frac{28}{48} - \frac{9}{48} = \frac{19}{48}$

Analysis.—Reduce both fractions to equivalent fractions having a common denominator.

2d Process.

$4\frac{1}{3} = 4\frac{2}{6}$

$2\frac{1}{2} = 2\frac{3}{6}$

Ans. $1\frac{5}{6}$

Analysis.—Reduce the fractional parts $\frac{1}{3}$ and $\frac{1}{2}$ to equivalent fractions having a common denominator. As $\frac{3}{6}$ cannot be taken from $\frac{2}{6}$, take $1 = \frac{6}{6}$ from the integral part of the minuend and add it to $\frac{2}{6}$. Then $\frac{3}{6}$ from $\frac{8}{6}$ leaves $\frac{5}{6}$, and 2 from 3 leaves 1.

Rule for Subtraction of Fractions.—1. *Change the fractions to equivalent fractions having a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and write the difference over the common denominator.*

2. *If there are mixed numbers and the numerator of the subtrahend is greater than the numerator of the minuend, take* 1 *from the integral part of the minuend, reduce it to the required fractional form, add it to the fractional part of the minuend, and proceed as in* 1 *above.*

From

2. $\frac{5}{8}$ take $\frac{5}{9}$.

3. $\frac{3}{8}$ take $\frac{3}{7}$.

4. $\frac{8}{13}$ take $\frac{3}{11}$.

5. $\frac{5}{9}$ take $\frac{2}{5}$.

6. $\frac{9}{10}$ take $\frac{7}{9}$.

7. $1\frac{3}{14}$ take $\frac{5}{42}$.

8. $40\frac{5}{8}$ take $32\frac{3}{4}$.

9. $8\frac{7}{16}$ take $5\frac{3}{7}$.

10. $9\frac{8}{17}$ take $4\frac{3}{4}$.

11. $75\frac{4}{5}$ take $60\frac{2}{3}$.

12. $15\frac{1}{20}$ take $13\frac{3}{10}$.

13. $22\frac{3}{8}$ take $19\frac{3}{5}$.

14. $13\frac{5}{6}$ take $7\frac{4}{9}$.

15. $18\frac{7}{11}$ take $15\frac{5}{6}$.

16. The sum of two fractions is $\frac{10}{16}$ and one of the fractions is $\frac{1}{8}$: what is the other?

17. From a farm containing $119\frac{3}{5}$ acres, the owner sold $79\frac{1}{6}$ acres: how many acres were left?

18. Thomas and Newton gathered $11\frac{1}{3}$ bushels of walnuts, and sold $7\frac{1}{4}$ bushels: how many bushels did they keep?

19. Mr. Gilman paid $\$46\frac{3}{8}$ for a cow, and sold it for $\$68\frac{1}{4}$: what was his gain?

20. Hugh expended $\$23\frac{1}{4}$ while earning $\$37\frac{1}{2}$: how much did he save?

21. The weight of 7 men was $1{,}680\frac{3}{4}$ pounds. If the weight of 6 of the men was $1{,}391\frac{1}{3}$ pounds, what was the weight of the seventh man?

22. A man owning $\frac{2}{3}$ of a ship sold $\frac{1}{4}$ of the ship: what part of the ship did he still own?

23. A man bought a horse for $\$140\frac{5}{8}$ and sold it for $\$175\frac{3}{8}$: how much did he gain?

24. A clerk's salary was $\$60\frac{1}{4}$ a month and his expenses $\$29\frac{1}{10}$: how much did he save?

25. A farmer having $121\frac{5}{8}$ acres of land sold $19\frac{3}{8}$ acres: how many acres did he retain?

26. From a piece of cloth containing $117\frac{3}{10}$ yards, a merchant sold $109\frac{5}{8}$ yards: how many yards remained?

27. From a pile of wood containing $27\frac{1}{7}$ cords, the owner sold $11\frac{1}{6}$ cords: how many cords did he retain?

28. A dealer bought $111\frac{3}{5}$ barrels of pork and sold $101\frac{5}{6}$ barrels: how many barrels remained?

29. From a barrel containing $99\frac{1}{3}$ pounds of sugar, a grocer sold $27\frac{1}{3}$ pounds: how many pounds were left?

30. A dealer bought $145\frac{5}{12}$ yards of silk and found at the close of his day's sales that he had $27\frac{3}{8}$ yards left: how many yards did he sell?

31. Illustrate by an original problem subtraction of fractions.

Problems combining Addition and Subtraction of Fractions.

ORAL EXERCISES.

1. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$, subtract $\frac{1}{6}$; subtract $\frac{1}{4}$.

2. From the sum of $\frac{1}{3}$ and $\frac{1}{4}$, subtract $\frac{1}{2}$; subtract $\frac{1}{3}$.

3. From the sum of $\frac{3}{4}$ and $\frac{1}{2}$, subtract $\frac{1}{3}$; subtract $\frac{1}{4}$.

4. From the sum of $\frac{4}{5}$ and $\frac{1}{10}$, subtract $\frac{1}{5}$; subtract $\frac{1}{20}$.

5. From the sum of $\frac{5}{6}$ and $\frac{1}{12}$, subtract $\frac{1}{6}$; subtract $\frac{1}{4}$.

6. $\frac{2}{3} - \frac{1}{4} + \frac{1}{2}$? $\quad \frac{3}{4} - \frac{1}{3} + \frac{1}{2}$? $\quad \frac{2}{4} - \frac{1}{2} + \frac{1}{4}$?

7. $\frac{3}{4} - \frac{1}{2} + \frac{1}{3}$? $\quad \frac{2}{3} - \frac{1}{6} + \frac{1}{3}$? $\quad \frac{1}{4} + \frac{1}{2} - \frac{1}{3}$?

8. $\frac{2}{5} + \frac{1}{10} - \frac{1}{20}$? $\quad \frac{3}{8} + \frac{1}{4} - \frac{1}{2}$? $\quad \frac{3}{4} - \frac{1}{2} + \frac{2}{3}$?

9. $\frac{3}{5} + \frac{1}{4} - \frac{1}{5}$? $\quad \frac{2}{5} - \frac{1}{4} + \frac{1}{3}$? $\quad \frac{2}{3} - \frac{1}{6} + \frac{1}{3}$?

10. $\frac{5}{6} - \frac{1}{2} + \frac{1}{3}$? $\quad \frac{3}{4} + \frac{1}{2} - \frac{1}{4}$? $\quad \frac{1}{3} + \frac{1}{3} - \frac{1}{4}$?

11. John received $\frac{1}{2}$ of a dollar from his father and $\frac{1}{4}$ of a dollar from his mother. He gave his sister $\frac{1}{5}$ of a dollar; how much had he left?

12. Susan paid $\frac{1}{4}$ of a dollar for a book and $\frac{1}{5}$ of a dollar for paper: how much change should she receive out of $\frac{1}{2}$ of a dollar?

13. Jane has $\frac{1}{5}$ of a dollar, and Mary has $\frac{1}{4}$ of a dollar: how much do they lack of having \$1?

14. A knife costs $\frac{4}{5}$ of a dollar and a pen $\frac{1}{20}$ of a dollar: what change would be left after paying for them with a two-dollar bill?

15. If the sum of two fractions is $\frac{5}{6}$ and the less is $\frac{1}{4}$, what is the greater?

16. From a piece of cloth containing 5 yards, two pieces were cut, one containing $1\frac{1}{2}$ yards and the other $2\frac{1}{3}$ yards: how much was left?

17. A dealer having 10 tons of coal, sold $2\frac{1}{2}$ tons to one customer and $2\frac{1}{4}$ to another customer: how many tons were left?

18. Maude having \$1, spent $\frac{1}{10}$ of it for candy and $\frac{3}{4}$ of it for a doll: how much had she left?

19. Three boys spent $\frac{3}{4}$ of a dollar apiece, and found they had $\frac{3}{4}$ of a dollar left: how much had they at first?

20. Lilian bought a pair of roller skates for \$3$\frac{1}{2}$ and a hat for \$2$\frac{1}{4}$, and found she had only \$5: how much more did she need to pay for the skates and hat?

21. George's watch loses $1\frac{3}{4}$ hours each day: at the end of two days what hour will it indicate, if the right time is 12 o'clock?

22. Thomas was given 3 hours in which to do some chores. He slept $1\frac{1}{3}$ hours and played $1\frac{1}{6}$ hours: how much time was left in which to do his chores?

23. If a man spends $\frac{1}{3}$ of a dollar daily, how much money at the end of 5 days will be left out of \$3?

WRITTEN EXERCISES.

1. $24\frac{2}{3} + 16\frac{1}{4} - 19\frac{1}{2}$.

2. $32\frac{1}{5} + 10\frac{2}{5} - 20\frac{3}{5}$.

3. $36\frac{2}{3} - 12\frac{1}{4} + 15\frac{5}{7}$.

4. $48\frac{5}{6} + 8\frac{1}{4} - 28\frac{4}{5}$.

5. $60\frac{4}{7} - 15\frac{1}{5} + 30\frac{3}{10}$.

6. $19\frac{1}{5} - 17\frac{1}{5} + 10\frac{1}{4}$.

7. $31\frac{1}{4} - 31\frac{1}{5} + 31\frac{1}{6}$.

8. $40\frac{5}{6} - 30\frac{2}{3} + 20\frac{1}{3}$.

9. $17\frac{3}{7} + 13\frac{8}{14} - 20$.

10. $23\frac{3}{4} + 45\frac{1}{6} + 24\frac{3}{4} - 50$.

11. $13\frac{1}{4} - 10\frac{1}{2} + 5\frac{4}{8}$.

12. $21\frac{2}{3} + 5\frac{1}{4} - 10\frac{3}{8}$.

13. $17\frac{1}{2} - 7\frac{1}{2} + 5\frac{1}{6}$.

14. $32\frac{3}{4} + 3\frac{1}{6} - 21\frac{1}{4}$.

15. $28\frac{1}{6} + 15\frac{4}{8} - 40$.

16. $31\frac{1}{7} - 10\frac{3}{8} + 7\frac{3}{7}$.

17. $35\frac{4}{6} - 12\frac{1}{3} + 11\frac{1}{15}$.

18. $42\frac{1}{3} + 4\frac{1}{6} - 20\frac{1}{12}$.

19. $48\frac{1}{2} - 6\frac{5}{6} - 20\frac{1}{3}$.

20. $64\frac{5}{6} - 30\frac{1}{2} + 17\frac{1}{4}$.

21. $7 - (\frac{2}{3} + \frac{3}{4})$; $6 - (\frac{1}{2} + \frac{1}{3})$; $10 + (\frac{2}{3} - \frac{1}{6})$.

22. From $3\frac{1}{4} - 2\frac{1}{2}$ take $\frac{1}{8}$; from $2\frac{1}{4} + 1\frac{1}{2}$ take $3\frac{1}{4}$.

23. From $\frac{1}{11}$ take $(\frac{3}{44} + \frac{1}{44})$; from $\frac{1}{6}$ take $(\frac{1}{3} - \frac{1}{4})$.

24. What is the value of $(\frac{5}{6} + \frac{3}{4}) - (\frac{2}{3} - \frac{1}{4})$?

25. What is the value of $(3\frac{1}{2} - \frac{1}{6}) - (2\frac{1}{2} - 2\frac{1}{4})$?

26. What is the value of $(\frac{3}{7} + \frac{3}{14}) - (\frac{1}{8} + \frac{1}{10})$?

27. What is the value of $17\frac{1}{2} + (2\frac{1}{4} - \frac{1}{2}) - (3\frac{1}{3} + 1\frac{2}{3})$?

28. A storekeeper had two rolls of cloth, one containing $13\frac{3}{5}$ yards and the other $19\frac{1}{3}$ yards. He sold $21\frac{1}{2}$ yards: how many yards had he left?

29. Sarah went shopping with a twenty-dollar bill. At the first store her purchases amounted to $\$7\frac{4}{5}$, and at the second to $\$9\frac{1}{4}$: how much change ought she to bring home?

30. A farmer bought three cows at $\$33\frac{1}{3}$ apiece and sold them all for $\$101\frac{1}{4}$: did he gain or lose, and how much?

31. In one barn I have stored $29\frac{1}{2}$ tons of hay, in another $31\frac{3}{4}$ tons, and in a third $20\frac{5}{6}$ tons. If I sell $37\frac{1}{2}$ tons to my neighbor, how many tons shall I have left?

32. A farmer picked $40\frac{2}{3}$ bushels of apples from one orchard, $37\frac{4}{5}$ bushels from another, and $61\frac{1}{6}$ bushels from a third. He sold 107 bushels: how many bushels did he keep?

33. If Wilson is $27\frac{1}{12}$ years old; Johnson, $23\frac{5}{8}$; Amos, $31\frac{1}{3}$; Jansen, $29\frac{3}{8}$; Higgens, $21\frac{4}{7}$; Wolcott, $20\frac{3}{5}$: how many years are their united ages less than 200 years?

34. Farmer Rogers sold 3 loads of produce. The first was worth $\$19\frac{1}{2}$, the second $\$21\frac{3}{4}$, and the third $\$18\frac{4}{5}$. He took in exchange groceries amounting to $\$37\frac{1}{2}$: how much was still due to him?

35. Illustrate by an original problem addition and subtraction of fractions.

Multiplication of Fractions.

ORAL EXERCISES.

ART. 133.—1. At $\frac{2}{5}$ of a dollar apiece how much will 2 melons cost?

Solution.—Two melons will cost 2 times $\frac{2}{5}$ of a dollar or $\frac{4}{5}$ of a dollar. *The result is obtained by multiplyiny the numerator of the fraction by the whole number and writing the product over the denominator.* Thus: $\frac{2}{5} \times 2 = \frac{4}{5}$.

GENERAL NOTE ON MULTIPLICATION OF FRACTIONS.—Multiply $\frac{3}{4}$ by $\frac{5}{6}$. The multiplier is $\frac{5}{6}$ of 1, or $\frac{1}{6}$ of 5. Multiplying $\frac{3}{4}$ by the whole of 5, we get $\frac{15}{4}$. But we were not to multiply by 5, but by $\frac{1}{6}$ of 5. Having multiplied by a multiplier 6 times the true multiplier, the product is 6 times as large as it ought to be. Hence we divide by 6 to obtain the true product, which is done by multiplying the denominator of the multiplicand by 6. Thus: $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$.

2. John walked $\frac{5}{8}$ of a mile an hour: how far did he walk in 2 hours? In 3 hours?

3. At $\$\frac{4}{5}$ a yard, what do 3 yards of lace cost?

4. If Hampden paid $\frac{3}{5}$ of a dollar for one book, what did 5 books cost? 4 books?

5. If 1 top costs $\$\frac{3}{4}$, what will 3 tops cost?

6. If a pound of sugar costs $\frac{1}{10}$ of a dollar, what do 6 pounds cost? 7 pounds?

7. How much will 7 crabs cost at $\$\frac{1}{8}$ apiece?

8. A farmer paid $\frac{1}{3}$ of a dollar each for setting 7 fence posts: how much did he pay?

9. What will 4 rabbits cost at $\$\frac{3}{10}$ each?

10. A newsboy bought 11 papers for which he paid $2\frac{1}{2}$ cents each: what was the cost?

11. How much is paid for 3 hens at $\$\frac{5}{6}$ each?

12. What are 9 qts. of milk worth at $5\frac{1}{2}$ cents a qt.?

13. How much do 12 pounds of blue-fish cost at $\frac{1}{20}$ of a dollar a pound?

14. At $4\frac{1}{2}$ cents a yard, what is the cost of 7 yards of calico?

15. Wilmot walked $\frac{3}{8}$ of a mile an hour: how far did he walk in 12 hours?

ART. 134.—16. Maria rode 11 miles and her sister rode $\frac{3}{5}$ as far: how far did her sister ride?

Solution.—Her sister rode $\frac{3}{5}$ of 11 miles, equal to $\frac{33}{5}$ miles, or $7\frac{3}{5}$ miles. *The result is obtained by multiplying the whole number by the numerator of the fraction and writing the product over the denominator.* Thus: $11 \times \frac{3}{5} = \frac{33}{5} = 7\frac{3}{5}$.

The word "of" between fractions is equivalent to the sign $\times$. Such expressions are generally called *Compound Fractions.*

17. How much is $\frac{3}{4}$ of a yard of broadcloth worth at \$4 a yard?

18. At \$3 a bushel what would $\frac{5}{8}$ of a bushel of apples cost?

19. How much is $\frac{5}{6}$ of a barrel of potatoes worth at \$2 a barrel?

20. What will $\frac{3}{5}$ of a pound of sugar cost at 10 cents a pound? At 12 cents?

21. How much is $\frac{3}{4}$ of a quart of milk worth at 6 cents a quart? At 8 cents?

22. What will $\frac{2}{3}$ of a yard of lace cost at \$1 a yard? At \$2?

23. What is $\frac{7}{8}$ of a gallon of syrup worth at 60 cents a gallon?

24. Myron worked $3\frac{1}{2}$ days at \$2 a day: what did he earn?

25. Multiply 3 by $\frac{3}{8}$; by $\frac{4}{7}$; by $\frac{5}{7}$; by $\frac{5}{8}$.

26. Multiply 6 by $\frac{2}{3}$; 8 by $\frac{3}{5}$; 10 by $\frac{3}{4}$; 12 by $\frac{5}{6}$.

27. Multiply 6 by $\frac{4}{5}$; by $\frac{5}{7}$; by $\frac{1}{5}$; by $\frac{9}{10}$.

28. Susan practiced 3 hours at the piano and Mary $\frac{3}{4}$ as long: how long did Mary practice?

29. Peggy paid \$2 for her doll, but Polly's doll cost only $\frac{3}{5}$ as much: what did Polly's doll cost?

30. How much is $\frac{2}{3}$ of the cost of a row boat, if the row boat cost \$12?

31. At $\frac{4}{5}$ of a dollar a gallon, how much does $\frac{2}{3}$ of a gallon of molasses cost?

Solution.—$\frac{2}{3}$ of a gallon costs $\frac{2}{3}$ of $\frac{4}{5}$ of a dollar. $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$ and $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$. *The result is obtained by multiplying the numerator of the multiplicand by the numerator of the multiplier and the denominator of the multiplicand by the denominator of the multiplier.* Thus: $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

32. If Frank studies $\frac{1}{2}$ an hour, and Harry $\frac{3}{4}$ as long, what part of an hour does Harry study?

33. If a yard of cloth cost $\frac{2}{3}$ of a dollar, how much does $\frac{5}{6}$ of a yard cost?

34. Jennie having $\frac{9}{10}$ of a dollar, gave her sister $\frac{1}{3}$ of it: what part of a dollar did she give her sister?

35. Multiply $\frac{2}{3}$ by $\frac{1}{2}$; $\frac{3}{2}$ by $\frac{1}{4}$; 2 by $\frac{3}{7}$; 3 by $\frac{1}{5}$.

36. Multiply $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{4}{4}$ by 3; $\frac{3}{4}$ by $\frac{5}{6}$; $\frac{5}{4}$ by $\frac{4}{5}$.

37. Multiply $\frac{3}{4}$ by $\frac{4}{6}$; $\frac{1}{6}$ by $1\frac{1}{2}$; 3 by $\frac{4}{6}$; 5 by $\frac{1}{3}$.

38. Multiply $\frac{6}{7}$ by $\frac{3}{4}$; $\frac{2}{5}$ by $2\frac{1}{4}$; $\frac{2}{3}$ by 7; $\frac{1}{4}$ by 8.

39. Multiply $\frac{5}{6}$ by $\frac{2}{5}$; $\frac{3}{7}$ by $\frac{2}{7}$; 5 by $\frac{3}{6}$; $\frac{1}{7}$ by 7.

40. How much money can a 'longshoreman earn in $3\frac{1}{2}$ days at $\$3\frac{1}{2}$ per day?

41. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$; $2\frac{1}{4}$ by $2\frac{1}{4}$; $3\frac{1}{3}$ by $4\frac{1}{3}$.

NOTE.—As a fraction is multiplied by multiplying the numerator or dividing the denominator (Arts. 99 and 100), the work may sometimes be shortened by dividing the denominator. Thus,

$$\frac{1}{8} \times 8 = \frac{1}{1} \text{ or } 7; \quad \frac{3}{4} \times 2 = \frac{3}{2} = 1\frac{1}{2}; \quad \frac{3}{16} \times 4 = \frac{3}{4}.$$

Since the numerator of a fraction represents the dividend and the denominator the divisor, the work may sometimes be shortened by cancellation (Art. 89).

Thus, in the example $\frac{2}{3} \times \frac{3}{5}$, we may cancel the first denominator 3 and the second numerator 3. In the former case we multiply the fraction by 3 (Art. 100), and in the latter we divide by 3 (Art. 102). By the long process $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$. By cancellation,

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}.$$

In the example $\frac{3}{7} \times \frac{5}{9}$, we may cancel by dividing the first numerator and the second denominator by 3. Thus,

$$\frac{\cancel{3}}{7} \times \frac{5}{\cancel{9}_{3}} = \frac{5}{21}.$$

A compound fraction, as we have learned, is changed to a simple one by multiplying the numerators together for a new numerator, and the denominators together for a new denominator. The process is the same as that of multiplying one fraction by another, and cancellation should be used wherever possible.

42. What is the value of $\frac{3}{4}$ of a melon, if the whole melon is worth $\frac{1}{6}$ of a dollar?

43. What will it cost to print $\frac{1}{2}$ page of a book at the rate of $\$1\frac{1}{2}$ a page?

44. Fred owns $\frac{2}{3}$ of a boat worth \$18: what is the value of his share?

45. What is $\frac{1}{3}$ of $\frac{3}{5}$ of a ton of coal screenings worth, if a ton is worth $\frac{2}{3}$ of a dollar?

46. What will $\frac{2}{3}$ of $\frac{3}{4}$ of a barrel of flour cost, when flour is worth 7 dollars per barrel?

To THE TEACHER.—It is clear from the foregoing solutions that the usual "three cases" in multiplication of fractions, which from time immemorial have confused the beginner, should be treated as one case. This can be done, when the multiplicand or multiplier is an integer, by writing 1 as the denominator.

Rule for Multiplication of Fractions.—*Change mixed numbers and integers to improper fractions, and then multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

WRITTEN EXERCISES.

NOTE.—In the following problems the pupils must use their ingenuity to discover the shortest methods of solution. Cancellation should be employed wherever possible.

1. Multiply $\frac{5}{7}$ by 5; $\frac{3}{4}$ by 3.

2. Multiply $\frac{7}{8}$ by 6; $\frac{5}{6}$ by 2.

3. Multiply $\frac{3}{10}$ by 5; $\frac{1}{3}$ by 7.

4. Multiply $\frac{6}{11}$ by 4; $\frac{5}{8}$ by 8.

5. Multiply $\frac{7}{8}$ by 4; $\frac{3}{5}$ by 8.

6. Multiply $\frac{7}{10}$ by 2; $\frac{6}{7}$ by 5.

7. Multiply $\frac{5}{6}$ by 3; $\frac{5}{9}$ by 6.

8. Multiply $\frac{3}{7}$ by $\frac{2}{3}$; $\frac{2}{3}$ by $\frac{5}{7}$.

9. Multiply $\frac{4}{5}$ by $\frac{4}{5}$; $\frac{3}{4}$ by $\frac{6}{8}$.

10. Multiply $\frac{5}{9}$ by $\frac{9}{5}$; $\frac{5}{6}$ by $\frac{9}{10}$.

11. Multiply $\frac{4}{12}$ by $\frac{3}{8}$; $\frac{1}{10}$ by $\frac{3}{10}$.

12. Multiply $\frac{5}{6}$ by $\frac{7}{10}$; $\frac{4}{7}$ by $\frac{8}{11}$.

13. Multiply $\frac{7}{13}$ by 7; $\frac{5}{8}$ by $\frac{26}{35}$.

14. Multiply $\frac{3}{4}$ by $\frac{3}{4}$; $\frac{9}{16}$ by $\frac{4}{3}$.

15. Multiply $\frac{5}{11}$ by $\frac{11}{15}$; $\frac{5}{10}$ by $\frac{20}{26}$.

16. Multiply $\frac{3}{10}$ by $\frac{10}{15}$; $\frac{9}{14}$ by $\frac{70}{81}$.

17. Multiply $8\frac{1}{3}$ by $3\frac{1}{3}$; $2\frac{1}{2}$ by 7.

18. Multiply $13\frac{1}{4}$ by 6; by $7\frac{3}{5}$; by $3\frac{3}{8}$.

19. Multiply $16\frac{7}{8}$ by $9\frac{8}{9}$; by $10\frac{1}{10}$; by 13.

20. Multiply $\frac{2}{3}$ of 5 by $\frac{3}{4}$ of 6; $\frac{3}{4}$ of $4\frac{1}{4}$ by $\frac{4}{5}$ of $6\frac{1}{2}$.

21. Rowland rode $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of a mile and paid at the rate of 12 cents a mile: how much did he pay?

22. A well digger excavated $\frac{1}{2}$ of $31\frac{1}{3}$ feet, for which he charged $16\frac{1}{2}$ cents a foot: what was his total charge?

23. What is the cost of $33\frac{1}{3}$ yards of gingham at $12\frac{1}{2}$ cents a yard?

24. A farmer sold 72 bushels of oats at $18\frac{3}{4}$ cents a bushel: how much did he receive?

25. A grocer sold $18\frac{3}{8}$ bushels of potatoes at $22\frac{1}{2}$ cents a bushel: how much did he receive?

26. A lady bought $16\frac{3}{4}$ yards of velvet at $\$4\frac{3}{8}$ a yard: how much did she pay?

27. What are $24\frac{1}{2}$ tons of hay worth at $\$18\frac{1}{8}$ a ton?

28. Joseph dug $\frac{2}{3}$ of $18\frac{3}{4}$ feet of a ditch and received $62\frac{1}{2}$ cents a foot; John dug $\frac{3}{4}$ of $20\frac{1}{3}$ feet and received $54\frac{2}{3}$ cents a foot: how much was earned by each?

29. Mr. Walker owning a farm of $303\frac{1}{2}$ acres, gave to his son Charles $\frac{2}{3}$ of the farm, and to his son Jackson $\frac{1}{3}$ of the farm: how much did each receive?

30. Mary bought $10\frac{1}{2}$ yards of calico at $6\frac{1}{2}$ cents a yard: how much did she pay?

31. Baltus bought $16\frac{1}{4}$ pounds of beef at $17\frac{1}{2}$ cents a pound: how much did he pay?

32. Wilson traveled $\frac{2}{3}$ of $13\frac{1}{3}$ miles, and Harry traveled $\frac{3}{4}$ of the distance traveled by Wilson: how far did Harry travel?

33. Illustrate by an original example multiplication of fractions.

Division of Fractions.

ORAL EXERCISES.

1. Annie divided $\frac{2}{3}$ of a quart of berries equally among 3 girls: what part of a quart did each girl receive?

Solution.—Each girl received $\frac{1}{3}$ of $\frac{2}{3}$ of a quart. $\frac{1}{3}$ of $\frac{1}{3}$ of a quart is $\frac{1}{9}$ of a quart, and $\frac{1}{3}$ of $\frac{2}{3}$ of a quart is $\frac{2}{9}$ of a quart, the part received by each girl. *The same result may be obtained by writing the divisor in the form of a fraction* (see note, p. 124), *inverting it, and proceeding as in multiplication.*

Thus: $\frac{2}{3} \div \frac{3}{1} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$.

GENERAL NOTE ON DIVISION OF FRACTIONS.—Divide $\frac{3}{4}$ by $\frac{5}{6}$. The divisor is $\frac{5}{6}$ of 1, or $\frac{1}{6}$ of 5. Dividing $\frac{3}{4}$ by the whole of 5, we get $\frac{3}{20}$. But we were not to divide by 5, but by $\frac{1}{6}$ of 5. Having divided by a divisor 6 times the true divisor, the quotient is only $\frac{1}{6}$ as large as it ought to be. Hence we multiply by 6 to obtain the true quotient. Inspection shows that the terms of our fractional divisor are inverted. Thus: $\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$.

2. Henry divided $\frac{3}{4}$ of a melon equally among 4 boys: how much did each receive?

3. Wharton shared $\frac{4}{5}$ of a peck of nuts equally with his 5 companions: how much did each receive?

4. If 5 knives are worth $\frac{5}{6}$ of a dollar, what is 1 knife worth?

5. A tract of land containing $\frac{12}{15}$ of an acre was divided into 6 equal lots: what part of an acre did each lot contain?

6. If 4 apples are worth $\frac{8}{10}$ of a dime, what is 1 apple worth?

7. A teacher bought 6 spelling books for $\frac{12}{15}$ of a dollar: what did each book cost?

8. A parent paid $\frac{15}{20}$ of a dollar for 5 tops for his boys: what did each top cost?

9. At $\frac{2}{3}$ of a dollar a yard, how many yards of cloth can be bought for $6?

Solution.—As many yards can be bought for \$6 as $\frac{1}{3}$ of a dollar is contained times in \$6. $\frac{1}{3}$ is contained in 6, 18 times, and $\frac{2}{3}$ is contained in 6, 9 times. Hence, 9 yards can be bought for \$6. *The same result may be obtained by inverting the divisor and proceeding as in multiplication.* Thus: $6 \div \frac{1}{3} = 6 \times \frac{3}{1} = 18$.

10. If $1\frac{1}{2}$ yards of cloth make 1 pair of trousers, how many pairs can be made from 9 yards of cloth?

11. At $\frac{4}{5}$ of a dollar a bushel, how many bushels of corn can bought for \$10?

12. Baltus earned \$6 by working for $\$1\frac{1}{2}$ a day: how many days did he work?

13. How many days can a person board at the sea shore for \$28, if he pays $\$3\frac{1}{2}$ a day?

14. At $\frac{3}{4}$ of a dollar a pound, how many pounds of spice can be bought for $\frac{4}{5}$ of a dollar?

Solution.—As many pounds can be bought as $\frac{3}{4}$ of a dollar is contained times in $\frac{4}{5}$ of a dollar. $\frac{3}{4}$ equals $\frac{15}{20}$, and $\frac{4}{5}$ equals $\frac{16}{20}$; $\frac{15}{20}$ is contained in $\frac{16}{20}$ as many times as 15 is contained in 16, or $1\frac{1}{15}$, equal to $1\frac{1}{15}$, the number of pounds that can be bought. *The same result may be obtained by inverting the divisor and proceeding as in multiplication.* Thus: $\frac{4}{5} \div \frac{3}{4} = \frac{4}{5} \times \frac{4}{3} = \frac{16}{15} = 1\frac{1}{15}$.

15. Divide $\frac{6}{7}$ by $\frac{1}{4}$; by $\frac{1}{6}$; by $\frac{3}{4}$; by $\frac{2}{3}$; by $\frac{1}{8}$.

16. Divide $\frac{3}{7}$ by $\frac{1}{5}$; by $\frac{1}{4}$; by $\frac{4}{5}$; by $\frac{3}{4}$; by $\frac{5}{6}$.

17. Divide $\frac{4}{5}$ by $\frac{1}{8}$; by $\frac{1}{5}$; by $\frac{4}{5}$; by $\frac{3}{5}$; by $\frac{2}{5}$.

18. Divide $\frac{5}{6}$ by $\frac{1}{3}$; by $\frac{1}{8}$; by $\frac{3}{8}$; by $\frac{2}{3}$; by $\frac{1}{6}$.

19. Divide $\frac{4}{7}$ by $\frac{3}{7}$; by $\frac{1}{6}$; by $\frac{2}{3}$; by $\frac{3}{4}$; by $\frac{4}{7}$.

20. How many books at $\frac{3}{10}$ of a dollar each can be bought for $\frac{6}{5}$ of a dollar?

21. If a pound of coffee is worth $\frac{1}{6}$ of a dollar, how many pounds can be bought for \$1?

22. If a hat is worth $\frac{5}{4}$ of a dollar, how many hats can be bought for \$5?

23. If the distance between Pelham and Reaville is

$11\frac{2}{3}$ miles, and Peter walked $\frac{1}{5}$ of the distance, how far did he walk?

To Teachers.—It is clear from the foregoing solutions that the usual "three cases" in division of fractions, which from time immemorial have confused the beginner, should be treated as one case. This can be done by writing 1 as the denominator, when the dividend or the divisor is an integer.

Rule for Division of Fractions.—*Change all the terms of the dividend and divisor to simple fractions, invert the divisor and proceed as in multiplication of fractions.*

Note.—Since the process in division of fractions is, after the inversion of the divisor, the same as in multiplication of fractions, pupils should use the same methods for shortening the work.

WRITTEN EXERCISES.

1. Divide $\frac{3}{4}$ by $\frac{2}{3}$.
2. Divide $\frac{7}{12}$ by $\frac{2}{21}$.
3. Divide $\frac{9}{10}$ by $\frac{16}{45}$.
4. Divide $\frac{33}{35}$ by $\frac{24}{35}$.
5. Divide $3\frac{2}{3}$ by $3\frac{2}{5}$.
6. Divide $2\frac{3}{4}$ by $2\frac{3}{5}$.
7. Divide $1\frac{5}{6}$ by $7\frac{2}{3}$.
8. Divide 15 by $\frac{3}{14}$.
9. Divide $\frac{16}{121}$ by 44.
10. Divide $\frac{15}{35}$ by 88.
11. Divide $9\frac{1}{7}$ by $8\frac{1}{7}$.
12. Divide $5\frac{2}{11}$ by $13\frac{3}{5}$.
13. Divide $16\frac{2}{3}$ by $5\frac{5}{7}$.
14. Divide $66\frac{2}{3}$ by $5\frac{5}{27}$.
15. Divide $7\frac{8}{9}$ by $16\frac{2}{3}$.
16. Divide $8\frac{11}{16}$ by $9\frac{2}{3}$.
17. Divide $48\frac{1}{2}$ by $12\frac{2}{3}$.
18. Divide $16\frac{2}{3}$ by $131\frac{1}{4}$.
19. Divide $8\frac{7}{9}$ by $8\frac{7}{8}$.
20. Divide $3\frac{1}{9}$ by $2\frac{2}{7}$.

21. How many bales of cotton at \$16 each can be bought for \$1,200?

22. At $\frac{3}{8}$ of a dollar a pound, how many pounds of tea can be bought for $\frac{9}{10}$ of a dollar?

23. I paid $\$25\frac{1}{2}$ for 17 baskets of peaches: what was the price per basket?

24. If a yard of silk is worth $\$1\frac{1}{2}$, how many yards can be bought for \$18?

25. If $11\frac{2}{3}$ bushels of corn weigh $622\frac{1}{2}$ pounds, how much does a bushel weigh?

26. When pork is sold at $\$10\frac{1}{5}$ a barrel, how many barrels can be bought for $\$94\frac{1}{2}$?

27. How many tons of hay at $\$11\frac{1}{2}$ a ton can be bought for $\$47$?

28. At $16\frac{2}{3}$ cents a pound, how many pounds of sugar can be bought for 100 cents?

29. If Jerry walks $2\frac{1}{2}$ miles an hour, how far can he walk in $\frac{4}{15}$ of an hour?

30. When iron is worth $2\frac{3}{4}$ cents a pound, how many pounds can be bought for 97 cents?

31. If a family consumes $\frac{3}{5}$ of a hundred weight of flour a month, how long will 3 hundred weight last?

32. A tailor bought $23\frac{1}{2}$ yards of broadcloth for $\$75\frac{1}{5}$: what was the price per yard?

33. When wood is worth $\$5\frac{4}{5}$ a cord, how many cords can be bought for $\$73\frac{1}{10}$?

34. If 19 yards of silk cost $\$61\frac{3}{4}$, what is the cost per yard?

35. At $\$5\frac{3}{5}$ a bushel, how many bushels of clover seed can be bought for $\$39\frac{1}{5}$?

36. If Carlos has $\frac{2}{3}$ of 27 cents and Horatio $\frac{3}{7}$ of 49 cents, how much do both lack of having $37\frac{1}{2}$ cents?

37. Divide the difference between $30\frac{2}{3}$ and $5\frac{1}{2}$ by 7 and give the result.

38. Three sailing vessels sailed a race lasting 2 days. The first went $130\frac{2}{5}$ and $131\frac{1}{4}$ miles respectively, the second $126\frac{1}{3}$ and $140\frac{1}{6}$ miles, and the third $128\frac{2}{7}$ and $141\frac{3}{5}$ miles: how many miles did each average per day, and what was the difference between the distance sailed by the fastest and that sailed by the slowest?

39. Illustrate by an original problem division of fractions.

Problems combining Multiplication and Division of Fractions.

ORAL EXERCISES.

1. 3 is $\frac{1}{3}$ of what number? 3 is 4 times what number?

2. 4 is $\frac{1}{6}$ of what number? 4 is 6 times what number?

3. 5 is $\frac{2}{3}$ of what number? 5 is 5 times what number?

4. 2 is $\frac{1}{4}$ of what number? 2 is 5 times what number?

5. 6 is $\frac{1}{7}$ of what number? 6 is 3 times what number?

6. 2 is $\frac{3}{4}$ of what number? 2 is 3 times what number?

7. 6 is $\frac{2}{7}$ of what number? 6 is 8 times what number?

8. 4 is $\frac{5}{8}$ of what number? 4 is 6 times what number?

9. 5 is 4 times what number? 6 is $\frac{3}{5}$ of what number?

10. 3 is 7 times what number? 8 is $\frac{4}{7}$ of what number?

11. I paid $\$2\frac{2}{5}$ for 4 books of equal price: how much did 3 of them cost?

12. If 12 years is $\frac{3}{4}$ of Susan's age, and if Susan is $\frac{1}{3}$ as old as her mother, what is her mother's age?

13. If I pay $\$2\frac{1}{2}$ for 2 baskets of pears, how much should I pay for 3 baskets?

14. A man owning $\frac{9}{10}$ of a ship, sold $\frac{1}{3}$ of his share to 3 friends: what part of the ship did he sell to each friend, if their shares were equal?

15. A boy having $\frac{3}{4}$ of a cake, shared it equally with 2 companions: how much did his two companions receive?

16. Fred paid $\$2\frac{1}{2}$ a pair for 5 pairs of roller skates and sold 3 pairs for the price he paid for all: how much did he receive per pair?

17. If $\frac{1}{3}$ of a farm is worth \$12,000, how much is $\frac{3}{4}$ of the farm worth?

18. If a laborer earns \$20 a month, how much does he earn in $1\frac{3}{5}$ months?

19. How many bushels are in 12 baskets of peaches if each basket holds $\frac{5}{8}$ of a bushel?

20. On Christmas day Mr. Brown divided $\$3\frac{1}{5}$ equally among 4 poor children: how much money was received by 3 of the children?

21. If a pound of butter is worth $\frac{1}{3}$ of a dollar, how many pounds can be bought for $\$2\frac{1}{3}$?

22. How much will 5 cords of wood cost, if 4 cords cost $\$5\frac{1}{3}$?

23. A horse and carriage are worth $240, and the carriage is worth $\frac{1}{2}$ as much as the horse: how much is each worth?

24. Mary had 48 cents, which was $\frac{6}{7}$ of her money: how much was $\frac{1}{8}$ of her money?

WRITTEN PROBLEMS.

Find the value of:

1. $18\frac{3}{4} \times 6 \div 3$; $15\frac{1}{4} \div 3 \times 5$; $18 \div 6 \times 7$.

2. $13\frac{1}{3} \div 4 \times 5$; $\frac{3}{5} \times 20\frac{1}{2} \div \frac{1}{2}$ of $\frac{1}{2}$; $16\frac{1}{2} \div 3 \times 9$.

3. $20\frac{1}{2} \times 2\frac{1}{4} \div \frac{2}{3}$; $\frac{4}{5} \times 11\frac{1}{3} \div \frac{2}{3}$ of $1\frac{1}{4}$.

4. $35\frac{2}{3} \times 3\frac{1}{2} \div \frac{5}{6}$; $\frac{2}{3} \div (\frac{1}{5} \times \frac{1}{4}$ of $1\frac{1}{2})$.

5. $75\frac{3}{5} \div 1\frac{1}{4} \times \frac{3}{4}$; $\frac{3}{4} \div (\frac{1}{4} \times \frac{3}{5}$ of $3)$.

6. There are 1,760 yards in 1 mile: how many yards in $\frac{9}{10}$ of a mile?

7. How many books at $\frac{3}{16}$ of a dollar each can be bought for $30?

8. A cotton broker sold some cotton for $1,800; his charges and total expenses were $\frac{1}{10}$ of the proceeds: if the shipper expended $\frac{1}{3}$ of his receipts for a pair of mules, how much did they cost?

9. There are 3,600 seconds in one hour, and $\frac{1}{10}$ of the number of seconds is 6 times the age in years of Mr. Jones: what is his age?

10. The difference between two numbers is 200 and the less is 400: what is $\frac{1}{20}$ of the greater number?

11. Three men bought land for \$1,248: their shares being equal, what was the value of $\frac{2}{3}$ of each share?

12. If 3 tons of hay are worth \$37½, how much are 5 tons worth?

13. If 5 bushels of corn are worth \$2¼, how much are 3 bushels worth?

14. When 7 pounds of sugar cost 87½ cents, how much do 3½ pounds cost?

15. If 2½ pounds of butter cost ½ of a dollar, how much does ⅓ of 6 pounds cost?

16. A farmer sold a horse for \$120½, and 12 sheep at \$3¾ apiece: how much change should he return from a five hundred dollar bill offered in payment?

17. If ¾ of a factory is worth \$27,321, how much is the whole worth?

18. A merchant lost $\frac{3}{7}$ of his capital by one creditor and $\frac{1}{10}$ by another and had \$66,000 left: what was his capital?

19. During one day a man paid out \$2,452¾, which was $\frac{3}{11}$ of the amount he had on deposit: how much had he left in bank?

20. A man had 3 plots, each containing 5¼ acres, which he divided into building lots, each containing ⅜ of an acre: how many lots did he have?

21. A cargo worth \$8,400 was insured for $\frac{5}{6}$ of its value. If the vessel is lost, how much insurance will each of the 3 equal owners of her cargo receive?

22. A farm containing 240⅕ acres was divided into 4 equal parts: how many acres are there in ⅔ of one of the parts?

23. Two men walked in opposite directions from the same point. One traveled 20½ miles, and the other 24¼

miles a day: what is $\frac{4}{5}$ of the distance they were apart at the end of 5 days?

24. An army numbering 24,000 men, lost $\frac{1}{6}$ in battle, $\frac{1}{8}$ by desertion, and $\frac{1}{12}$ by sickness: how many companies, numbering 75 men each, were left in the army?

25. Illustrate by an original problem multiplication and division of fractions.

REVIEW QUESTIONS.

What is a fraction? What is a common fraction? What is the denominator, and what does it show? What is the numerator, and what does it show? What are the terms of a fraction? Name several fractions.

What is a proper fraction? An improper fraction? A simple fraction? A compound fraction? A complex fraction? A mixed number? Give an example of each. What are the terms of a fraction?

What is the unit of a fraction? What is a fractional unit? What is a collection of fractional units? How do you analyze a fraction?

When is a fraction changed to its lowest terms? Give the rule for changing fractions to their lowest terms.

How are whole or mixed numbers changed to improper fractions? How are improper fractions changed to whole or mixed numbers? How are compound fractions changed to simple ones?

How does it affect the value of a fraction to multiply its numerator? Why? To multiply its denominator? Why? To divide its numerator? Why? To divide its denominator? Why?

How does it affect the value of a fraction to multiply its terms by the same number? Why? To divide its terms by the same number? Why?

When are fractions changed to a common denominator? Give the rule. When are they changed to the least common denominator? Give the rule.

What is addition of fractions? Give the rule.

What is subtraction of fractions? Give the rule.

What is multiplication of fractions? Give the rule. What is meant by cancellation? On what principle is it based?

What is division of fractions? Give the rule.

What are the three cases of multiplication and of division of fractions? How may each be simplified? Explain.

PART III.

Decimal Fractions.

If a unit be divided into 10 equal parts, what is one of the parts called?

If a tenth be divided into 10 equal parts, what is one of the parts called?

If a hundredth be divided into 10 equal parts, what is one of the parts called?

If a thousandth be divided into 10 equal parts, what is one of the parts called?

How many thousandths make a hundredth? How many hundredths make a tenth? How many tenths make a unit, or one?

ART. 135.—A **Decimal Fraction** is one whose denominator is 10, or 10 multiplied by itself one or more times.

ART. 136.—A decimal fraction, for convenience, is written without the denominator, and with a point after the units' place. Thus $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, and $\frac{5}{10000}$, are written decimally .5, .05, .005, .0005.

The point is called the *decimal point*, and the fraction thus written is called a *decimal*.

The decimal point separates the decimals on the right from whatever whole number may be written on the left of it.

A **Mixed Decimal** is an integer and decimal written together; as, 5.05; 164.25.

A **Complex Decimal** is a decimal fraction and common fraction written together; as, $.51\frac{1}{6}$; $.615\frac{7}{8}$; $25.23\frac{1}{3}$.

For brevity, mixed decimals and complex decimals are called *decimals*.

ART. 137.—As we proceed from left to right in whole numbers, their value decreases in a tenfold ratio. The same law governs the value of decimal fractions.

Beginning at the decimal or unit point, the first place to the right is *tenths;* the second, *hundredths;* the third, *thousandths;* the fourth, *ten-thousandths;* the fifth, *hundred-thousandths;* the sixth, *millionths,* and so on.

5.	units.	5.	units.
.5	tenths.	.5	tenths.
.05	hundredths.	.15	hundredths.
.005	thousandths.	.115	thousandths.
.0005	ten-thousandths.	.1115	ten-thousandths.

ART. 138.—In reading decimals, first read the figures representing them, as whole numbers; then give the decimal order of the last figure as a name for the entire decimal. Thus, .105 is first read as 105; then as the 5 or last figure stands in *thousandths* place, we read the decimal as 105 *thousandths.* Use the word *and* only between the whole number and the decimal. Thus, 12.056 is read twelve *and* fifty-six thousandths.

Read the following:

1.	.5	**10.**	.07	**19.**	.0416	**28.**	7.0101
2.	5.	**11.**	.013	**20.**	.101	**29.**	16.005
3.	.15	**12.**	.007	**21.**	.009	**30.**	200.002
4.	15.	**13.**	.08	**22.**	5.03	**31.**	1.01
5.	15.05	**14.**	.101	**23.**	11.0109	**32.**	96.096
6.	3.03	**15.**	.09	**24.**	6.006	**33.**	7.003
7.	1.015	**16.**	.033	**25.**	10.0008	**34.**	1001.1001
8.	40.04	**17.**	.007	**26.**	3.707	**35.**	108.072
9.	6.6	**18.**	.0008	**27.**	210.0101	**36.**	10001.10001

Write the following:

1. Six.
2. Five tenths.
3. Eight hundredths.
4. Seven tenths.
5. Four hundredths.
6. Thirteen hundredths.
7. Nine tenths.
8. Three hundredths.
9. Seven thousandths.
10. One hundred and nineteen thousandths.
11. One thousandth.
12. One hundred and one thousandth.
13. Seven hundredths.
14. Five thousandths.
15. Sixty-two thousandths.
16. Twenty-four thousandths.
17. Two thousandths.
18. Thirteen hundredths.
19. Seven and seven thousandths.
20. Ten and one thousandth.
21. Two hundred and one ten-thousandth.
22. Thirty and three thousandths.
23. Three hundred and twenty-one thousandths.
24. Five hundred and five hundredths.
25. Two hundred and two ten-thousandths.
26. One thousand and seven thousandths.
27. Seven hundred and seven hundred-thousandths.
28. Eight hundred eight and eight ten-thousandths.
29. Seven hundred and one tenth.
30. Nine thousand nine and four thousandths.
31. Sixteen and sixteen hundred-thousandths.
32. Thirteen and thirteen hundredths.
33. Three and three hundredths.
34. Fourteen and four thousandths.
35. Eight hundred and one tenth.
36. Seventy thousand and seventy-four thousandths.
37. Twenty-one and twelve hundred-thousandths.
38. One thousand one and one thousand one ten-millionths.
39. Seven billionths.

ART. 139.—Changing Decimals to Common Fractions and Common Fractions to Decimals.

Change .75 to a common fraction of equivalent value.

Process.

$.75 = \frac{75}{100} = \frac{3}{4}$.

Analysis.—.75 written as a common fraction is $\frac{75}{100}$, which, changed to its lowest terms, is $\frac{3}{4}$.

Rule for changing a decimal to a common fraction of equivalent value.—*Write the decimal as a common fraction, and change this fraction to its lowest terms.*

Change the following to equivalent common fractions:

1. .15	**9.** .18	**17.** .065
2. .025	**10.** .006	**18.** .0125
3. .45	**11.** .05	**19.** .07
4. .045	**12.** .8	**20.** .002
5. .175	**13.** .01	**21.** .0545
6. .0005	**14.** .008	**22.** .011
7. .875	**15.** .95	**23.** .705
8. .9	**16.** .004	**24.** .625

Change $\frac{5}{8}$ to an equivalent decimal.

Process.

$\frac{5}{8} = \frac{5000}{8000}$; $\frac{5000 \div 8}{8000 \div 8} = \frac{625}{1000} = .625$.

Analysis.—$\frac{5}{8} = \frac{5000}{8000}$, which changed to thousandths is $\frac{625}{1000}$. This expressed decimally is .625.

Rule for changing a common fraction to a decimal of equivalent value.—*Annex ciphers to the numerator, divide by the denominator, and point off as many decimal places as there are ciphers annexed.*

Change the following to equivalent decimals:

1. $\frac{1}{3}$.

NOTE.—If, after annexing ciphers, the numerator is not exactly divisible by the denominator, the decimal may be completed by writing the remainder as a common fraction, or the sign + may be annexed to the quotient to show that the division is not exact. Thus, $\frac{1}{3} = .3\frac{1}{3}$ or $.33\frac{1}{3}$ or $.333\frac{1}{3}$ or $.333+$.

2. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{1}{8}$; $\frac{3}{8}$. 4. $\frac{4}{8}$; $\frac{7}{8}$; $\frac{9}{8}$; $\frac{15}{8}$.

3. $\frac{3}{6}$; $\frac{5}{20}$; $\frac{3}{6}$; $\frac{1}{25}$; $\frac{4}{10}$. 5. $\frac{8}{25}$; $\frac{6}{14}$; $\frac{4}{15}$; $\frac{8}{18}$.

Addition of Decimals.

ART. 140.—1. What is the sum of $\frac{2}{10}$ and $\frac{5}{10}$? Of .3 and .6?

2. What is the sum of $\frac{4}{100}$ and $\frac{15}{100}$? Of .06 and .08?

3. What is the sum of $\frac{3}{10}$ and $\frac{7}{100}$? Of .8 and .35?

4. What is the sum of $\frac{1}{10}$ and $\frac{24}{1000}$? Of .7 and .012?

5. What is the sum of .18 and .213?

WRITTEN EXERCISES.

1. What is the sum of .75, 12.5, 3.05?

Process.

.75	**Analysis.**—We write the numbers so that units
12.5	of the same order stand in the same column and
3.05	add as in whole numbers.
16.30	

Rule for Addition of Decimals.—*Write the numbers so that units of the same order shall stand in the same column, and add as in whole numbers.*

Add the following:

2. 73.07, 49.17, 71.71, 303.0176, 117.0224.
3. 96.006, 88.181, 185.101, 8004.381, 711.017.

4. 111.09, 93.004, 321.9, 1001.548, 803.6.

5. 384.95, 201.07, 222.8, 9053.067, .94001.

6. Twenty-seven hundredths; seven hundred and eight ten-thousandths; two hundred and two hundred six hundred-thousandths.

7. Seventy-one hundredths; seventy-one thousandths; seventy-one hundred-thousandths.

8. One hundred one; one hundred one ten-thousandths; fifty-seven and one hundred nine hundred-thousandths; one hundred and five millionths.

9. Eighty-three hundredths; two thousand; one hundred and eleven millionths.

10. Fifty-three thousandths; fifteen hundredths; two hundred five; one hundred one hundred-thousandths.

11. Illustrate by an original problem addition of decimals.

Subtraction of Decimals.

ART. 141.—1. From $\frac{9}{10}$ take $\frac{5}{10}$. From .7 take .4.

2. From $\frac{7}{100}$ take $\frac{3}{100}$. From .15 take .01.

3. From $\frac{12}{100}$ take $\frac{12}{1000}$. From .14 take .023.

4. From 5 take $\frac{5}{10}$. From 6 take .6.

5. From 4 take $1\frac{3}{10}$. From 12 take 5.4.

WRITTEN EXERCISES.

1. From 15.205 take 9.006.

Process.

15.205
9.006
6.199

Analysis.—We write the numbers so that units of the same order stand in the same column, and subtract as in whole numbers.

Rule for Subtraction of Decimals.— *Write the subtrahend under the minuend, so that units of the same order shall stand in the same column, and subtract as in whole numbers. If there are more decimal places in the subtrahend than in the minuend, fill the vacant orders of the minuend with ciphers.*

2. From 67.72 take 54.66.

3. From 85.73 take 66.72.

4. From 45.005 take 25.08.

5. From 94.09 take 91.009.

6. From 101.095 take 100.086.

7. From 258.006 take .707.

8. From 500. take .005.

9. From 1001.01 take 303.005.

10. From three tenths take one hundred three thousandths.

11. From five and five tenths take five hundred five thousandths.

12. From three tenths take three hundredths.

13. From twenty-five and five thousandths take five and fifteen hundredths.

14. From seventeen thousand six take three hundred three and thirty-five thousandths.

15. From one hundred thousand and forty-two thousandths take ten thousand and twenty-seven thousandths.

16. From seventeen million take forty-four hundred-thousandths.

17. From sixty-five thousand two hundred sixty-four take one hundred twenty-eight and eight tenths.

18. From eighty-three thousand and two hundred forty-nine hundred-thousandths take two hundred ninety-four and forty-four hundredths.

19. From one hundred fifteen take one hundred one and twenty-one hundredths.

20. Illustrate by an original problem subtraction of decimals.

Multiplication of Decimals.

ART. 142.—1. What is the product of $\frac{2}{10} \times 4$? Of $.3 \times 3$?

2. What is the product of $\frac{3}{100} \times 3$? Of $.02 \times .4$?

3. What is the product of $\frac{2}{10} \times \frac{4}{10}$? Of $.3 \times .2$?

4. What is the product of $\frac{7}{10} \times \frac{6}{100}$? Of $.4 \times .03$?

5. What is the product of $\frac{3}{100} \times \frac{8}{100}$? Of $.06 \times .07$?

WRITTEN EXERCISES.

1. Multiply .017 by .15.

Process.

.017
.15
85
17
.00255

Analysis.—Since multiplying thousandths by hundredths gives hundred-thousandths, the product is 255 hundred-thousandths, or .00255. It is seen that the decimal places in the product equal the number in the multiplicand and multiplier. The same result may be obtained thus: $\frac{17}{1000} \times \frac{15}{100} = \frac{255}{100000} = .00255$.

Rule for Multiplication of Decimals.—*Multiply as in whole numbers, and point off as many decimal places in the product as there are in the multiplicand and multiplier. In case there are not the required number of decimal places in the product, prefix as many ciphers as are necessary to make the required number of decimal places.*

Multiply:

2. .15 by .05.
3. 3.14 by .15.
4. 21.12 by .35.
5. 16.006 by .9.
6. 45.09 by .015.
7. 38.38 by .37.
8. 3.015 by .123.
9. 4.012 by 3.02.
10. 5.73 by .09.
11. 12.31 by .401.
12. 25.25 by 1.305.
13. .101 by .11.
14. 31.03 by 6.021.
15. 17.07 by 16.37.

16. 105.5 by 7.008.
17. 321.8 by 25.07.
18. 303.062 by 18.41.
19. 41.37 by 47.38.
20. 140.014 by .01.
21. 19.301 by 3.05.
22. 210.21 by 31.031.
23. 124.051 by 7.07.

24. Multiply twenty-nine thousandths by seventeen hundredths.

25. Multiply one hundred and two hundred twelve ten-thousandths by one hundred and one thousandth.

26. Multiply two hundred ten and eighteen thousandths by sixty-five and eight tenths.

27. Multiply eight hundred eighteen and forty-seven ten-thousandths by thirteen and forty-eight thousandths.

28. Illustrate by an original problem multiplication of decimals.

Division of Decimals.

ART. 143.—1. What is the product of $.7 \times 8$?

2. What is the quotient of $5.6 \div 8$?

3. What is the product of $.4 \times .7$?

4. What is the quotient of $.28 \div .7$?

5. What is the product of $.05 \times .9$?

6. What is the quotient of $.045 \div .9$?

WRITTEN EXERCISES.

1. Divide .625 by .25.

Process.

$$\begin{array}{r} .25).625(2.5 \\ 50 \\ \hline 125 \\ 125 \\ \hline \end{array}$$

Analysis.—Since the dividend is the product of the divisor by the quotient, the number of decimal places in the quotient must equal the excess of those in the dividend over those in the divisor. The same result may be obtained thus:

$$\frac{625}{1000} \div \frac{25}{100} = \frac{625}{1000} \times \frac{100}{25} = \frac{62500}{25000} = 2\frac{1}{2} = 2.5.$$

Rule for Division of Decimals.—*Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceed the number in the divisor.*

NOTE.—When the dividend has fewer decimal places than the divisor, annex enough ciphers to the dividend to make them equal. When the number of quotient figures is less than the excess of decimal places in the dividend over those in the divisor prefix as many ciphers as there are places lacking.

ART. 144.—Divide :

2. 75.273 by 22.2.

3. 29.07 by 29.07.

4. 70.25 by 702.5.

5. 100.47 by 1004.7.

6. 848.04 by 305.05.

7. 32.13 by 11.001.

8. 14.09 by 1409.

9. 907.30 by .125.

10. 708.06 by 5.5.

11. 40. by 650.

12. 30.05 by 3.003.

13. 150.15 by .15.

14. Divide seventy-five and forty-seven thousandths by twenty-three and five tenths.

15. Divide seventeen hundred and thirty-five ten-thousandths by one hundred.

16. Divide one hundred thirty-four and thirteen hundredths by sixty-seven and nine tenths.

17. Divide one thousand seven hundred twenty-eight by fourteen and four tenths.

18. Divide two hundred fifty-six by twenty-five and six tenths.

19. Divide twelve thousand three hundred forty-eight by three hundred thirty-two and twelve thousandths.

20. Illustrate by an original problem division of decimals.

NOTE.—Additional problems in decimal fractions will be found in the exercises under United States Money.

United States Money.

ART. 145.—United States Money is based on the decimal system. Calculations involving United States Money are performed in the same manner as in whole numbers and decimals.

ART. 146.—The dollar is the unit; the dime is one tenth of a dollar; the cent one tenth of a dime, and the mill one tenth of a cent.

There is no coin known as the *mill;* it is used only in making calculations.

10 mills (m.) = 1 cent, ct.	10 dimes = 1 dollar, $.
10 cents = 1 dime, d.	10 dollars = 1 eagle, E.

The demands of business have compelled the subdivision of coins, so that those now used are:

Bronze, 1 and 2 ct. pieces; *Nickel,* 3 ct. and 5 ct. pieces; *Silver,* 5 ct., 10 ct., 25 ct., 50 ct., and $1 pieces; *Gold,* $1, $2½, $5, $10, $20, and $50. Some of the branch mints have coined 50 ct. and 25 ct. pieces of gold, but such coinage is illegal. The $10 piece is the eagle, the $5 piece the half eagle, and the $2½ piece the quarter eagle.

Paper money consists of bills of the denomination of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

1. John sold a horse for $240.50, a carriage for $170.25, and a set of harness for $37.75: how much did he receive for all?

2. Three men bought a tract of land for $7,308. The first paid $2,150.25, and the second $1,940: how much did the third pay?

3. A customer bought a coat for $25.75, a vest for

$7.50, and a pair of trousers for $9.25. He handed the clothier a $50 bill: how much change should he receive?

4. A grocer purchases flour at $7.62½ a barrel, and sells it at $9 a barrel: what is his gain on each barrel?

5. The sum of two drafts is $1,015 and the less draft is $495½: what is the greater draft?

6. If 5 horses cost $800, how much at the same rate will 11 horses cost?

7. A merchant bought a barrel of sugar for $29.50 and a barrel of flour for $9.62½. He sold them for $40; what was his profit?

8. A laborer earns $1.62½ a day and his weekly expenses are $6½: how much does he save each week?

9. A man paid $2,836 for a house and $1,500 for a lot. He sold the two for $4,500: how much was his profit?

10. A table was bought for $17.25, a mirror for $21.50, a bedstead for $19.40, and a chair for $11.25: how much did they all cost?

11. Lucien sold a farm of 140 acres for $6,380.50: what price did he receive per acre?

12. A gentleman bought a carriage for $160.50. He paid $17.25 for repairing it, and then sold it for $170: how much did he lose?

13. What part of a dollar is 5 cts.? 10 cts.? 12½ cts.? 25 cts.? 75 cts.? 50 cts.?

14. At $.33⅓ per yard, how much must be paid for 5 pieces of dress goods, each piece containing 28 yards?

15. How much must be paid for 1,176 pounds of confectioner's sugar at $.16⅔ a pound?

16. How much will 3,856 pounds of cheese cost at \$.12½ per pound?

17. How much will 28.75 acres of land cost at \$28.50 per acre?

18. A dairyman sells 36.25 pounds of butter for \$4.875: how much does he receive per pound?

19. If a party of men earn \$110.11 in 9.15 days, how much do they earn in one day?

20. If 13.5 yards of broadcloth cost \$75.75, what is the cost per yard?

21. What is the cost of 28 reams of paper weighing 40 pounds each, at 16 cents a pound?

22. What is the freight on 178 tons of coal at \$1.30 a ton?

23. A farmer sold 36 pounds of butter at 25 cents a pound, and was paid in muslin at 12½ cts. a yard: how many yards did he receive?

24. If an office-boy earns \$.75 per day, how long will it take him to earn \$9?

25. A speculator bought 2 farms, one containing 130 acres at \$20.50 per acre, the other containing 125 acres at \$19.75 per acre: what was the difference in their cost?

26. Mr. Moore sold 18 cords of wood at \$4.25 a cord and received 8 barrels of flour in payment: what did the flour cost him per barrel?

27. If 55 pounds of tea cost \$32.50, what will 11 pounds cost?

28. Walter paid \$68.50 for a wagon and 3 times as much for a horse: what was the cost of both?

29. If a merchant buys 10,000 bushels of wheat at \$1.25 a bushel and sells it at \$1.33 a bushel, how much is his gain?

30. A laborer receives $1.40 a day and his expenses average $.95 a day: how much can he save in one year if he works 311 days?

31. A man bought molasses at $46.75 a cask and sold it at $50.25 a cask, and thereby gained $35: how many casks did he sell?

32. If $\frac{1}{5}$ of a barrel of flour cost $\frac{9}{10}$ of a dollar, how much is one barrel worth?

33. Emmet divided $20 among some workmen, giving them $\frac{4}{5}$ of a dollar apiece: how many workmen were there?

34. A farmer exchanged 45 pounds of butter at $.24 a pound and 6 dozen eggs at $.12$\frac{1}{2}$ a dozen, for cloth at $.20 a yard: how many yards did he receive?

35. If 10 yards of tape cost $\frac{2}{5}$ of a dollar, how much will 1 yard cost?

36. A set of harness cost $15, which was $\frac{2}{5}$ of the cost of a carriage: what was the cost of both?

37. At $\frac{3}{4}$ of a dollar a bushel, what will $\frac{3}{4}$ of a bushel of potatoes cost?

38. A newsboy paid $3.25 a hundred for papers and sold them at 5 cents apiece: how much did he make on 100 papers?

39. What will $\frac{2}{3}$ of 18 bushels of apples cost at $\frac{1}{2}$ of 4\frac{1}{2}$ a bushel?

40. How many pounds of butter at $.20 a pound will pay for 65 yards of cloth at $\frac{3}{4}$ of a dollar per yard?

41. A boy spent $\frac{3}{5}$ of his money, and had $.20 left; how much had he at first?

42. Maria paid $1.80 for a hat, and $\frac{1}{4}$ as much for a doll; what did she pay for both?

43. If 9 yards of cloth cost $\frac{2}{3}$ of \$27, what is the cost of $\frac{3}{4}$ of $16\frac{1}{2}$ yards?

44. If $\frac{3}{4}$ of a pound of tea cost \$.60, what is the cost of $\frac{7}{8}$ of 13 pounds?

45. How many times must a measure holding $1\frac{5}{8}$ bushels be emptied into a bin containing $16\frac{1}{4}$ bushels of grain in order to fill the bin?

46. St. Louis, January 4, 1886.

Mr. Washington Irving,

Bought of Scruggs, Vandervoort & Barney.

2	neckties	@	\$	.50	
6	handkerchiefs	@		.60	
6	pair socks	@		.50	
8	collars	@		.25	
10	yards shirting	@		.09	
6	" linen	@		.75	

Received payment,

Scruggs, Vandervoort & Barney.

47. St. Louis, Mo., Jan. 14, 1886.

Edw. S. Ellis,

Bought of David Nicholson.

25	lb. Java coffee	@	\$	.25	
50	" sugar	@		.11	
5	" cotton twine	@		.25	
3	" tea	@		.75	
2	gal. vinegar	@		.15	

Received Payment,

David Nicholson.

48. St. Louis, Mo., Jan. 28, 1886.

Mr. Thomas H. Benton,

Bought of J. O. Gibson.

1885.				
Oct. 28.	4	bbl. potatoes, ...	@ $1.75	$ 7.00
" "	25	head cabbage, ...	@ .05	1.25
" "	5	bbl. apples, ...	@ 2.00	10.00
" "	2	bbl. sweet potatoes, . . .	@ 2.50	5.00
" "	1	bush. turnips, ...		.50
" "	1	basket tomatoes, ...		.40
" "	1	bbl. melons, ...		1.50
" "	5	bush. corn, ...	@ .60	3.00
" "	5	" wheat, ...	@ 1.00	5.00
				$

Received payment,

J. O. Gibson.

49. Make out a bill, containing 6 items, against one of your classmates, compute the amount and receipt the same.

REVIEW QUESTIONS.

What are decimal fractions? How is a decimal fraction usually written? What is the point called? What does it do? What is the law of decrease of decimals?

What is the first place to the right of the decimal point? The second? Third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth?

How do you read decimals?

How do you change decimals to the form of common fractions? How are common fractions changed to decimals?

What is addition of decimals? Give the rule. Subtraction of decimals? Give the rule. Multiplication of decimals? Give the rule. Division of decimals? Give the rule.

On what is United States money based? How does it increase and decrease? How, then, is it governed? What is the unit? What is said of *mills*? Repeat the table.

Why do we have a variety of coins? What coins are made of bronze? Of nickel? Of silver? Of gold? What is the eagle? The half eagle? The quarter eagle?

Of what denominations does paper money consist?

PART IV.

Denominate Numbers.

ART. 147.—A Denominate Number is a concrete number composed of one or more denominations: as 10 pounds, 3 ounces.

ART. 148.—Denominate Numbers are either Simple or Compound.

ART. 149.—A Simple Denominate Number is composed of units of the same kind or denomination: as 5 days, 7 cents, 4 inches, 8 books.

ART. 150.—A Compound Denominate Number consists of two or more denominate numbers of the same nature: as 4 feet 3 inches; 2 pounds 10 ounces; 15 hours 20 minutes.

Operations in denominate numbers embrace:

1. Changing Denominate Numbers to lower denominations, called *Reduction Descending.*

2. Changing Denominate Numbers to higher denominations, called *Reduction Ascending.*

Tables and Measures.

AVOIRDUPOIS WEIGHT.

ART. 151.—Avoirdupois Weight is used in weighing

everything in common use, and the metals, except gold and silver.

16 ounces (oz.)	=	1 pound, lb.
100 pounds	=	1 hundred weight, cwt.
20 hundred weight	=	1 ton, T.

NOTE.—Formerly 28 pounds equaled a quarter, 112 pounds a hundred weight, and 2,240 pounds equaled a ton, known as the long or gross ton. The long or gross ton is used in weighing coal at wholesale and at the United States Custom House.

1. Change 5 T. 3 cwt. 18 lb. 12 oz. to oz.

Process.

T.	cwt.	lb.	oz.
5	3	18	12
20			
103			
100			
10318			
16			
61908			
10318			
12			
165100			

Analysis.—Since there are 20 cwt. in 1 ton, in 5 tons there are 5 times 20 cwt. = 100 cwt., to which the 3 cwt. given are added = 103 cwt.

Since there are 100 lb. in 1 cwt., in 103 cwt. there are 103 times 100 lb. = 10300 lb., to which the 18 lb. given are added = 10318 lb.

Since there are 16 oz. in 1 lb., in 10318 lb. there 10318 times 16 oz. = 165088 oz., to which the 12 oz. given are added = 165100 oz.

Therefore, 5 T. 3 cwt. 18 lb. 12 oz. = 165100 oz.

Rule for Reduction Descending.—*Multiply the highest denomination by the number of units of the next lower denomination required to make a unit of the higher; add to the product the given number, if any, of the next lower denomination.*

Proceed in like manner with each successive denomination until the required denomination is reached.

2. Change 165100 oz. to higher denominations.

Process.

16)165100
100)10318 + 12 oz.
20)103 + 18 lb.
5 T. + 3 cwt.

Analysis.—Since there are 16 oz. in 1 lb., in 165100 oz. there are as many lb. as 16 oz. is contained times in 165100 oz., which is 10318 times, with 12 oz. remaining.

Since there are 100 lb. in 1 cwt., in 10318 lb. there are as many cwt. as 100 lb. is contained times in 10318 lb., which is 103 times with 18 lb. remaining.

Since there are 20 cwt. in 1 T., in 103 cwt. there are as many tons as 20 cwt. is contained times in 103 cwt., which is 5 times, with 3 cwt. remaining.

Therefore, 165100 oz. = 5 T. 3 cwt. 18 lb. 12 oz.

Rule for Reduction Ascending.—*Divide the given denominate number by the number of units required to make one of the next higher denomination. Divide the quotient in like manner, and continue the operation until the required denomination is reached. The last quotient, with the several remainders annexed in their proper order, is the answer.*

3. Change 7 T. 2 cwt. 9 lb. to lb.

4. Change 2850 lb. to higher denominations.

5. Change 7 T. 1 cwt. 20 lb. 11 oz. to oz.

6. Change 18750 lb. to T.

7. Change 2300 lb. to oz.

8. Change 7200 lbs. to T.

9. 3 T. 1 lb. to oz.

10. 2500 lb. to T.

11. 13 T. 1 cwt. to oz.

12. Change 3 T. 13 lb. to oz.; 32000 oz. to T.

13. Show by an original problem in avoirdupois weight the method of changing denominate numbers to lower denominations.

14. Show by an original problem in avoirdupois weight the method of changing denominate numbers to higher denominations.

TROY WEIGHT.

ART. 152.—Troy Weight is used in weighing gold, silver and jewels.

24 grains (gr.) = 1 pennyweight, pwt.
20 pennyweights = 1 ounce, oz.
12 ounces = 1 pound, lb.

APOTHECARIES' WEIGHT.

ART. 153.—Apothecaries' Weight is used in compounding medicines.

20 grains (gr.) = 1 scruple, ℈. | 8 drams = 1 ounce, ℥.
3 scruples = 1 dram, ʒ. | 12 ounces = 1 pound, ℔.

NOTE.—Since Troy Weight and Apothecaries' Weight are used only in special cases, it is suggested that their study be deferred until "The Complete Arithmetic" is reached.

DRY MEASURE.

ART. 154.—Dry Measure is used in measuring grain, fruit, salt, and like articles.

The standard bushel is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2,150.42 cubic inches.

In measuring grain, seeds, and small fruits, the measure must be *even full;* but in measuring corn in the ear, apples and coarse vegetables, the measure must be *heaped.* A heaped bushel contains $\frac{1}{4}$ of an even or stricken bushel. A pint dry measure equals $1\frac{1}{6}$ pints liquid measure.

2 pints (pt.) = 1 quart, qt.
8 quarts = 1 peck, pk.
4 pecks = 1 bushel, bush.

1. How many pints in 5 qts.? In 2 pks.? In 1 bushel?

2. In 60 pts. how many quarts? How many pks.?

3. How many pecks in 2 bush.? How many quarts? How many pints?

4. Change 12 bush. 1 pk. 3 qts. 1 pt. to pints.

5. Change 1148 pts. to bushels.

6. If I buy 3 bush. of dried apples at $1.25 a bushel and sell them at 6 ct. a quart, how much do I gain?

7. Change 19 bush. 1 pt. to pints.

8. A farmer sold 10 bush. 3 pks. of potatoes at $.75 a bushel: how much did he receive for them?

9. Hiram bought 6 pks. of berries at 50 cents a peck and sold them at 15 cents a quart: what was his profit?

10. Show by an original problem in dry measure the method of changing denominate numbers to lower denominations.

11. Show by an original problem in dry measure the method of changing denominate numbers to higher denominations.

LIQUID MEASURE.

ART. 155.—Liquid Measure is used in measuring all kinds of liquids.

4 gills (gi.) = 1 pint, pt.
2 pints = 1 quart, qt.
4 quarts = 1 gallon, gal.

NOTE.—The gallon contains 231 cubic inches. The barrel and hogshead were formerly fixed measures, but their contents are now estimated by gallons. Generally, the barrel contains 31½ gallons, and the hogshead 63 gallons.

1. What part of a gallon in 1 qt.? 2 qts.? 6 pts.?

2. At 32 cents a gallon, how much will 3 pints of cider cost?

3. If a gill of molasses costs 2 cents, how much will 5 gal. cost?

4. Change 64 gal. 3 qts. 1 pt. to pints.

5. How many cubic inches in a hogshead containing 63 gal.?

6. How many bottles holding 2 gills each can be filled from $4\frac{1}{2}$ gal. of paregoric?

7. What will 4 gal. 2 qts. of molasses cost at 20 cents a gallon?

8. What will 6 gal. of milk cost at 6 cents a pint?

9. A grocer bought 60 gal. of maple syrup for $30 and sold it at 16 ct. a quart: how much did he gain?

10. If I buy 3 bush. of nuts at $1.75 a bushel and sell them at 4 ct. a quart, how much do I lose?

11. Show by an original problem in liquid measure the method of changing denominate numbers to lower denominations.

12. Show by an original problem in liquid measure the method of changing denominate numbers to higher denominations.

LINEAR OR LONG MEASURE.

ART. 156.—Linear or **Long Measure** is used in measuring lengths and distances.

12 inches (in.)	= 1 foot, ft.
3 feet	= 1 yard, yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod, rd.
320 rods	= 1 mile, mi.

1. How many rods in 2 mi.?

2. How many inches in 2 yds.? In 7 ft.? In 1 rd.?

3. How far can I ride in 11 days of 24 hours each, at the rate of 30 miles an hour?

4. If James rides 60 rds. on his bicycle in 1 minute, how long will it take him to ride a mile?

5. If sound travels at the rate of 1,100 feet a second, and a thunder clap is heard 3 seconds after the flash is seen, how many rods distant is the lightning?

6. Fred uses a vaulting pole 12 ft. 11 in. long: how many inches is it in length?

7. A lot is 30 rods long and 20 feet wide: how many feet is it around the 4 sides?

8. How many rods in $\frac{3}{4}$ of a mile?

9. How many inches in half a mile?

10. Change 720 ft. to higher denominations.

11. A lot is 40 rods long: what part of a mile is its length?

12. Illustrate by an original problem in long measure, the method of changing denominate numbers to lower denominations.

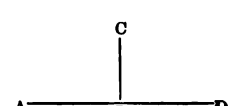
13. Illustrate by an original problem in long measure, the method of changing denominate numbers to higher denominations.

SURFACE OR SQUARE MEASURE.

ART. 157.—Square Measure is used in measuring surfaces.

A surface has length and breadth but no thickness.

An angle is the difference in the direction of two lines that meet at a point called the vertex.

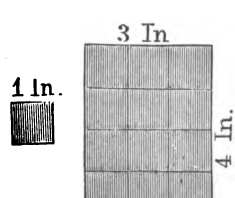


When a line so meets another line as to make the adjacent angles equal, each angle is a right angle.

Thus, A B C and C B D are right angles.

A rectangle is any plane figure bounded by four straight lines and whose angles are all right angles.

A square is a rectangle with four equal sides.



The larger figure represents a surface 4 inches long and 3 inches wide. The dividing lines of the figure show that its surface contains as many times 3 sq. inches as there are inches in the length; that is, 4 × 3 sq. in. or 12 sq. in. *The length and breadth of a rectangular surface being given in the same denomination, their product is the number of superficial units in the area.*

144 square inches (sq. in.)	= 1 square foot, sq. ft.
9 square feet	= 1 square yard, sq. yd.
30¼ square yards	= 1 square rod, sq. rd.
160 square rods	= 1 acre, A.
640 acres	= 1 square mile, sq. mi.

1. How many square rods in 10 A.?

2. How much will 5 sq. mi. of land cost at $40 an A.?

3. How many sq. rds. in 11 A.?

4. Change 180 sq. rds. to sq. ft.

5. A rectangular piece of land is 360 ft. long and 320 ft. wide: how many square feet does it contain?

6. How many acres in a field 160 rds. long and 60 rds. wide?

7. Change 5,280 sq. rds. to A.

8. A floor is 18 ft. long and 3 yds. wide: what will it cost to cover it with linoleum at $1.25 per sq. yd.?

NOTE.—The applications of square measure are numerous. In carpeting rooms, in papering, plastering and painting, in the measurement of boards, and wherever the surface area is desired, square measure is used.

9. What will it cost at $.20 per sq. yd. to kalsomine the four walls of a parlor 21 feet long, 15 feet wide, and 12 feet high?

10. What will it cost to plaster the ceiling of a room 18 ft. long, 16 ft. wide, and 10 ft. high, if the cost is $.25 per square yard? What will it cost to cover the walls with Lincrusta-Walton at $3¼ per square yard?

11. A man was paid 1 cent a square yard for whitewashing a barn 48 ft. long, 24 ft. wide, and 30 ft. from the ground to the eaves: how much money did he receive?

12. Mr. Warren's parlor is 42 ft. long and 27 ft. wide. What will it cost to cover the floor with Axminster carpet $\frac{3}{4}$ of a yd. wide, the price being \$2 per yd.?

13. How many yards of carpet 1 yd. wide will it take to cover a floor $18\frac{1}{2}$ ft. long and $16\frac{1}{4}$ ft. wide.?

14. How many feet of lumber in a board 14 ft. long, 10 inches wide, and an inch thick?

15. How many feet of lumber in 12 joists each 12 ft. long by 12 in. by 2 in.?

NOTE.—Boards no more than an inch in thickness are sold by square measure. If they are more than an inch in thickness, they are measured by taking $\frac{1}{12}$ of the continued product of the length in feet, and the width and thickness in inches.

16. How many feet in a plank 17 ft. long, 9 inches wide, and 2 inches thick?

17. What is the surface of a board 12 ft. long, 8 inches wide at one end and 6 inches wide at the other end and tapering uniformly?

NOTE.—The average width of a board of the shape named is one half the sum of the width of the broad and the narrow ends. Thus, $\frac{1}{2}$ (8 + 6) is 7, which is the average width of the board named.

18. How many feet of lumber in 12 boards, each 15 ft. long, 9 in. wide at one end and 5 in. wide at the other, and 1 in. thick?

19. How many ft. of lumber in 24 scantlings, each 18 ft. long by 3 in. by 2 in.?

20. What will it cost to cover with Brussels carpet at \$1.25 a yard, a parlor 40 ft. long by 18 ft. 9 in. wide, if the carpet is $\frac{3}{4}$ of a yard wide, and $4\frac{1}{2}$ yards are wasted in matching the pattern?

21. What will it cost to cover a yard 100 ft. long and 32 ft. wide, with sod, at 25 ct. per sq. yd.?

22. What will it cost to plaster the walls and ceiling of a room 32 ft. 6 in. by 18 ft., and 10 ft. high, at 40 ct. per sq. yd.?

23. Illustrate by an original problem the method of finding how much carpet will cover a room.

24. Illustrate by an original problem the method of finding how many sq. yd. of papering or kalsomining a room will require.

25. Illustrate by an original problem the method of finding how much it will cost to plaster a room.

26. Illustrate by an original problem the method of measuring boards an inch in thickness.

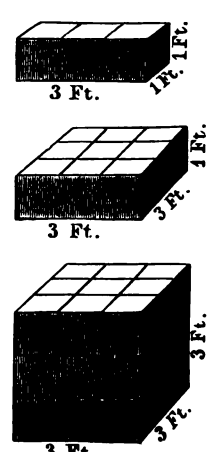
27. Illustrate by an original problem the method of measuring boards which are more than an inch in thickness.

28. Illustrate by an original problem the method of measuring boards which are broader at one end than at the other.

CUBIC MEASURE.

ART. 158.—Cubic Measure is used in measuring solids.

A cubic foot is a cube or solid, one foot wide, one foot deep, and one foot long.



Suppose we take 3 blocks, each of which is a foot long, a foot wide, and a foot deep. Now, if they are laid on the table beside each other, you see that it is 3 feet from one side to the other. We will lay 3 similar blocks beside the first row, and 3 more beside the second row. We now have a layer consisting of 3 rows. Above this layer let us put 2 other layers exactly like it. How many feet is it across the front of the pile? How wide, then, is the pile? How high? How deep?

The pile thus formed being 3 feet each way, there are 9 sq. ft. on the top; but each layer is a foot deep, so that each must contain 9 cu. ft., or 27 cu. ft. in the three layers. Hence, *the product of the number of linear units in length, breadth, and thickness of any rectangular solid is the number of cubic units in volume.*

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot, cu. ft.
27 cubic feet	= 1 cubic yard, cu. yd.
16 cubic feet	= 1 cord foot, cd. ft.
8 cord feet, 128 cubic feet	= 1 cord, cd.

NOTE.—The cord foot is rarely used.

1. How many cubic ft. in 3 cu. yds.?

2. How many cu. ft. does a box contain which is 6 ft. long, 3 ft. wide, and 4 ft. deep?

3. How many cords in a pile of wood 12 ft. long, 6 ft. high, and 4 ft. wide?

4. Change 12 cu. yds. 12 cu. ft. 1236 cu. in. to cu. in.

5. At $5.25 per cord, what is the value of a pile of wood 24 ft. long, 12 ft. high, and 8 ft. wide?

6. How many cubic inches in a tank 5 ft. long, 5 ft. wide, and 3 ft. deep?

7. At $1.50 a cubic yard, what will it cost to dig a cellar 18 ft. long, 12 ft. wide and 8 ft. deep?

8. How many cubic feet of air in a schoolroom 30 ft. long, 18 ft. wide, and 10.5 ft. high?

9. How many cubic yards of earth must be removed in digging a cellar, 16 ft. 8 in. long, 10 ft. 9 in. wide, and 5 ft. 4 in. deep?

10. What is the value of a pile of pine wood 100.5 ft. long, 10 ft. high, and 4.5 ft. wide, if it is worth $2 per cord?

11. How many bricks each 8 in. long, 4 in. wide, and 2 in. thick will it take to build a wall 12 ft. long, 8 in. wide, and 10 ft. high?

12. How many cords of wood in a pile containing 640 cu. ft.? 704 cu. ft.? 800 cu. ft.? 1280 cu. ft.?

13. What is the value of a pile of hickory wood 24 ft. long, 8 ft. high, and 4 ft. wide, if it is worth $4.75 per cord?

14. Illustrate by an original problem the use of cubic measure.

ENGLISH CURRENCY.

ART. 159.—English Currency or **Sterling Money** is the legal currency of Great Britain and Ireland.

The sovereign or pound sterling is the standard unit of English money. Its value in U. S. money is $4.8665.

The coins of Great Britain in general use are: *Gold*, the sovereign and half sovereign; *Silver*, the crown, half crown, florin, shilling, six-penny, and three-penny pieces; *Copper*, the penny, half-penny, and farthing.

TABLE.

4 farthings (far.)	=	1 penny, d.
12 pence	=	1 shilling, s.
2 shillings	=	1 florin, fl.
5 shillings	=	1 crown, cr.
20 shillings	=	1 sovereign, sov. 1 pound, £.

1 How many farthings in 4 d.? In 7 d.? In 12 d.?

2. How many pence in 20 far.? In 60 far.? In £1?

3. How many pence in 1 florin? In 1 crown? In 1 sov.?

4. Change £6 6 s. to pence; to farthings.

5. Change 982 d. to higher denominations.

6. Change 1440 s. to higher denominations.

7. Change 17 florins to pence; to farthings.

8. Change 24 crowns to sovereigns.

9. Change 2,848 sovereigns to florins.

10. Illustrate by English currency the method of changing denominate numbers to lower denominations.

11. Illustrate by English currency the method of changing to higher denominations.

TIME.

ART. 160.—Time Measure is used in measuring time.

TABLE.

60 seconds (sec.)	= 1	minute, min.
60 minutes	= 1	hour, h.
24 hours	= 1	day, d.
7 days	= 1	week, wk.
365 days	= 1	year, yr.
366 days	= 1	leap year.
100 years	= 1	century.

NOTE.—The exact length of a year is slightly less than 365¼ days. Every fourth year, therefore, has 366 days, the extra day being added to the shortest month, February.

In business transactions 12 months are considered a year, and 30 days a month.

February has 28 days except in leap year, when it has 29.

April, June, September, and November have 30 days each.

January, March, May, July, August, October, and December have 31 days each.

1. Change 4 d. 30 min. to seconds.

2. Change 1,051,200 minutes to years.

3. How many seconds are there in $1\frac{1}{2}$ years?

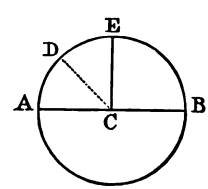
4. How many minutes from half past one in the morning to a quarter of 12 noon?

5. Lilian, with the type-writer, averages 50 words a minute, and receives 5 cents for each hundred words; how much does she earn in an hour?

6. Illustrate time measure by an original problem.

CIRCULAR OR ANGULAR MEASURE.

ART. 161.—Circular or **Angular Measure** is used in measuring angles and arcs of circles, and in finding the latitude and longitude of places.



A Circle is a plane figure bounded by a curved line every part of which is equally distant from a point within called the center.

The *Circumference* is the boundary line of a circle; the *Diameter* is the distance from one side to the other through the center; the *Radius* is a straight line drawn from the center to the circumference.

An **Arc** is any part of the circumference.

In the figure of a circle, what line is the circumference? What the diameter? Point out an arc; an angle; the vertex of an angle.

Every circle, no matter what its size, is divided into 360 equal parts called *Degrees*.

TABLE.

60 seconds (″)	= 1 minute, (′).
60 minutes	= 1 degree, (°).
90 degrees	= 1 quadrant or fourth of a circle.
360 degrees	= 1 circumference.

1. The length of a degree of longitude on the equator is 69.16 miles; how many miles does a ship sail in moving $3\frac{1}{2}$ degrees eastward along the equator?

2. How many miles does a ship sail in going $5\frac{1}{4}$ degrees westward on the equator?

3. Through how many degrees does the hour hand of a clock move in 6 hours?

4. Change 5° 5′ 25″ to seconds.

5. Change 32,360″ to higher denominations.

6. In sailing three fourths of the way around the earth, through how many degrees would you sail?

7. In 1616 William Baffin discovered the bay named for him and penetrated to 74° north latitude; Parry in 1827 reached 79° north latitude: how many minutes farther north did Parry go than Baffin?

8. In 1854 Kane reached 80° 30′ north latitude, and in 1861 Hayes reached 81° 30′; how many seconds farther north did Hayes go than Kane?

9. In 1876 Nares reached 83° 20′, and in 1882 Lieutenant Lockwood of the Greely expedition reached 83° 25′; how much farther north did Lieutenant Lockwood go than Nares?

10. The most northern land known is Cape Robert Lincoln, lying in latitude 83° 35′: how many degrees is the Cape from the North Pole?

11. Illustrate circular or angular measure by an original problem.

Miscellaneous Measures.

PAPER.

ART. 162.—Paper is measured by the following

TABLE.

24 sheets = 1 quire, qr. | 2 reams = 1 bundle.
20 quires = 1 ream. | 5 bundles = 1 bale.

COUNTING.

ART. 163.—The following table is used in counting certain articles:

TABLE.

12 units = 1 dozen, doz.
12 dozen = 1 gross, gro.
12 gross = 1 great gross.

A book made of sheets of paper each folded in 2 leaves is a *folio*; in 4 leaves, a *quarto* or 4*to*; in 8 leaves, an *octavo* or 8*vo*; in 12 leaves, a *duodecimo* or 12*mo*; in 18 leaves, an 18*mo*.

1. If I buy a gross of pens for $.80, and sell them at the rate of $1\frac{1}{2}$ cents apiece, how much profit do I make?

2. If foolscap paper costs $1.25 a ream, what is the profit per ream when sold for a cent a sheet?

3. A cabinet maker finds that each of 18 desks which he is making requires $5\frac{1}{4}$ dozen screws: how many screws are needed to complete the 18 desks?

4. If a scrivener uses 13 sheets of paper a day, at that rate how much will he use in 129 days?

5. If I purchase $7\frac{3}{4}$ gross of lead pencils and sell 50 dozen, how many will I have left?

6. Illustrate by an original problem the measurement of paper.

Addition of Denominate Numbers.

WRITTEN EXERCISES.

1. What is the sum of 3 gal. 1 qt. 1 pt.; 5 gal. 3 qts.; 8 gal. 2 qts. 1 pt. ?

Process.

gal.	qt.	pt.
3	1	1
5	3	0
8	2	1
17	3	0

Analysis.—We write like denominations in the same column, and begin with the lowest denomination to add.

1 pt. and 1 pt. are 2 pts., equal to 1 qt. We write 0 under the column of pts. and add 1 qt. to the column of qts. 1 qt., 2 qts., 3 qts. and 1 qt. are 7 qts. = 1 gal. and 3 qts. We write 3 under the column of qts., and add 1 to the column of gallons. 1 gal., 8 gal., 5 gal. and 3 gal. are 17 gal., which we write under the column of gallons.

NOTE.—The operations in denominate numbers differ from those in simple numbers merely in that they increase and decrease by an irregular scale, instead of by the decimal scale.

Find the sum of the following:

2.

pk.	qt.	pt.
5	7	0
6	3	1
4	6	1
		0

3.

yd.	ft.	in.
5	0	6
7	2	5
6	1	11
	1	

4.

d.	h.	min.
7	17	40
8	16	32
19	11	52
35		

5. What is the sum of 5 mi. 15 rds.; 13 mi. 13 rds.; 21 mi. 1 rd.; and 27 mi. 27 rds. ?

6. What is the sum of 13 wks. 6 d. 16 h. 37 min. 27 sec.; 11 wks. 5 d. 25 min. 28 sec.; 17 wks. 4 d. 10 h. 48 min. 48 sec.; 12 wks. 4 d. 9 h. 29 sec.; 7 h. 17 min. 17 sec.?

7. What is the sum of 5 d. 9 h. 20 min. 40 sec.; 12 d. 12 h. 18 min. 35 sec.; 16 d. 16 h. 18 min. 15 sec.?

8. What is the sum of 24 cu. yds. 19 cu. ft.; 16 cu. yds. 13 cu. ft.; 13 cu. yds. 16 cu. ft.; 11 cu. yds. 10 cu. ft.?

9. What is the sum of 5 ℔. 5 ℥ 3 ʒ 2 ℈ 11 gr.; 7 ℔. 9 ℥ 7 ʒ 1 ℈ 18 gr.; 10 ℔. 10 ℥ 5 ʒ 1 ℈ 11 gr.?

10. Mr. Gatzmer owns four farms. The first contains 140 A. 140 sq. rds. 11 sq. yds.; the second contains 162 A. 100 sq. rds. 5 sq. yds.; the third contains 150 A. 132 sq. rds. $1\frac{1}{2}$ sq. yds.; the fourth contains $120\frac{1}{2}$ A.: how much land does Mr. Gatzmer own?

11. Mr. Taylor purchased 5 prize pigs, whose respective weights were 3 cwt. 73 lb. 12 oz.; 4 cwt. 99 lb. 15 oz.; 5 cwt. 12 oz.; 3 cwt. 99 lb. 8 oz.; 4 cwt. 77 lb. 7 oz.: how much did all the pigs weigh?

12. Illustrate by an original problem addition of denominate numbers.

Subtraction of Denominate Numbers.

1. From 25 gal. 1 qt. 1 pt. 3 gi. take 18 gal. 2 qts. and 2 gi.

Process.

gal.	qt.	pt.	gi.
25	1	1	3
18	2	0	2
6	3	1	1

Analysis.—We write like denominations in the same column and begin with the lowest to subtract. 2 gi. from 3 gi. leaves 1 gi., which we write under the column of gi. 0 pts. from 1 pt. leaves 1 pt. which we write under the column of pts. 2 qts. cannot be taken from 1 qt., so we increase it by 1 gal. which equals 4 qts. 2 qts. from 5 qts. leaves 3 qts. which we write under the column of qts. Since we increased the minuend by 1 gal. we increase the subtrahend by 1 gal. 19 gal. from 25 gal. leaves 6 gal. which we write under the column of gallons.

2. Mr. Havens bought a barrel of cider containing 30 gal. 1 qt. When he reached home, 5 gal. 2 qts. 1 pt. had leaked away; how much remained in the barrel?

3. From a pile of wood containing 9 cds. a teamster hauled away 5 cds. 5 cu. ft.: how much remained?

4. George Washington was born February 22, 1732, and died December 14, 1799: what was his age when he died?

5. America was discovered Oct. 14, 1492, and the centennial of the evacuation of the city of New York by the British was celebrated November 26, 1883: how much time elapsed between the events?

6. Vice-President Thomas A. Hendricks was born September 7, 1819, and died November 25, 1885: what was his age when he died?

7. General George B. McClellan was born December 3, 1826, and died October 29, 1885: what was his age when he died?

8. Find out your age in years, months, and days.

9. Gold was discovered in California on the 2d of February, 1848: how much time has elapsed since then?

10. The Revolution began April 19, 1775, and ended January 20, 1783: how long did it last?

11. The civil war began April 11, 1861, and closed April 9, 1865: how long did it continue?

12. What is the time a note has to run, dated January 10, 1886, and payable March 15, 1886?

13. From the sum of 11 ℔. 5 ℥ 5 ʒ; 9 ℔. 6 ℥ 7 ʒ; 9 ℔. 8 ℥ 7 ʒ subtract the sum of 5 ℔. 9 ℥ 6 ʒ; 4 ℔. 4 ℥ 5 ʒ; 10 ℔. 2 ℥ 3 ʒ.

14. Illustrate by an original problem subtraction of denominate numbers.

Multiplication of Denominate Numbers.

1. Multiply 12 gal. 3 qts. 1 pt. by 4.

Process.

gal.	qt.	pt.
12	3	1
		4
51	2	0

Analysis.—4 times 1 pt. is 4 pts., equal to 2 qts. and 0 pts. We write 0 in the place of pts. and add 2 to the product of qts. 4 times 3 qts. are 12 qts. and 2 qts. added are 14 qts., equal to 3 gal. and 2 qts. We write 2 in the place of qts. and add 3 to the product of gal. 4 times 12 gal. are 48 gal., and 3 gal. added are 51 gal., which we write in the place of gal.

2. Multiply £ 10 5 s. 9 d. by 6.

3. Multiply 5 gal. 2 qts. 1 pt. by 12.

4. Multiply 5 T. 2 cwt. 15 lb. 11 oz. by 7.

5. Zechariah is 9 yr. 8 mo. 15 d. old, and his uncle is 4 times as old: how old is his uncle?

6. Mr. Jones piled on his wagon 27 sacks of wheat, each containing 3 bush. 1 pk. 1 qt.: how large a load did his wagon contain?

7. Multiply 15 cu. yds. 25 cu. ft. by 10.

8. Multiply 12 sq. yds. 7 sq. ft. 84 sq. in. by 9.

9. Multiply 4 reams 11 quires 11 sheets by 8.

10. Multiply 7 gal. 3 qts. 1 pt. 2 gi. by 11.

11. Multiply 5 d. 9 h. 20 min. 25 sec. by 7.

12. Multiply 18 mi. 150 rds. by 16.

13. If a pipe discharges 231 gal. 2 qts. 1 pt. of water an hour, how much will it discharge in 24 hours?

14. Mr. B owned 3 farms, each containing 35 A. 120 sq. rds.: how much land did he own?

15. A farmer sold 6 loads of hay, each containing 16 cwt. 85 lb.: how much did he sell?

16. Illustrate by an original problem multiplication of denominate numbers.

Division of Denominate Numbers.

1. Divide 37 bush. 2 pks. 6 qts. by 5.

Process.

bu.	pk.	qt.
5)37	2	6
7	2 .	$1\frac{1}{5}$

Analysis.—$\frac{1}{5}$ of 37 bush. is 7 bush., and 2 bush. remaining, which is 8 pks. 8 pks. and 2 pks. = 10 pks. $\frac{1}{5}$ of 10 pks. = 2 pks. $\frac{1}{5}$ of 6 qts. is 1 qt. and 1 qt. remaining, which is $\frac{1}{5}$ of 5 qts.

2. Divide 60 bush. 3 pks. 7 qts. by 8.

3. Divide 17 cwt. 80 lb. 8 oz. by 9.

4. Divide 7 gal. 3 qts. 1 pt. by 6.

5. Divide $42\frac{1}{2}$ gal. by 7.

6. Divide 12 gal. 3 qts. 1 pt. by 5.

7. Divide 18 h. 30 min. 40 sec. by 4.

8. Divide 18 cwt. 38 lb. 10 oz. by 6.

9. Divide 20 lb. 10 oz. 9 pwt. by 7.

10. Divide 15 bush. 3 pks. 6 qts. by 8.

11. Divide 20 cu. yds. 20 cu. ft. by 6.

12. Divide 30 sq. yds. 8 sq. ft. 100 sq. in. by 5.

13. How many sacks, each holding 2 bush. 3 pk., can be filled from a bin containing 27 bush. 2 pk. of wheat?

14. How many bottles, each holding 2 qts. 1 pt., can be filled from a demijohn holding 7 gal. 2 qts. of wine?

15. Illustrate by an original problem division of denominate numbers.

REVIEW QUESTIONS.

What is a denominate number? Give an example. Name the two kinds of denominate numbers. What is a simple denominate number? Illustrate. What is a compound denominate number?

For what is avoirdupois weight used? Repeat the table. How many pounds formerly equaled a quarter? How many pounds a hundred weight? How many pounds make the long or gross ton?

Give the rule for changing denominate numbers to lower denominations; for changing denominate numbers to higher denominations.

For what is troy weight used? Repeat the table. For what is apothecaries' weight used? Repeat the table.

For what is dry measure used? What are the dimensions of the standard bushel? How many cubic inches does it contain? Repeat the table. When must even measure be used? When must heaped measure be used?

For what is liquid measure used? Repeat the table. How many cubic inches in a gallon? What is said of the barrel and hogshead?

For what is linear or long measure used? Repeat the table.

For what is square measure used? What is a surface? What is a square inch? A square foot? What does the area of a square or rectangle equal? Repeat the table.

For what is cubic measure used? What is a cubic foot? How do you ascertain the cubic contents of a body? Repeat the table.

For what is time measure used? Repeat the table. What is said of the exact length of a year? What years are leap years?

What is considered a year in business transactions? Which is the shortest month? When does it have 29 days?

For what is circular or angular measure used? What is a circle? What is the circumference of a circle? Its diameter? Its radius? What is an arc? An angle? Into how many parts is every circle supposed to be divided? Why is the number 360 used?

Give the table for the measurement of paper. For counting certain articles. What is meant by a folio? A quarto? An octavo? A duodecimo? An 18mo?

How are denominate numbers added? How are denominate numbers subtracted?

How are denominate numbers multiplied? How are denominate numbers divided? What is the process when the dividend and divisor are denominate numbers?

What is English currency? What is the standard unit? What is its value in Federal money? What English coins are made of gold? Of silver? Of copper? Repeat the table.

PART V.

Percentage.

ART. 164.—Per centum or per cent. means *by the hundred.* One per cent. means one hundredth, expressed also $\frac{1}{100}$ or .01; five per cent., or five hundredths, is expressed $\frac{5}{100}$ or .05; ten per cent. is expressed $\frac{10}{100}$ or .10 or .1. Per cent. is usually written %.

Per cent. is expressed in several ways, thus:

5	per cent., or	5 %	=	$\frac{5}{100}$	=	.05.
4	per cent., or	4 %	=	$\frac{4}{100}$	=	.04.
15	per cent., or	15 %	=	$\frac{15}{100}$	=	.15.
$12\frac{1}{2}$	per cent., or	$12\frac{1}{2}$ %	=	$\frac{25}{200}$	=	.125.
$6\frac{1}{4}$	per cent., or	$6\frac{1}{4}$ %	=	$\frac{25}{400}$	=	.0625.
$\frac{1}{2}$	per cent., or	$\frac{1}{2}$ %	=	$\frac{1}{200}$	=	.005.
$\frac{1}{4}$	per cent., or	$\frac{1}{4}$ %	=	$\frac{1}{400}$	=	.0025.
125	per cent., or	125 %	=	$\frac{125}{100}$	=	1.25.

ART. 165.—Every number is $\frac{2}{2}$ or $\frac{3}{3}$ or $\frac{5}{5}$ or $\frac{10}{10}$ or $\frac{50}{50}$ or $\frac{100}{100}$ of itself, since each of these fractions is equal in value to 1. It follows, therefore, that when we speak of 100 per cent. of any amount, we mean the whole amount.

ORAL EXERCISES.

1. Tell what part of any sum is meant by 50% of it; by 25%; by 75%; by 20%; by 40%; by 60%; by 80%; by 90%; 125%; 150%; 175%; 200%.

2. What per cent. of a number is $\frac{1}{100}$ of it? $\frac{5}{100}$? $\frac{10}{100}$? $\frac{20}{100}$? $\frac{25}{100}$? $\frac{30}{100}$? $\frac{65}{100}$? $\frac{70}{100}$? $\frac{85}{100}$?

3. What is 5% of 100? 200? 500? 1000?

4. What is 10% of 50? 80? 90? 190?

5. What is 25% of 200? 300? 400? 600?

6. What is 50% of 300? 180? 250? 300?

7. What is 100% of 500? 400? 100? 750?

8. Express decimally 1%; 5%; 6%; 8%; 10%; $12\frac{1}{2}\%$; 20%; 25%; $33\frac{1}{3}\%$; 50%; $62\frac{1}{2}\%$; 75%; 80%; 100%; 125%; 150%; 200%.

DEFINITIONS.

ART. 166.—Percentage is the result obtained by taking a certain per cent. of the base.

ART. 167.—The **Base** is the number on which the percentage is computed.

ART. 168.—The **Rate** per cent. is the number of hundredths of the base to be taken.

ART. 169.—The **Amount** is the sum of the base and percentage.

ART. 170.—The **Difference** is the remainder after subtracting the percentage from the base.

Thus, if we take 12% of \$500, the result is \$60. The *percentage* is \$60; the *base* \$500; the *rate* .12; the *amount* \$560, and the *difference* \$440.

ART. 171.—The percentage, base and rate bear such relations to each other that if any two are given, the third can be found.

ART. 172.—To find the percentage, the base and rate being given.

ORAL EXERCISES.

1. What is 6% of 100 bushels?

Solution.—6% of any number is $\frac{6}{100}$ of that number; $\frac{1}{100}$ of 100 bushels is 1 bushel, and $\frac{6}{100}$ is 6 times 1 bushel, which is 6 bushels. Therefore, 6% of 100 bushels is 6 bushels.

2. What is 1% of \$300? 4%? 5%? 6%?

3. What is 1% of 250 gallons? 2%? 5%? 6%?

4. What is 6% of 100 years? 8%? 7%? 5%?

5. What is 20% of 100 days? 2,000 days? 1,500 days?

6. What is 5% of 120 feet? 200 feet? 300 feet?

7. What is 8% of 100 pounds? 250 pounds? 300 pounds?

WRITTEN EXERCISES.

1. What is 8% of \$425?

Process.

\$425
.08
\$34.00

Analysis.—8% of any number is .08 of it, and .08 of \$425 is \$34.

Rule for finding the Percentage, the base and rate being given.—*Multiply the base by the rate per cent., expressed decimally.*

2. What is 7 % of 650 hours? 8%? 9%?

3. What is 6 % of 725 days? 5%? 7%?

4. What is 9 % of 848 weeks? 7%? 3%?

5. What is 11 % of 66 degrees? 13%? 18%?

6. What is 15 % of 6.5 inches? 16%? 25%?

7. What is 18 % of \$75.40? 20%? 35%?

8. What is $3\frac{1}{3}$% of 125 ounces? $4\frac{1}{2}$%? 99%?

9. What is $5\frac{1}{4}$% of 160 feet? $16\frac{2}{3}$%? $37\frac{1}{2}$%?

10. What is $62\frac{1}{2}$% of \$250? $87\frac{1}{2}$%? 100%?

11. What is 10 % of 10 miles? 100%? 1%?

12. What is $\frac{3}{8}$% of 200 lbs.? $\frac{1}{4}$%? 110%?

13. I bought a horse for \$150 and sold it at an advance of 15%: what did I gain?

14. My salary of $95 a month was reduced 10%: what was it after the reduction?

15. Mr. Smith owned a farm of 175 acres, and gave 33⅓ per cent. of it to his son: how many acres had he left?

16. In a school containing 120 pupils, 35% of them are boys: how many girls are in the school?

17. George bought a knife for $2.25, and a toy wagon for $3.50. He sold the knife at a gain of 8% and the wagon at a loss of 6%: did he gain or lose, and how much?

18. Find 8⅓% of 120 yards; 16⅔% of $840; 33⅓% of 64 T.

19. Find 50% of 75.5; 75% of 24.56; 95% of 125.4.

20. Which is greater, 5% of $4,500 or 4% of $5,200?

21. Which is less, 12% of $48,000 or 10% of $57,600?

22. A man having $8,200 in bank drew out 10% at one time and then 10% of the remainder: how much had he left on deposit?

23. Illustrate by an original problem the method of finding the percentage when the base and rate are given.

ART. 173.—To find the rate when the base and percentage are given.

ORAL EXERCISES.

1. What per cent. of 10 is 5?

Solution.—10 is 100% of 10; and since 5 is ½ of 10, it is ½ of 100%, or 50% of 10.

2. What per cent. of 12 is 4? What per cent. of 40 is 5?

3. What per cent. of 60 is 12? What per cent. of 72 is 8?

4. What per cent. of 80 is 10? What per cent. of 96 is 24?

5. What per cent. of 144 is 48? What per cent. of 180 is 60?

6. Simpson had 60 peaches and ate 20 of them: what per cent. did he eat?

7. A carriage bought for $75 was sold for $15 less: what per cent. was lost?

8. Of eighteen boys engaged in a game of ball, 3 were injured: what per cent. of the players were injured?

9. Fifteen girls were invited to a tea-party, but 5 did not go: what per cent. of the number invited went to the party?

WRITTEN EXERCISES.

1. What per cent. of 900 is 150?

Process.

$150.00 \div 900 = .16\frac{2}{3}$.

Analysis.—900 is 100% of 900; and since 150 is $\frac{1}{6}$ of 900, it is $\frac{1}{6}$ of 100%, $= .16\frac{2}{3}$% of 900.

REMARK.—To obtain hundredths in the quotient, the dividend must contain two more decimal places than the divisor.

Rule for finding the rate when the base and percentage are given.—*Divide the percentage by the base, and the quotient, expressed decimally, is the rate per cent. required.*

2. What per cent. of 360 is 90? Is 40? Is 60?

3. What per cent. of 50 is 40? Is 30? Is 20?

4. What per cent. of 60 is 50? Is 40? Is 60?

5. What per cent. of 80 is 100? Is 50? Is 90?

6. What per cent. of $500 is $10? Is $50? Is $100?

7. What per cent. of $5 is 50 cts.? Is 25 cts.? Is 75 cts.?

8. What per cent. of 120 is 80? Is 90? Is 100?

9. What per cent. of $\frac{3}{4}$ is $\frac{1}{2}$? Is $\frac{1}{4}$? Is 1?

10. John having $28, expended $7: what per cent. of his money did he expend?

11. In a certain school of 100 pupils, 60 study history: what per cent. of the number study history?

12. What per cent. of \$6.80 is \$.25? Is \$3.40?

13. Mr. Lansing bought a house for \$1,200, and paid \$800 cash: what per cent. of the cost remained due?

14. If silk is bought at \$2.25 per yard, and sold at \$2.50, what per cent. profit is made?

15. If John bought an apple for two cents and sold it for three cents, what per cent. profit did he make?

16. A boy having \$1.10 gave 90 cents for a knife: what per cent. did he spend and what per cent. had he left?

17. If a miller takes 3 quarts of every bushel he grinds, what per cent. does he take?

18. A farmer having 48 hogs sold $\frac{1}{4}$ to one man, and $\frac{1}{4}$ of the remainder to another: what per cent. remained?

19. In a school there are 36 boys and 24 girls: what per cent. of the whole number are girls?

20. After 42 gallons had leaked from a hogshead containing 63 gallons, what per cent. remained?

21. Illustrate by an original problem the method of finding what per cent. one number is of another.

ART. 174.—To find the base when the percentage and rate are given.

ORAL EXERCISES.

1. 50 is 25% of what number?

Solution.—25% of any number is $\frac{25}{100}$ or $\frac{1}{4}$ of it; and since $\frac{1}{4}$ of a certain number = 50, $\frac{1}{4}$, or that number, is 4 times 50 = 200.

2. 60 is 30% of what number? 40%? 50%?

3. 75 is 75% of what number? 20%? 50%?

4. 80 is 20% of what number? 10%? 15%?

5. 70 is 35% of what number? 25%? 40%?

6. 100 is 50% of what number? 25%? 60%?

7. 120 is 80% of what number? 50%? 200%?

8. 150 is $33\frac{1}{3}\%$ of what mber? 75%? 150%?

9. 200 is 40% of what number? 60%? 100%?

10. If 20% of the cost of a coat is \$3, what is the whole cost?

11. Mr. Howell sold 100 acres of land, which was $33\frac{1}{3}\%$ of the whole number of acres in his farm: how many acres were in his farm at first?

WRITTEN EXERCISES.

1. 240 is 5% of what number?

Process.

$240 \div .05 = 4800.$

Analysis.—Since 5% or $\frac{1}{20}$ of a certain number = 240, $\frac{20}{20}$, or that number, is 20 times 240 = 4800.

Rule for finding the base when the percentage and rate are given.—*Divide the percentage by the rate per cent. expressed decimally, and the quotient is the required number.*

2. 300 miles is 20% of how many miles?

3. \$38.50 is 25% of how many dollars?

4. 165 gallons is 28% of how many gallons?

5. \$6.25 is 15% of how many dollars?

6. \$16.50 is 2% of how many dollars?

7. \$28.75 is $12\frac{1}{2}\%$ of how many dollars?

8. \$4.70 is $16\frac{2}{3}\%$ of how many dollars?

9. 17 rods is 7% of how many rods?

10. 148 feet is 70% of how many feet?

11. Mr. Kenyon rents his house for \$250, which is 6% of its value: what is its value?

12. 30% of the entire number of boys enrolled in school is 33: how many boys are enrolled?

13. A clerk spends 10% of his salary for board, 30% for clothing, and saves \$300: what is his salary?

14. In July, 1864, a gold dollar was worth \$2.85 currency: what was the gold value of a dollar in currency?

15. A merchant sold \$7,200 worth of goods, and had 60% left: how much was his entire stock worth?

16. A man owned 50% of a steamboat and sold 50% of his share for \$16,200: what was the value of the steamboat?

17. A clerk spends \$480 and has 60% of his salary left: what is his salary?

18. A man drew $16\frac{2}{3}\%$ of the money he had in bank to pay a debt of \$1,248: how much had he left in bank?

19. A farmer raised 284 bushels of wheat, which was $33\frac{1}{3}\%$ of his crop of corn: how much corn did he raise?

20. A man who owned 60% of a factory sold 25% of his share for \$15,000: what was the value of the factory?

21. A creditor paid me \$32, which was 25% of the amount he owed me: how much did he still owe?

22. A speculator sold a house for \$12,000, which was 90% of its cost: how much did he lose?

23. A farm cost \$6,000. One fourth of this sum is 75% of what the house cost: what was the cost of the house?

24. A man owning $\frac{1}{4}$ of a ship sold 50% of his share for \$2,250: what was the value of the ship?

25. Illustrate by an original problem the method of finding the base when the percentage and rate are given.

Interest.

DEFINITIONS.

ART. 175.—Interest is money paid for the use of money.

ART. 176.—The **Principal** is the money for which interest is paid.

ART. 177.—The **Rate of Interest** is the per cent. of the principal paid for its use for a year or month.

ART. 178.—The **Time** is the period during which the money is on interest.

ART. 179.—The **Amount** is the sum of the principal and interest.

ART. 180.—Simple Interest is interest on the principal only.

ART. 181.—Compound Interest is interest on the principal and the interest.

ART. 182.—Legal Interest is interest at a rate fixed by law.

ART. 183.—Usury is interest at a rate higher than that fixed by law.

ORAL EXERCISES.

1. What is the interest on \$100 for 1 yr. at 6%?

Solution.—The interest on \$1 for 1 year at 6% is 6 cents = \$.06; and the interest on \$100, for the same time and at the same rate, is 100 times \$.06 = \$6.

2. What is the interest on \$200 for 1 yr. @ 5%?
3. What is the interest on \$50 for 2 yr. @ 6%?

4. What is the interest on \$300 for $2\frac{1}{2}$ yr. @ 6%?
5. What is the interest on \$400 for 3 yr. @ 5%?
6. What is the interest on \$500 for 5 yr. @ 4%?
7. What is the interest on \$25 for 5 yr. @ 6%?
8. What is the interest on \$600 for 2 yr. @ 6%?
9. What is the interest on \$10 for 6 yr. @ 6%?
10. What is the interest on \$1500 for 1 yr. @ 6%?
11. What is the interest on \$200 for 1 yr. and 6 mo. @ 6%?

WRITTEN EXERCISES.

1. What is the interest on \$560 for 2 yr. 9 mo. and 27 days at 7%?

Process.

\$560
.07
12)39.20 Int. for 1 yr.
\$3.266 Int. for 1 mo.
33.9
29394
9798
9798
\$110.717

Analysis.—The interest for 1 yr. at 7% is .07 of the principal, and the interest for 1 mo. is $\frac{1}{12}$ of that sum; for 2 yr. 9 mo. (33 mo.) and 27 da. (.9 mo.), it must be 33.9 times \$3.266, which is \$110.717. Therefore, etc.

Rule for computing interest.—*Multiply the principal by the rate per cent., divide the product by* 12, *and multiply the quotient by the total number of months and tenths of a month for which interest is required.*

NOTE.—Since 30 days make an interest month, days divided by 3 give tenths of a month. Thus, 9 days = .3; 12 days = .4; 13 days = .$4\frac{1}{3}$; 26 days = .$8\frac{2}{3}$, and so on. This method being universally applicable and simple, it is deemed best not to confuse the learner by giving other methods.

2. What is the interest on \$145 for 1 yr. 5 mo. 24 da. @ 6%? @ 5%? @ 8%? @ 10%?

3. What is the interest on $284.75 for 1 yr. 3 mo. 18 da. @ 6%? @ 7%? @ 8%? @ 10%?

4. What is the interest on $695.23 for 2 yr. 2 mo. 6 da. @ 6%? @ 7%? @ 5%? @ 4%?

5. What is the interest on $384.71 for 3 yr. 3 mo. 3 da. @ 6%? @ 5%? @ 3%? @ 7%?

6. What is the interest on $1238.19 for 1 yr. 1 mo. 4 da. @ 6%? @ 3%? @ 5%? @ 7%?

7. What is the interest on $97.38 from Oct. 23, 1881 to April 12, 1885, at 5%?

8. What is the amount of $981.49 for 2 yr. 11 mo. 3 da. @ 4%? @ 3%? @ 7%? @ 6%?

9. What is the amount on $491.03 for 3 yr. 7 mo. 5 da. @ 3%? @ 5%? @ 7%? @ 6%?

10. What is the amount on $222.22 for 7 yr. 7 mo. 4 da. @ 6%? @ 5%? @ 4%? @ 3%?

Find the *amount* of

11. $125.25 from Oct. 13, 1881, to March 19, 1883, @ 6%.

12. $250.48 from Feb. 19, 1881, to Jan. 13, 1882, @ 5%.

13. $240.50 from Jan. 16, 1883, to July 1, 1884, @ 4%.

14. $242.21 from March 1, 1882, to Aug. 16, 1885, @ 7%.

15. $360.75 from July 14, 1883, to Feb. 11, 1884, @ 8%.

16. $178.78 from Sept. 11, 1880, to Dec. 19, 1881, @ 5%.

17. $341.58 from Aug. 28, 1884, to Nov. 26, 1885, @ 3%.

18. Illustrate by an original problem the method of computing interest.

Problems for Review.

ORAL EXERCISES.

1. What will $4\frac{1}{2}$ bushels of apples cost at $2 a bushel? At $6 a bushel? At $7\frac{1}{2}$ a bushel?

2. What will $2\frac{1}{2}$ gallons of oil cost at 30 cents a gallon? At 40 cents? At 50 cents?

3. Charles picked a half bushel of cherries and sold them at 10 cents a quart: how much did he receive?

4. If an express train travels a mile in $\frac{5}{6}$ of a minute, how far, at that rate, will it go in ten minutes?

5. If a boy earns $14 a month and his expenses are $6 a month, how much will he save in a year?

6. How many quart baskets in $2\frac{1}{2}$ pks. of berries?

7. The sum of two numbers is 30, and their difference is 6: what are the numbers?

8. The sum of two numbers is 24 and their difference 16: what is the less number?

9. How much is $\frac{7}{8}$ of 56? $\frac{5}{6}$ of 72?

10. How much is $\frac{2}{3}$ of $18\frac{3}{4}$? $\frac{1}{8}$ of $16\frac{2}{3}$

11. Mr. Grimes sold his cow for $36, which was $\frac{3}{4}$ of what he paid for her: what did the cow cost him?

12. Mr. Simpson sold his horse for $120, which was $\frac{1}{3}$ more than he paid for him: what did he gain?

13. Mr. Lansom planted 30 acres in corn. If that is $\frac{3}{10}$ of the number of acres in his farm, how many acres does his farm contain?

14. Mary, having some ice cream, gave $\frac{1}{4}$ to her brother, and $\frac{1}{3}$ of what was left to her sister: what part did she keep for herself?

15. A boy having $\frac{1}{2}$ of a watermelon, shared it equally with 3 of his friends: what part of a melon did each receive?

16. If a pole $15\frac{2}{3}$ feet long is cut into 5 equal pieces, how long will each piece be?

17. If it takes 5 men $\frac{3}{4}$ of a day to dig a ditch, how long will it take 1 man to do the same work?

18. A man owning $\frac{5}{7}$ of a farm, sold $\frac{3}{5}$ of his share:

what part of the whole farm did he sell, and what part had he remaining?

19. A boy spent $\frac{1}{2}$ of his time in eating and sleeping, $\frac{1}{2}$ the remainder in play, $\frac{1}{3}$ the remainder in idleness, and the rest in study: how many hours did he study daily?

20. Jennie wrote 20 words on her slate, and misspelled 5 per cent.: how many words did she misspell?

21. If Jennie had missed 5 out of 20 words, what per cent. would she have missed?

22. A person bought a team for $200, and sold it for $250: what per cent. did he gain?

23. For what must butter which cost 20 cents a pound be sold, so as to gain 10%?

24. What will 4 pounds of beef cost at $6\frac{1}{4}$ cents a pound?

25. A silk hat costing $4 was sold for $4\frac{1}{2}$: what per cent. was gained?

26. How much is 20% of 20% of $75?

27. If I buy apples at 2 cents apiece and sell them at the rate of 2 for 5 cents, what per cent. do I gain?

28. I have a wagon that cost me $48. What must I ask for it, that I may drop 25% and still gain 25%?

29. In a certain school 48 pupils study history, which is $33\frac{1}{3}$% of the number of pupils in the school: how many pupils in the school?

30. The number of men employed in Mr. Brown's factory is 360, which is 80% of the number employed in Mr. Smith's factory: how many men does Mr. Smith employ?

31. If New York has 50% more population than Philadelphia, what per cent. has Philadelphia less than New York?

WRITTEN EXERCISES.

1. If Richard rides 7.25 miles an hour on his bicycle, how far will he ride in 9.75 hours?

2. If 7.5 acres produce 250.25 bushels of corn, what is the average yield an acre?

3. At $\$.16\frac{2}{3}$ a dozen, how many eggs can be bought for $3?

4. There are 1,760 yards in a mile: how many yards in $\frac{3}{20}$ of a mile?

5. Mr. Clayton sold $\frac{3}{7}$ of his farm for $8,400: at that rate, what is his entire farm worth?

6. Mr. Godfrey, owning $\frac{2}{11}$ of a silver mine, sold $\frac{2}{3}$ of his share for $13,000: at that rate, what was the entire mine worth?

7. What will $3\frac{1}{2}$ pecks of nuts cost at $2\frac{1}{2}$ cents a pint?

8. What will a hogshead of molasses containing 63 gallons cost at $.10 a quart?

9. Lucius and Owen started from points 50 miles apart, and walked toward each other. Lucius walked $4\frac{1}{4}$ miles the first hour; $5\frac{1}{2}$ miles the second hour, and $4\frac{3}{4}$ miles the third. Owen, the first hour walked $5\frac{1}{2}$ miles; the second, $5\frac{1}{4}$, but during the third hour he walked $4\frac{1}{6}$ miles back toward the point from which he started. How far apart are Lucius and Owen at the end of three hours?

10. How many yards of cloth at $\$.18\frac{3}{4}$ cents a yard can be bought for $11\frac{1}{4}$ dozen eggs at $.16 a dozen?

11. If Maud S. trots a mile in 2 min. and $9\frac{1}{4}$ seconds, how far would she trot in an hour if it were possible for her to maintain the same speed?

12. From a roll of cloth containing $30\frac{3}{4}$ yards, a tailor cuts a suit containing $11\frac{1}{3}$ yards, another containing $9\frac{5}{8}$ yards, and a third containing $9\frac{1}{7}$ yards: how many yards were left?

13. Mr. Darrow finds that it takes him $3\frac{3}{4}$ days to cut a certain pile of wood: what part of it can he cut in $2\frac{1}{4}$ days?

14. If the driving wheels of an express engine are 20 feet and 6 inches in circumference, how many times will each one turn around in going from Philadelphia to Jersey City, the distance being 89.7 miles?

15. If a clock tick 172,800 times in 1 day, how many times will it tick in 6 h. 6 min. 6 sec.?

16. If $6\frac{1}{2}$ sheets of tin weigh 30 lb. 10 oz., how much tin is there in each sheet?

17. Mr. Freeland bought a load of hay weighing 2,250 pounds at the rate of $13.75 a ton: what did the hay cost him?

18. I own a pile of wood 40 feet long and 4 feet wide: how high is it if it contains 9.5 cords?

19. How many spoons weighing 2 oz. 4 pwt. each can be made from 5 lb. 6 oz. of silver?

20. A drove of 8,450 cattle on their way north from Texas was stampeded by Indians and 30% stolen; the hoof disease carried off 20% of what were left: how many cattle remained?

21. A grocer having 580 bbl. of flour, sold $\frac{2}{29}$ of it: what per cent. of the whole remained?

22. A watch and chain cost $250; the chain cost $80: what % is that of the cost of the watch?

23. Coal bought at $6.50 a ton was sold at $6.45 a ton: what was the loss per cent.?

24. A gentleman having damaged his watch, sold it at a loss of $10, which was $7\frac{1}{2}\%$ of the cost: what was the cost?

25. An oil well flows at the rate of 4,250 gallons a day, but $12\frac{1}{2}\%$ of the yield is lost by leakage: how much oil is saved daily?

26. A clerk saves $700, which is 37½% of his income: what is his income?

27. A letter carrier during the month of June traveled 800 miles, which was 25% more than ½ of what he traveled in July: how far did he travel in July?

28. Martin sold a farm for $6,976, and thereby gained 9%: what did the farm cost him?

29. The number of pupils enrolled in a certain school is 360, and 7½% are absent: how many pupils are present?

30. A broker sold $18,500 worth of cotton, and charged ¼% for his services: what was his commission?

31. Wilkins owned ⅔ of a farm worth $16,400; he sold ½ of his share to James and deducted 6% for cash: how much did he receive?

32. The area of California is 158,360 square miles, and the estimated population of the earth is 1,471,607,808 people. If this land were equally divided among all the inhabitants of the earth, how many square rods would each receive?

REVIEW QUESTIONS.

What does *per cent.* mean? Show the different ways in which *per cent.* may be expressed? What is percentage? What is meant by the base? By the rate per cent.? By the amount? By the difference?

How do you find the percentage when the base and rate per cent. are given? How do you find the rate when the base and percentage are given? How do you find the base when the percentage and rate per cent. are given?

What is interest? What is the principal? What is rate of interest? The time? The amount? What is simple interest? Compound interest? Legal interest? Usury?

Give the rule for computing interest.

ANSWERS TO WRITTEN PROBLEMS.

No answers are given to the oral exercises, and in some instances answers to the written problems are omitted in order that the pupil may learn to rely on himself.

Page 36.

1. A. D. 1799.
2. A. D. 1607.
3. $7077.
4. A. D. 1848.

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5. $20,886.
6. 3466 acres.
7. 2578 miles.
8. 35,246,633.
9. 50,155,783.
10. 6,679,943.
11. 58,605.
12. 216,576.

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13. 4,698,098.
14. 9,908,799.
15. $37,426,262.
16. $53,000.
17. $134,800.
18. $32,600.
19. 12,544.
20. $27,800.

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14. 4897.
15. 3123.
16. 6084.
17. 17228.
18. 3,917,910.
19. 904,418.
20. 198 acres.
21. 391.

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22. 174 years.
23. $387.
25. $1090.
26. 67 years.
27. 5850 miles.
28. 13,210 sq. m.
29. 107,420 sqr. miles.
30. 311,610 sqr. miles.
31. 5,626,284 sq. miles.
32. 7,788,195 sq. miles.
33. $295.
34. 588,330,036.

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1. $490.76.

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2. $2,618.
3. $12,960.
4. $14.
5. $43.50.
6. 16 years.
7. $6,365.
8. 8680.
9. $3,950.
10. $3,046.
11. 2175 sq. m.

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12. 103,910sq.m
13. 1,230,917.
14. 1,330,913less than Paris.
15. 2,854,250 sq. miles.
16. 360,276more.
17. 36,843,291.
18. 70,345 miles.

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20. $87.21.
21. $47163.
22. 129297.
23. 202476.

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24. 417677.
25. $7,637,232.
26. 22995.
27. 27132 wks.
28. $21252.
29. $9176.
30. $16731.
31. 9184.
32. $5888.
33. $53025.
34. $14896.
35. 42823 miles.
36. 98790.
37. 4134240.
38. 1632960.
39. $113391.

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1. 2400 miles.

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2. $30.
3. $540.
4. Gained $80.
5. $260.68.
6. $2100.
7. None.
8. Lost $100.
9. 9 feet.
10. 336.
11. $4581.

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12. Gained $100
14. 849.
15. 570.

16. $297.
17. 3960000 ft.
18. $275.
19. 216000
20. James, 20 c.
21. 5200 miles.

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22. $54,850.
23. $10,892.
24. The same.
25. Gained $213
26. $8,000.
27. $8,900.
28. Gained $855
29. 185 miles.

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30. 1 mile.
32. 44,425,600.
33. $22.
34. $185.
35. $25220.

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1. 24 weeks.
2. $17\frac{2}{16}$ yards.
3. 16 mos.
4. $30.
5. Brown lost $25; Jones lost $210.
6. $35.

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7. 570.
8. $12.
9. $46.80.
10. 51 days.
13. $346\frac{28}{432}$.
15. 74,040.
16. 9.
17. 9.
18. 11.
19. $1344.

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20. $2500.
21. $346.
22. $322.
23. 63 cents.
24. 128.
25. 81.
26. 16.
27. $98.
28. 49 cents.

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12. $233\frac{7}{20}$.
13. $278\frac{7}{15}$ tons.
14. $247\frac{37}{60}$ acres.
15. $152\frac{3}{4}$ miles.
16. $187\frac{21}{40}$.
17. $81\frac{1}{8}$ cords.
18. $185\frac{23}{40}$ yards.

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19. $1277\frac{169}{845}$ m.
20. $31\frac{53}{80}$.
21. $138\frac{49}{60}$ tons.
22. $483\frac{1}{18}$ yards.
23. $827\frac{13}{16}$ miles.
24. $591\frac{53}{80}$ acres.
25. $362\frac{11}{20}$.
26. $3,466\frac{37}{40}$.
27. $426\frac{7}{16}$ gal.
28. $131\frac{128}{315}$ miles

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16. $\frac{1}{2}$.
17. $40\frac{1}{2}$ acres.
18. $3\frac{6}{20}$ bushels.
19. $21\frac{17}{20}$.
20. $14\frac{1}{4}$.
21. $289\frac{23}{48}$ lbs.
23. $34\frac{38}{40}$.
24. $31\frac{3}{20}$.
25. $101\frac{17}{18}$.
26. $7\frac{7}{15}$ yards.
27. $15\frac{1}{2}$ cords.
28. $9\frac{23}{30}$ barrels.
29. $71\frac{7}{8}$ pounds.
30. $117\frac{3}{4}$ yards.

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23. $\frac{7}{60}$.
24. $1\frac{1}{8}$.
25. $3\frac{1}{14}$.
26. $\frac{1}{280}$.
27. $14\frac{1}{4}$.
28. $11\frac{13}{30}$ yards.
29. $2\frac{19}{20}$.
30. Gained $\frac{3}{4}$.
31. $44\frac{7}{12}$ tons.
32. $32\frac{19}{30}$ bush.
33. $46\frac{1073}{2520}$ yrs.
34. $22\frac{11}{20}$.

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18. $47\frac{7}{16}$.
19. $219\frac{3}{8}$.
20. $16\frac{23}{40}$.
21. 5 cents.
22. 2.58\frac{1}{2}$.
23. 4.16\frac{4}{5}$.
24. $13.50.
25. 4.18\frac{1}{2}$.
26. 77\frac{1}{20}$.
27. 460\frac{3}{5}$.
28. Joseph $7.-81$\frac{1}{4}$, John 8.28\frac{1}{5}$.
29. Charles $202\frac{1}{3}$ acres, Jackson, $101\frac{1}{6}$.
30. $68\frac{1}{4}$c.
31. 2.84\frac{3}{8}$.
32. $6\frac{3}{4}$ miles.

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21. 75 bales.
22. $1\frac{1}{2}$ lbs.
23. 1\frac{1}{2}$.
24. 12.

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25. $53\frac{5}{14}$ pounds.
26. $9\frac{9}{34}$ barrels.
27. $4\frac{2}{25}$ tons.
28. 6 pounds.
29. $\frac{2}{3}$ miles.
30. $35\frac{3}{11}$ pounds.
31. 5 months.
32. 3\frac{1}{5}$.
33. $12\frac{28}{48}$ cords.
34. 3\frac{1}{4}$.
35. 7 bushels.
36. $5\frac{1}{2}$ cents.
38. $8\frac{33}{140}$ miles.

Page 132.

5. $1\frac{2}{3}$.
6. 1584 yards.
7. 160 books.
8. $180.
9. 60 years.

Page 133.

10. 30.
11. 277\frac{1}{3}$.
12. 62\frac{1}{2}$.
13. 1\frac{7}{20}$.
14. $43\frac{3}{4}$ cents.
15. $\frac{2}{5}$.
16. 334\frac{1}{2}$.
17. $364\frac{2}{8}$.
18. $140000.
19. $6540.66\frac{2}{3}$.
20. 42 lots.
21. 2333\frac{1}{3}$.
22. $40\frac{1}{30}$ acres.
23. 179 miles.

Page 134.

24. 200.

Page 140.

6. 900.27286.
7. .78171.
8. 258.011195.
9. 2100.830011.
10. 205.20401.

Page 141.

10. .197.
11. 4.995.

12. .27.
13. 19.855.
14. 16702.965.
15. 90000.015.
16. 16999999.99956.
17. 65135.2.
18. 82705.56249.
19. 13.79.

Page 143.

24. .00493.
25. 10002.2200212.
26. 13819.1844.
27. 10673.3253256.

Page 144.

14. 3.193 +.
15. .17000035.
16. 1.975 +.
17. 120.
18. 10.
19. 37.19 +.

Page 145.

1. $448.50.
2. $3217.75.

Page 146.

3. $7.50.
4. $1.37½.
5. $519½.
6. $1760.
7. 875.
8. $3.25.
9. $164.
10. $69.40.
11. $45.575.
12. $7.75.
14. $46.66⅔.
15. $196.

Page 147.

16. $482.
17. $819.375.
18. $.134 +.
19. $12.03 +.
20. $5.61 +.
21. $179.20.
23. 72 yards.
24. 12 days.
25. $196.25.
26. $9.56¼.
27. $6.50.
28. $274.
29. $800.

Page 148.

30. $139.95.
31. 10 casks.
32. $4.50.
33. 25.
34. 57¾ yards.
35. $.04.
36. $52.50.
37. $.56¼.
38. $1.75.
39. $27.
40. 243¾ lbs.
41. $.50.
42. $2.25.

Page 149.

43. $24.75.
44. $9.10.
45. 10 times.

Page 153.

3. 14209 lbs.
4. 1 T. 8 cwt. 50 lbs.
5. 225931 oz.
6. 9 T. 7 cwt. 50 lbs.
7. 36800 oz.
8. 3⅗ T.
9. 96016 oz.
10. 1¼ T.
11. 17600 oz.
12. 96208 oz; 1 T.

Page 155.

5. 17 bush. 3 pks. 6 qts.
6. $2.01.
7. 1217 pts.
8. $8.06¼.
9. $4.20.

Page 156.

5. 14553 cu. in.
6. 72.
7. $.90.
8. $2.88.
9. $8.40.

Page 157.

5. 200 rds.
6. 155 in.
7. 1030 ft.
8. 240 rds.
9. 31680 in.
10. 43 rds. 3½ yds.
11. ⅛.

Page 158.

5. 115,200 sqr. ft.
6. 60 A.
7. 33 A.
8. $22.50.
9. $19.20.
10. $245.56.

Page 159.

12. $336.
13. 33.4 yds.
14. 11⅔ ft.
15. 288 ft.
16. 25½ sq. ft.
17. 7 sq. ft.
18. 105 sq. ft.
19. 216 sq. ft.
20. $144. 51.
21. $88.89.

Page 160.

22. $70.89.

Page 161.

5. $94.50.
6. 129600 cu.in.
7. $96.
8. 5670 cu. ft.
9. $35\frac{25}{243}$ cubic yds.

Page 162.

10. $70.66 +.
11. 2160 bricks.
12. 10 cords.
13. $28.50.

Page 163.

5. £4 1s. 10d.
6. £72.
7. 1632 far.
8. 6 sov.
9. 28480 fl.

Page 164.

2. 2 yr.
3. 47304000 sec.
4. 615 min.
5. $1.50.

Page 165.

5. 8 hr. 59′,20″.
6. 270°.
7. 300′.
8. 3600″.
9. 5′.
10. 6° 25′.

Page 166.

1. $1.36.
2. $3.55.
3. 1134.
4. 1677 sheets.
5. 43 doz.

Page 167.

5. 66 mi. 56 rds.
6. 55 w. 6 da. 20 hr. 9 min. 29 sec.

Page 168.

10. 574 A. 132 sq. rd. 17½ sq. yd.
11. 22 cwt. 51 lb. 6 oz.

Page 169.

6. 66 yr. 2 mo. 18 da.
7. 58 yr. 10 mo. 26 da.
13. 10 lb. 4 3 5 3

Page 170.

6. 88 bush. 2 pks. 3 qts.
10. 87 gal. 1 qt. 2 gi.
11. 37 da. 17 hr. 22 min. 55 sec.

Page 171.

10. 1 bush. 3 pks. 7¾ qts.
11. 3 cu. yds. 12⅓ cu. ft.
12. 6 sq. yds. 1 sq. ft. 106⅔ sq. in.

Page 175.

13. $22.50.

Page 176.

14. $85.50.
15. 116⅔ A.
16. 78 girls.
17. Lost 3 cts.
22. $6642.

Page 177.

10. 25%.

Page 178.

11. 60%.
13. 33⅓%.
14. 11⅛%.
15. 50%.
16. $18\frac{2}{11}$ left.
17. 9⅜%.
18. 56¼%.
19. 40%.

Page 178.

20. 33⅓%.

Page 179.

10. $15.
11. 300 A.

Page 180.

11. $4166⅔.
12. 110 pupils.
13. $500.
14. $.35+.
15. $18000.
16. $64,800.
17. $1200.
18. $6,240.
19. 852 bush.
20. $100,000.
21. $96.
22. $1,333⅓.
23. $2000.
24. $18,000.

Page 183.

3. $22.21@6%.
4. $91.08@6%.
5. $75.21@6%.
6. $81.30@6%.
7. $16.89.
8. $1153.74+ @6%.
9. $597.01@6%.
10. $272.85@6%.
12. $261.75.
14. $300.85.
15. $377.34.
17. $354.33.

Page 186.

1. 70.6875 mi.
2. 33.36+bush.
3. 18 doz.
4. 264 yds.
5. $19,600.
6. $107,250.
7. $1.40.
8. $25.20.
9. $28\frac{19}{40}$ mi.
10. 9⅜ yds.
11. 27 mi. 272 rds. 15 ft. 9+in.
12. $\frac{37}{84}$ yds.

Page 187.

13. ⅜.
14. 23,103+ times.
15. 43,932 times.
16. 4 lbs. $11\frac{4}{15}$ oz.
17. $15.47.
18. 7.6 ft.
19. 30 spoons.
20. 4732 cattle.
21. $93\frac{3}{25}$%.
22. $47\frac{1}{17}$%.
23. $.007\frac{9}{13}$%.
24. $133.34.
25. $3718\frac{1}{4}$ gal.

Page 188.

26. $1866.66⅔.
27. 1280 miles.
28. $6400.
29. 333 pupils.
30. $46.25
31. $3083.20.
32. 11.01+rods.