THE

MODEL ELEMENTARY

ARITHMETIC.

ORAL AND WRITTEN.

PART I.

ARRANGED TO SUIT PRESENT GRADATION OF SCHOOL WORK

BY

EDWARD GIDEON, A.M.,

SUPERVISING PRINCIPAL OF THE GEORGE G. MEADE SCHOOL, PHILADELPHIA.

REVISED EDITION.

PHILADELPHIA:

ELDREDGE & BROTHER.

1904.

EDITION OF 1904.

PREFACE.

IN this book the treatment of each topic is so full as to insure facility and accuracy in performing such computations as are most likely to occur in ordinary business transactions.

The peculiar features of this, and of the other books of the series, are embraced in the following statements:

First. Forms or models of analysis, correct in expression and logical in reasoning.

Second. Definitions, clear and brief.

Third. In each new subject full solutions of typical examples, combined with proper analytical explanations.

Fourth. Frequent drill exercises to insure accuracy and rapidity.

Fifth. Numerous examples showing the practical application of rules to business and mechanical pursuits.

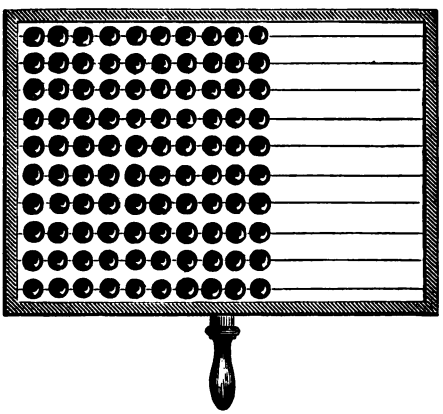
Sixth. Frequent reviews, in order that pupils may be helped to organize knowledge previously acquired.

CONTENTS

THE

MODEL

ELEMENTARY ARITHMETIC.



SECTION I.

NUMBERS.

1. How many are one and one? Two and one? Three and one? Four and one? Five and one?

2. How many things are in a collection of six things and one thing? Seven and one? Eight and one? Nine and one?

3. How many balls are on one row? On two rows? On three rows? On four? On five? On six? On seven? On eight? On nine? On ten?

4. How many ones in ten? In two tens? In three tens? In four tens? In five tens? In six tens? In seven tens? In eight tens? In nine tens? In ten tens?

5. How many tens in ten? How many tens in twenty? In thirty? In forty? In fifty? In sixty? In seventy? In eighty? In ninety? In one hundred?

6. How many hundreds in ten tens? In twenty tens? In thirty tens? In forty tens? In fifty tens? In sixty tens? In seventy tens? In eighty tens? In ninety tens?

7. How many tens in one hundred? In two hundred? In three hundred? In four hundred? In five hundred? In six hundred? In seven hundred? In eight hundred? In nine hundred?

8. Name every number between twenty and thirty. Between thirty and forty. Between forty and sixty. Between sixty and eighty. Between eighty and one hundred.

9. Name every number between two hundred and two hundred twenty. Four hundred fifty and four hundred seventy. Six hundred seventy and six hundred ninety. Eight hundred ninety and nine hundred.

SECTION II.

NOTATION AND NUMERATION.

The Ones' Period.

1. A ***Unit*** is one, or a single thing.

Thus, *one boy, one book,* or *one,* is a unit.

2. A ***Number*** is a unit, or a collection of units.

Thus, *one ball, two blocks, ten cents, one, two,* or *ten,* is a number.

3. A ***Figure*** is a character used to express a number.

There are ten figures used to express numbers: —

FIGURES.	*0,*	*1,*	*2,*	*3,*	*4,*	*5,*	*6,*	*7,*	*8,*	*9.*
NAMES.	*Naught,*	*one,*	*two,*	*three,*	*four,*	*five,*	*six,*	*seven,*	*eight,*	*nine.*

4. The figure 0 denotes *nothing*, or the absence of number.

5. In naming numbers a single thing is called ***one.***

6. When a number is expressed by a single figure it denotes *ones.*

1. How many ones are expressed by 3? By 5? By 7? By 4? By 6? By 8?

7. In naming numbers ***ten ones*** taken together are called ***one ten.***

Tens are written: —

Ten,	*10,*	5 tens, or *fifty,*	*50,*
2 tens, or *twenty,*	*20,*	6 tens, or *sixty,*	*60,*
3 tens, or *thirty,*	*30,*	7 tens, or *seventy,*	*70,*
4 tens, or *forty,*	*40,*	8 tens, or *eighty,*	*80,*

9 tens, or *ninety,* *90.*

8. To express a number of tens, or a number of tens and ones, at least two figures are needed.

1 ten is written	*10,*	named	*ten.*
1 ten and 1 one are written	*11,*	"	*eleven.*
2 tens and 3 ones are written	*23,*	"	*twenty-three.*
Thirty-four denotes	3 tens 4 ones,	written	*34.*
Fifty-six "	5 tens 6 ones,	"	*56.*
67 is named	*sixty-seven,* and expresses	6 tens 7 ones.	
89 "	*eighty-nine* "	"	8 tens 9 ones.

9. When a number is expressed by two figures written side by side, the right-hand figure denotes *ones,* and the left-hand figure denotes *tens.*

2. In the number 45, which figure denotes ones? Which figure denotes tens?

3. How many tens and how many ones are there in the number 13? In 24? In 35? In 46? In 57? In 60?

10. In naming numbers ***ten tens*** taken together are called ***one hundred.***

Hundreds are written: —

One hundred,	*100,*	*Five hundred,*	*500,*
Two hundred,	*200,*	*Six hundred,*	*600,*
Three hundred,	*300,*	*Seven hundred,*	*700,*
Four hundred,	*400,*	*Eight hundred,*	*800,*

Nine hundred, 900.

11. To express a number of hundreds, or a number of hundreds and tens, or a number of hundreds, tens, and ones, at least three figures are needed.

1 hundred 1 one are written *101*, named *one hundred one.*

2 hundred 2 tens are written *220* named *two hundred twenty.*

3 hundreds 3 tens 3 ones are written *333*, named *three hundred thirty-three.*

Four hundred fifty-six denotes 4 hundreds 5 tens 6 ones, written *456.*

789, named *seven hundred eighty-nine*, expresses 7 hundreds 8 tens 9 ones.

12. When a number is expressed by three figures written side by side, the right-hand figure denotes *ones*, the middle figure denotes *tens*, and the left-hand figure *hundreds.*

4. In the number 357, which figure denotes ones? Which figure denotes tens? Which denotes hundreds?

5. How many hundreds, how many tens, and how many ones are there in the number 123? In 200? In 304? In 506? In 468? In 579? In 680?

13. In naming or reading numbers expressed by two figures, the tens and the ones are read together as so many ones.

Thus, 34 is named or read *thirty-four ones*, or, simply *thirty-four.*

In naming or reading numbers expressed by three

figures, the hundreds, the tens, and the ones are read together as so many ones.

Thus, 234 is named or read *two hundred thirty-four ones*, or, simply *two hundred thirty-four*. 987 is named or read *nine hundred eighty-seven.*

Definitions.

14. Orders of Units are the classes of units formed by grouping together ten lower or smaller units.

Thus, *ones* are units of the *first order; tens* are units of the *second order;* and *hundreds* are units of the *third order.*

15. A figure in the ones' place denotes *units of the first order;* in the tens' place, *units of the second order;* and in the hundreds' place, *units of the third order.*

Thus, in the number 789, 9 denotes *ones*, or units of the first order; 8 denotes *tens*, or units of the second order; and 7 denotes *hundreds*, or units of the third order.

16. A ***Period*** is a group of three orders of units, beginning with ones and counting from right to left.

17. The ***First,*** or ***Ones' Period,*** is formed of the first, the second, and the third order of units, and comprises ones, tens, and hundreds.

Thus, 345 is a number forming the *ones' period.*

Exercises.

18. Express by figures the number *eight hundred and six.*

Model. — Since eight hundred and six is 8 *hundreds* 6 *ones*, it is expressed by writing *8* in the hundreds' place and *6* in the ones' place, and filling the tens' place by *0*, giving *806*, the expression required.

1. Express by figures the following numbers: —

Forty,	Three hundred,	Nine hundred five,
Eighty,	Seven hundred,	One hundred eight,
Thirty-four,	Two hundred ten,	Seven hundred two,
Eighty-one,	Four hundred ninety,	Three hundred six,
Fifty-three,	Six hundred twenty,	Six hundred sixteen,
Forty-nine,	Eight hundred eighty,	One hundred eleven.

19. Name the number expressed by the figures *468*.

Model. — Since 468 expresses four *hundreds* six *tens* eight *ones*, it is named *four hundred sixty-eight.*

2. Name the numbers expressed by the figures: —

10	*45*	*19*	*200*	*370*	*208*	*911*	*293*	*123*
30	*54*	*37*	*800*	*730*	*802*	*823*	*475*	*345*
70	*67*	*73*	*190*	*550*	*307*	*735*	*657*	*567*
12	*76*	*91*	*910*	*440*	*703*	*376*	*839*	*789*
21	*89*	*66*	*280*	*102*	*406*	*468*	*999*	*987*
23	*98*	*22*	*820*	*304*	*604*	*900*	*817*	*777*

20. Name the different numbers that may be expressed by combining *4*, *5*, and *6*.

Model. — The three figures *4*, *5*, and *6* combined, may express the numbers *456*, *465*, *546*, *564*, *645*, and *654*.

3. Name the different numbers that may be expressed by combining

1 and *2*	*9* and *1*	*1*, *2*, and *3*	*6*, *0*, and *4*	*3*, *0*, and *7*.
4 and *5*	*7* and *3*	*9*, *0*, and *1*	*5*, *6*, and *7*	*8*, *9*, and *0*.
5 and *6*	*6* and *5*	*3*, *4*, and *5*	*5*, *0*, and *4*	*2*, *0*, and *8*.
7 and *8*	*5* and *7*	*7*, *0*, and *3*	*6*, *7*, and *8*	*1*, *3*, and *5*.

4. In the preceding exercises, tell which figure in each number expressed by two figures denotes *tens*, and which denotes *ones*; and in the numbers expressed by three figures, tell which figure in each number denotes *hundreds*, which *tens*, and which *ones*.

The Thousands' Period.

21. In naming numbers ***ten hundreds*** taken together are called ***one thousand.***

Thousands are written: —

One thousand,	*1000,*	*Five thousand,*	*5000,*
Two thousand,	*2000,*	*Six thousand,*	*6000,*
Three thousand,	*3000,*	*Seven thousand,*	*7000,*
Four thousand,	*4000,*	*Eight thousand,*	*8000,*

Nine thousand, 9000.

22. To express a number of thousands, or a number of thousands, hundreds, tens, and ones, at least four figures are needed.

23. When a number is expressed by four figures written side by side, the figure in the first place denotes *ones*, in the second *tens*, in the third *hundreds*, and in the fourth *thousands*.

1. How many thousands in 2000? In 4300? In 3300? In 5500? In 6600? 8600? 7090? 9010? 3008?

2. Read the ones' period in the numbers 9100; 2210; 8320; 3430; 7501; 4609; 6080; 5070; 1009; 7707.

24. In naming numbers ***ten thousands*** taken together are called ***one ten-thousand.***

Ten-thousands are written: —

Ten thousand,	*10000,*	*Fifty thousand,*	*50000,*
Twenty thousand,	*20000,*	*Sixty thousand,*	*60000,*
Thirty thousand,	*30000,*	*Seventy thousand,*	*70000,*
Forty thousand,	*40000,*	*Eighty thousand,*	*80000,*

Ninety thousand, 90000.

25. To express a number of ten-thousands, at least five figures are needed.

26. When a number is expressed by five figures, the figure in the fourth place denotes *thousands*, and in the fifth *ten-thousands.*

3. How many ten-thousands in 10000? In 90000? In 20100? 80210? 30320? 70430? 40540? 60650? 50760? 21060? 43070? 65009?

4. How many ten-thousands and how many thousands in 98000? In 12900? In 87100? 20800? 65200? 30700? 54300? 70102? 40432?

5. Read the units' period in 98100; 87120; 76230; 65304; 54405; 43060; 32078; 21009.

27. In naming numbers ***ten ten-thousands*** taken together are called ***one hundred-thousand.***

One hundred thousand	is written	*100000,*
Two hundred thousand	"	*200000,*
Five hundred thousand	"	*500000,*
Eight hundred thousand	"	*800000,*
Nine hundred thousand	"	*900000.*

28. To express a number of hundred-thousands, at least six figures are needed.

29. When a number is expressed by six figures, the figure in the fourth place denotes *thousands,* in the fifth *ten-thousands,* and in the sixth *hundred-thousands.*

6. How many hundred-thousands are in 200000? In 900100? In 300800? In 800300? In 400700? In 700400? In 500600?

7. How many hundred-thousands, ten-thousands, and thousands in 300102? In 901304? In 403450? 810560? 540067? 751080? 676009?

8. Read the ones' period in 901100; 208200; 802030; 307040; 730105; 406260; 640037; 570046; 705006.

30. The ***Second,*** or ***Thousands' Period,*** is formed of the fourth, the fifth, and the sixth order of units, and comprises thousands, ten-thousands, and hundred-thousands.

31. In reading numbers, ten-thousands and thousands, and hundred-thousands, ten-thousands, and thousands are read together as so many thousands.

Thus, 36000 is read *thirty-six thousand,* and 258000 is read *two hundred fifty-eight thousand.*

Exercises.

32. **Express by figures the number *nine thousand eight.***

Model. — Write *9* for the number of thousands and *8* for the number of ones, and fill the hundreds' place and the tens' place with ciphers, giving *9008*, the expression required.

1. Express by figures the numbers: —

Two thousand; six thousand; nine thousand one hundred; seven thousand three hundred; nine thousand one hundred twenty; one thousand nine hundred eighty; three thousand eight hundred two.

Four thousand six hundred four; one thousand nineteen; nine thousand eighty-two; two thousand thirty-seven; eight thousand sixty-four; nine thousand ten; one thousand eighty; eight thousand twenty; two thousand seventy; seven thousand three; three thousand six.

33. Express in words the number *908070*.

Model. — Separate the first three figures of the given number into a period, and the remaining three figures will form the second period, giving *908,070*.

The first period, *070*, expresses tens, and the absence of hundreds and ones, forming the ones' period; and the remaining figures, *908*, express hundred-thousands and thousands, and the absence of ten-thousands, forming the thousands' period. The whole number is read *nine hundred eight thousand seventy*.

2. Express in words the numbers: —

3000	*9180*	*1902*	*9028*	*2090*	*9002*	*2300*	*5090*
5000	*1920*	*9103*	*8037*	*3010*	*7004*	*3450*	*4087*
7000	*8270*	*2804*	*7046*	*4080*	*5006*	*4506*	*2708*
8100	*2830*	*8205*	*6055*	*5020*	*3008*	*5056*	*3600*
6400	*7360*	*3706*	*5064*	*6007*	*1005*	*6070*	*4003*

3. Express by figures the numbers: —

Thirty thousand; fifty thousand; seventy thousand one hundred; ninety thousand three hundred; forty thousand seven hundred ninety; sixty thousand nine hundred eighty; eighty thousand two hundred seventy; ninety thousand four hundred six.

Seventy thousand five hundred seven; fifty thousand six hundred eight; seventy thousand ten; ninety thousand thirty; ten thousand twenty-three; twenty thousand thirty-four; thirty thousand ninety-eight; ninety thousand one; seventy thousand three; fifty thousand five.

4. Express in words the numbers: —

10000	*91800*	*29180*	*90900*	*19020*	*90082*	*80002*
80000	*18200*	*48270*	*10700*	*82040*	*80073*	*10004*
30000	*82700*	*57360*	*80500*	*37060*	*70064*	*70003*
71000	*27300*	*66450*	*20300*	*64080*	*60055*	*20006*
49000	*73600*	*75540*	*70100*	*55070*	*50046*	*60004*

5. Express by figures the following numbers: —

Two hundred thousand; four hundred thousand; three hundred five thousand; five hundred seven thousand; seven hundred nine thousand; nine hundred thousand one hundred; eight hundred thousand three hundred; seven hundred thousand five hundred.

Two hundred nine thousand ten; four hundred seven thousand thirty; one hundred thousand nine hundred eighty; three hundred thousand eight hundred seventy; five hundred thousand sixty; seven hundred thousand fifty; nine hundred thousand ten; two hundred thousand eight; four hundred thousand six; six hundred thousand four.

6. Express in words the following numbers: —

300000	*901000*	*100800*	*209030*	*100080*	*900001*
500000	*109000*	*800700*	*803050*	*200060*	*200008*
700000	*802000*	*200600*	*308070*	*300040*	*700003*
410000	*208100*	*710560*	*700490*	*900108*	*407080*
630000	*703500*	*330070*	*400710*	*800702*	*500607*
850000	*307900*	*650080*	*600680*	*700306*	*600075*

7. In each of the preceding Exercises, tell what order of units each figure in each number denotes.

Read in each the ones' period, then the thousands' period, then both the thousands' and the ones' period.

The Millions' Period.

34. In naming numbers ***ten hundred-thousands*** taken together are called ***one million.***

35. To express a number of millions, at least seven figures are needed.

36. When a number is expressed by seven figures, the seventh figure denotes *millions.*

1. How many millions in 9008000? In 2003000? In 8007003? 3009004? 7090080? 4700600?

2. Read the thousands' period and the ones' period in 1020300; 9080700; 2004050; 8009060; 3008603; 7006005; 5500505; 6006606; 4040040.

37. In naming numbers ***ten millions*** taken together are called ***one ten-million.***

38. To express a number of ten-millions, at least eight figures are needed.

39. When a number is expressed by eight figures, the seventh figure denotes *millions,* and the eighth *ten-millions.*

3. How many ten-millions in 20000000? In 40000000? In 60600606? 80080080? 10001010? 30030030?

4. How many ten-millions and how many millions in 12030400? 98070600? 34005060? 70006050? 40004005?

5. Read the thousands' period and the ones' period in 20000800; 90405600; 31560708; 89007084; 46008090; 70900098; 53010002; 60008007; 35700008.

40. In naming numbers ***ten ten-millions*** taken together are called ***one hundred-million.***

41. To express a number of hundred-millions, at least nine figures are needed.

42. When a number is expressed by nine figures, the seventh figure denotes *millions,* the eighth denotes *ten-millions,* and the ninth *hundred-millions.*

6. How many hundred-millions in 200300400? In 908070600? 400005060? 870060050? 605004032?

7. How many hundred-millions, ten-millions, and millions in 999800700? In 109208307? 870650430? 207306450? 706054300?

8. Read the thousands' period and the ones' period in 100200300; in 909808707; in 203450560; 807065043; 500500604; 7077007070.

43. The ***Third,*** or ***Millions' Period,*** is formed of the seventh, the eighth, and the ninth order of units, and comprises millions, ten-millions, and hundred-millions.

44. In reading numbers, ten-millions and millions, and hundred-millions, ten-millions, and millions are read together as so many millions.

Thus, 98000000 is read *ninety-eight million;* and 579000000 is read *five hundred seventy-nine million.*

Definitions.

45. There are three methods of expressing numbers:

1. By *words;* as, *one, ten, one hundred,* etc.

2. By *figures;* called the *Arabic* method.

3. By *letters;* called the *Roman* method. (See page 23.)

In practical arithmetic, only the *Arabic* method of expressing numbers is used.

46. ***Notation*** is the method of expressing numbers by figures.

Thus, the expressing of the number one hundred thirty-five by the figures *135*, is the notation of *one hundred thirty-five.*

47. ***Numeration*** is the method of naming numbers that are expressed by figures.

Thus, the naming of the number 246 as *two hundred forty-six,* is the numeration of *246.*

48. ***Significant Figures*** are the figures that express a number of units.

The significant figures are 1, 2, 3, 4, 5, 6, 7, 8, and 9.

49. Figures have two values: a *simple value* and a *local value.*

The ***Simple Value*** of a figure is the value which it expresses when taken alone.

The simple value of a figure depends only upon the *number* of units that it expresses.

Thus, the simple value of 4 is 4 ones, or *four;* of 3 is 3 ones, or *three;* of 2 is 2 ones, or *two.*

The ***Local Value*** of a figure is the value which it expresses when combined with other figures.

The local value of a figure depends upon both the *number* and the *order* of units that it expresses.

Thus, in the number 432, the local value of 4 is 4 *hundreds*, or *four hundred;* of 3 is 3 *tens*, or *thirty;* of 2 is 2 *ones*, or *two.*

50. The simple value of a figure is always the same; but the local value varies according to the place in which the figure stands.

Thus, in the numbers 3, 30, 300, each 3 expresses *three units;* but the *value* of the first is 3 *ones*, or *three*, of the second 3 *tens*, or *thirty*, and of the third 3 *hundreds*, or *three hundred.*

Orders and Periods.

NAMES OF PERIODS.	Millions' Period.			Thousands' Period.			Ones' Period.		
NAMES OF ORDERS.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Ones.
	9	8	7	6	5	4	3	2	1
PERIODS.	Third Period.			Second Period.			First Period.		

The fourth period is *billions*, and the fifth is *trillions.*

51. A full period contains ones, tens, and hundreds of the group of orders forming that period.

Thus, a number forming three full periods contains ones, tens, and hundreds; ones, tens, and hundreds of *thousands;* and ones, tens, and hundreds of *millions.*

52. A ***number is written*** by placing the figure expressing the number of each order of units in each period as it is named.

In writing numbers, the places of all orders and periods not named must be filled by ciphers.

Thus, sixty million six hundred six is written *60000606*—the the significant figures being written in their proper places, and the other places filled by ciphers.

53. A ***number is numerated*** by naming each order of units commencing at ones, and separating the number into periods.

Thus, 505005 is numerated, *ones, tens, hundreds* forming *ones' period; thousands, ten-thousands, hundred-thousands* forming *thousands' period.*

Periods may be separated from each other by commas.

Thus, 155050005 separated into periods is written *155,050,005.*

54. A ***number is read*** by commencing at the left and naming the number of units of each order in each period, together with the name of that period.

In reading numbers, the names of places or periods filled by ciphers are omitted.

Thus, 7000809 is read *seven million eight hundred nine.*

The name of one's period is generally omitted.

Thus, 809, in the last number, is read *eight hundred nine,* not eight hundred nine *ones.*

From preceding definitions, explanations, and exercises, are deduced the following

55. Principles of Numeration and Notation.

I. *Ten units of any order are one unit of the next higher order.*

II. *The successive orders of units increase in value tenfold from right to left.*

56. Rule for Notation.

Write the figures expressing the number of hundreds, tens, and ones of each period, in the order in which they are named.

57. Rule for Numeration.

I. *Begin with the lowest order of units and separate the figures of the given number into periods of three figures each.*

II. *Begin with the highest period, read the figures expressing the number of hundreds, tens, and ones of each period, and give the name of the period after reading the ones of that period, omitting the name of the ones' period.*

Exercises.

58. Express by figures, or notate, the number *thirty million four thousand five.*

Model. — Write 30 for the millions' period, and place a comma after it, giving *30,;* then write 4 in the thousands' period, filling the hundred-thousands' place and the ten-thousands' place with ciphers, giving *30,004,;* then write 5 in the ones' period, filling the hundreds' place and the tens' place with ciphers, giving *30,004,005*, the expression required.

59. Express in words, or numerate and read, the number *303030300.*

Model. — Separate the number into periods, commencing at the right, giving *303,030,300.*

Begin with ones to numerate, — ones, tens, hundreds, forming the *ones' period;* thousands, ten-thousands, hundred-thousands, forming the *thousands' period;* millions, ten-millions, hundred-millions, forming the *millions' period.*

The whole number is read *three hundred three million thirty thousand three hundred.*

1. Express by figures, or notate, the following numbers:—

One million; nine million; two million eight hundred thousand; seven million three hundred thousand; four million six hundred fifty thousand; one million nine hundred twenty thousand.

Eight million thirty-seven thousand; four million sixty-five

thousand; nine million one thousand eight hundred; two million seven thousand three hundred; six million forty thousand five hundred; two million forty thousand six hundred; five million five thousand five.

2. Express in words, or numerate and read, the numbers: —

2000000	*1920000*	*1020300*	*9010802*	*2000803*
4000000	*8370000*	*4050600*	*7030604*	*7000406*
6000000	*4065000*	*7008090*	*5009001*	*5003029*
3200000	*9018000*	*9008070*	*8002007*	*3008047*
5400000	*2007300*	*6050040*	*3006040*	*8000027*
7600000	*6004500*	*3020010*	*5001090*	*3000064*

3. Express by figures, or notate, the following numbers: —

Thirty million; fifty million; twenty-eight million; seventy-three million; forty-six million five hundred thousand; nineteen million two hundred thousand.

Eighty million three hundred seventy thousand; forty million six hundred fifty thousand; ninety million eighteen thousand; twenty million seventy-three thousand; sixty million four hundred five thousand seventy; fifty million fifty thousand fifty.

4. Express in words, or numerate and read, the numbers: —

30000000	*10300000*	*20040060*	*90009009*	*20202020*
50000000	*50700000*	*80060040*	*80800808*	*30303030*
70000000	*10080000*	*90700500*	*70070007*	*40040040*
82000000	*60040000*	*30100800*	*60006006*	*50050050*
64000000	*20103000*	*60042001*	*50505505*	*77000077*
46000000	*50709000*	*20034005*	*40000404*	*99000099*

5. Express by figures, or notate, the following numbers: —

Six thousand seventy; twenty thousand three hundred; thirty thousand forty; forty thousand five; five hundred thousand; six thousand six hundred; nine hundred thousand seven hundred; eight hundred six thousand forty.

Two million; three million four thousand five hundred; four million fifty thousand sixty; five million five; sixty million seventy thousand eight hundred; seventy million eight hundred thousand ninety; eight hundred eight million seven hundred seven thousand six hundred six.

6. Express in words the following numbers: —

9009; 20000; 81000; 34500; 40560; 60070; 50008.

900000; 750000; 654000; 543200; 432190; 321981.

109876; 200345; 300045; 400006; 5000000; 60708; 70008.

5050000; 5005000; 5000500; 5000050; 5000005; 505050.

60500000; 60060006; 60006006; 60000606; 60000066.

700000000; 760000000; 765000000; 706500000; 700650065.

800760000; 800706000; 800007600; 800000760; 800070060.

900900900; 909090090; 909009909; 900009009; 909090909.

7. In the preceding Exercises, tell what order of units each figure in each number expresses.

Tell how many of each order of units are expressed in each number.

Read first the ones' period, then the thousands' period, then the millions' period, and then the entire number.

Roman Notation.

60. Roman Notation is the method of expressing numbers by capital letters.

61. There are seven capital letters used to express numbers: —

LETTERS.	I	V	X	L	C	D	M.
VALUES.	*1*	*5*	*10*	*50*	*100*	*500*	*1000.*

62. All numbers may be expressed by the seven capital letters, by repeating or combining them, as follows: —

I. When a letter is repeated, its value is repeated.

Thus, III represents *3;* XXX, *30;* CCC, *300.*

II. When a letter is placed *after* another letter of greater value, the less value is *added to* the greater value.

Thus, VI represents *6;* XI, *11;* CX, *110.*

III. When a letter is placed *before* another letter of greater value, the less value is *taken from* the greater value.

Thus, IV represents *4;* IX, *9;* XC, *90.*

Exercises.

63. Express in figures the number *DXC.*

Model. — Since D expresses 500, and XC expresses 90, DXC expresses *590.*

64. Express by letters the number *175.*

Model. — Since 100 is expressed by C, 70 by LXX, and 5 by V, 175 is expressed by *CLXXV.*

1. Express by figures the following numbers: —

XIX	LXV	CX	CLX	DCXXIX.
XXI	LXIV	CXIX	CXC	DCCCXI.
X	LXXIX	C	CCLX	DCLX.
XLV	LXXXV	CXIV	CCXC	MCCLIX.

2. Express by letters the following numbers: —

50	*56*	*100*	*200*	*400*	*500*	*700*	*800*
60	*65*	*120*	*230*	*440*	*560*	*785*	*909*
70	*78*	*109*	*209*	*409*	*614*	*704*	*937*
80	*87*	*148*	*348*	*547*	*626*	*875*	*990*
90	*99*	*167*	*367*	*579*	*649*	*869*	*1000*

3. Express the numbers in Exercise 1 and in Exercise 2 by words, by figures, and by letters.

Review.

1. What is a ***Unit?*** A ***Number?*** Give five examples of each. What is a ***Figure?*** Name the figures.

2. What is ***Numeration?*** In naming numbers, what are ten ones called? Ten tens? Ten hundreds? Ten thousands? Ten millions? What make one ten? One hundred? One thousand? One million? One ten-thousand? One hundred-million?

3. What is ***Notation?*** When a number is expressed by two figures written side by side, what does each figure denote? When expressed by three figures? By four? By six? By eight? How many figures are needed to express ones? To express tens? Hundreds? Ten-thousands? Ten-millions? Hundred-millions?

4. What are ***Orders of Units?*** Name the first nine orders of units. Name the first order; the fourth; the seventh; the second; the fifth; the eighth. What order of units does a figure in the first

place denote? In the second? The fifth? The eighth? The third? The sixth? The ninth? What order of units is in the second place? The fifth? The eighth? The third? The sixth? The ninth?

5. What is a ***Period?*** Name the first period; the second; the third. Of what orders is ones' period composed? Millions'? Thousands'? Of what orders is each full period composed? In which place of which period are ones? Thousands? Millions? Tens? Ten-thousands? Ten-millions? Hundred-thousands? Hundred-millions?

6. What are ***Significant Figures?*** Name the significant figures. How many ones does each express? What two values have significant figures? Define ***Simple Value*** and ***Local Value.*** Which value is always the same? Upon what does the local value depend?

7. How is a ***number written?*** Give an example. How numerated? Give an example. How read? Give an example. Give the principles of numeration and notation. Recite the rule for notation. The rule for numeration.

8. What is ***Roman Notation?*** In what three ways may numbers be expressed? How are numbers expressed by the Arabic method? By the Roman method? What capital letters are used to express numbers? What is the value of each? Give the principles of Roman notation.

To make pupils familiar with the arrangement of figures in numbers, and to enable them to write and to read numbers with facility, they may, at first, be allowed to prepare diagrams having the places of the orders indicated, and the periods of figures separated from each other.

Billions.			Millions.			Thousands.			Ones.		
H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.
							3	0	3	0	3
					3	4	0	0	0	5	0
			4	0	0	0	0	5	0	0	0
		6	5	0	0	6	0	7	0	8	9

By placing the significant figures in their proper places, and by filling all other places with ciphers, the numbers can be readily recognized and named.

The aid of such table or diagram should, however, be discontinued as soon as possible, and the pupil should be required to depend upon his knowledge of the *principles* of notation and numeration, which should be impressed upon him by frequent and continued *drill.*

SECTION III

ADDITION.

1. Harry had 4 marbles and found 3 marbles. How many marbles had he then? How many are 4 and 3?

2. A man rode 5 miles and then walked 3 miles further. How many miles did he go altogether?

3. How many dollars are 4 dollars and 6 dollars? 7 dollars and 5 dollars are how many dollars?

4. How many ones are 3 ones and 4 ones? 6 ones and 2 ones are how many? 5 ones and 4 ones?

5. What number contains as many ones as 6 ones and 4 ones? As many as 5 ones and 6 ones?

6. What number is produced by uniting the ones in 5 ones and 7 ones? In 8 ones and 6 ones? In 7 ones and 9 ones?

7. How many are 3 and 4? 4 and 5? 7 and 6? 6 and 9? 8 and 7? 9 and 8? 4 and 6 and 8?

8. If Charles has a 10-cent piece and a 5-cent piece, what sum of money has he?

9. What amount of money is earned by a boy who earns 6 dollars one week, 8 dollars the next, and 7 dollars the next?

10. If Thomas is 9 years old, and James is 10 years old, what is the sum of their ages?

11. What sum of money is equal to the number of dollars in 6 dollars, 8 dollars, and 10 dollars united?

12. What is the unit of 7 yards? Of 9 yards? Are the units of 7 yards and 9 yards like, or unlike?

13. Can 7 yards be added to 9 yards? What is the sum of 7 yards and 9 yards? What is the unit of their sum?

14. What is the unit of 10 dollars? Of 8 miles? Are

their units like, or unlike? Can they be united in one sum? Why?

15. What kind of units only can be added together? What kind of numbers only can be added together?

In each of the preceding questions, two or more numbers are given to find one number which shall contain as many units as are in all the numbers taken together.

Definitions.

65. Addition is the process of uniting two or more numbers to find their sum.

66. The ***Sum,*** or ***Amount,*** is the number obtained by adding two or more numbers together.

Thus, 17 dollars is the *sum* of 9 dollars and 8 dollars.

67. The numbers that are added together to form the sum are called ***Parts.***

Thus, 4 feet, 6 feet, and 8 feet are the *parts* whose *sum* is 18 feet.

The sum always contains as many units as are in all the parts taken together.

68. The ***Sign of Addition*** is a short upright cross, +. It is named *plus*, and means *more.*

When the sign of addition is placed between two numbers, it shows that the number placed after it is to be added to the number placed before it.

69. The ***Sign of Equality*** is two short straight lines, =. It is read *equals*, or *equal.*

When the sign of equality is placed between two numbers or sets of numbers, it shows that the number or set of numbers placed before it is equal to the number or set of numbers placed after it.

Thus, 17 yards + 8 yards = 25 yards, is read 17 yards *plus* 8 yards *equal* 25 yards; and shows that 8 yards are to be added to 17 yards, and that, taken together, they equal 25 yards.

70. Similar Numbers are numbers whose units are alike.

Thus, 4 yards and 5 yards are *similar numbers;* so, also, are 5 ones and 7 ones; 6 tens and 9 tens; 7 and 9.

71. Dissimilar Numbers are numbers whose units are not alike.

Thus, 4 yards and 5 days are *dissimilar numbers;* so, also, are 5 ones and 7 tens; 6 tens and 8 hundreds.

72. Addition Tables.

0 and	*1* and	*2* and	*3* and	*4* and
1 are *1*	*1* are *2*	*1* are *3*	*1* are *4*	*1* are *5*
2 " *2*	*2* " *3*	*2* " *4*	*2* " *5*	*2* " *6*
3 " *3*	*3* " *4*	*3* " *5*	*3* " *6*	*3* " *7*
4 " *4*	*4* " *5*	*4* " *6*	*4* " *7*	*4* " *8*
5 " *5*	*5* " *6*	*5* " *7*	*5* " *8*	*5* " *9*
6 " *6*	*6* " *7*	*6* " *8*	*6* " *9*	*6* " *10*
7 " *7*	*7* " *8*	*7* " *9*	*7* " *10*	*7* " *11*
8 " *8*	*8* " *9*	*8* " *10*	*8* " *11*	*8* " *12*
9 " *9*	*9* " *10*	*9* " *11*	*9* " *12*	*9* " *13*
10 " *10*	*10* " *11*	*10* " *12*	*10* " *13*	*10* " *14*

5 and	*6* and	*7* and	*8* and	*9* and
1 are *6*	*1* are *7*	*1* are *8*	*1* are *9*	*1* are *10*
2 " *7*	*2* " *8*	*2* " *9*	*2* " *10*	*2* " *11*
3 " *8*	*3* " *9*	*3* " *10*	*3* " *11*	*3* " *12*
4 " *9*	*4* " *10*	*4* " *11*	*4* " *12*	*4* " *13*
5 " *10*	*5* " *11*	*5* " *12*	*5* " *13*	*5* " *14*
6 " *11*	*6* " *12*	*6* " *13*	*6* " *14*	*6* " *15*
7 " *12*	*7* " *13*	*7* " *14*	*7* " *15*	*7* " *16*
8 " *13*	*8* " *14*	*8* " *15*	*8* " *16*	*8* " *17*
9 " *14*	*9* " *15*	*9* " *16*	*9* " *17*	*9* " *18*
10 " *15*	*10* " *16*	*10* " *17*	*10* " *18*	*10* " *19*

Exercises.

1. Find the sum of 8 and 1; 18 and 1; 28 and 1; 38 and 1; 48 and 1; 6 and 2; 16 and 2; 26 and 2; 36 and 2; 46 and 2; 54 and 3; 64 and 3; 74 and 3; 84 and 3; 94 and 3.

2. Tell the amount of 6 and 4; 16 and 4; 26 and 4; 36 and 4; 46 and 4; 7 and 5; 17 and 5; 27 and 5; 37 and 5; 47 and 5; 58 and 6; 78 and 6; 88 and 6; 98 and 6.

3. How many are 7 and 7? 27 and 7? 47 and 7? 67 and 7? 87 and 7? 8 and 8? 18 and 8? 28 and 8? 38 and 8? 48 and 8? 59 and 9? 69 and 9? 79 and 9? 89 and 9? 99 and 9?

4. Count by twos from 10 to 20. From 20 to 40. From 40 to 60. From 60 to 80. From 80 to 100. From 11 to 23. From 34 to 46. From 53 to 69. From 63 to 81. From 79 to 99.

5. Count by 3's from 10 to 22. From 20 to 35. From 30 to 51. From 50 to 74. From 31 to 58. From 42 to 72. From 53 to 77. From 61 to 88. From 66 to 96. From 55 to 97.

6. Add 4 to every fourth number from 12 to 24. From 20 to 36. From 30 to 50. From 40 to 64. From 23 to 51. From 34 to 66. From 41 to 77. From 51 to 95.

7. Add 5 to every fifth number from 10 to 25. From 20 to 40. From 40 to 65. From 50 to 80. From 17 to 32. From 23 to 43. From 41 to 66. From 52 to 77. From 61 to 91. From 59 to 99.

8. Add by 6's from 10 to 28. From 20 to 44. From 30 to 60. From 23 to 41. From 34 to 76. From 42 to 90. By 7's from 10 to 31. From 20 to 48. From 30 to 65. From 21 to 49. From 33 to 68. From 47 to 89. From 49 to 98.

9. Add by 8's from 10 to 26. From 20 to 44. From 40 to 72. From 23 to 47. From 31 to 71. From 44 to 100. By 9's from 10 to 28. From 20 to 47. From 30 to 66. From 29 to 56. From 37 to 73. From 41 to 86. From 44 to 98. By 10's from 10 to 40. From 20 to 60. From 70 to 90. From 21 to 61. From 33 to 73. From 45 to 95.

Mental Exercises.

1. If a man drives 6 miles one hour, and 8 miles the next, how far does he drive in the two hours?

73. Model.—If a man drives 6 miles one hour, and 8 miles the next, he drives in the two hours the sum of 6 miles and 8 miles, which is 14 miles.

2. A tinsmith made for a dairyman 9 large milk-pans and 7 small ones. How many milk-pans did he make?

3. A pole is 6 feet in the earth and 12 feet in the air. How long is the pole?

4. A stationer sold 8 dozen lead-pencils to one man, and 15 dozen crayons to another. How many dozen did he sell?

5. If you go 16 miles north from your home, and your father goes 9 miles south from it, how far apart will you be?

6. A farmer sold a cow for 28 dollars, and her calf for 7 dollars. How much did he get for both?

7. I bought 32 yards of carpet for my parlor, and 8 yards for my hall. How many yards did I buy?

8. If John's mother was 34 years old 9 years ago, how old is she now?

9. If John's father is 46 years old, how old will he be in 9 years?

10. A farmer has 65 acres of cleared land, and 8 acres of woodland. How many acres are there in his farm?

11. If I give 6 apples to one boy, 8 to another boy, and keep 5, how many apples had I at first?

74. Model. — If I give 6 apples to one boy, 8 to another, and keep 5, I had at first the sum of 6 apples and 8 apples and 5 apples. 6 apples and 8 apples are 14 apples, and 5 apples are 19 apples.

12. What is the sum of 3 reams + 5 reams + 7 reams? 4 quires + 6 quires + 8 quires? 3 sheets + 7 sheets + 11 sheets?

13. 5 years + 7 years + 9 years = how many years? 6 months + 8 months + 10 months are how many months?

14. If Frank pays 10 dollars for an overcoat, 6 dollars for a pair of boots, and 3 dollars for a hat, what do they all cost him?

15. The head of a fish is 6 inches long, its body is 12 inches, and its tail is 4 inches. How long is the fish?

16. At a dry-goods store a lady bought 15 yards of dress-goods, 8 yards of muslin, and 7 yards of linen. How many yards did she buy?

17. What is the length of a pole which stands 18 feet in the air, 6 feet in the water, and 8 feet in the earth?

18. A grocer bought a barrel of mackerel for 17 dollars, a barrel of extra flour for 10 dollars, and a cheese for 6 dollars. What did all cost him?

19. A saddler sold a set of harness for 20 dollars, fly-straps for 8 dollars, and a bridle for 7 dollars. What did he receive for all?

20. A young man had 45 dollars in a savings-bank, and he put in 18 dollars more. How many dollars had he in then?

75. Model. — If a young man had 45 dollars in a savings-bank, and he put in 18 dollars, he then had in the sum of 45 dollars and 18 dollars.

18 dollars equal 10 dollars and 8 dollars. 45 dollars and 10 dollars are 55 dollars, and 8 dollars are 63 dollars.

21. How many acres are there in a farm of 52 acres of cleared land and 15 acres of meadow and woodland?

22. I paid 55 dollars for a cow, and 19 dollars for a heifer. What did I pay for them both?

23. How many miles did James travel in going 68 miles by railroad and 23 by stage?

24. A farmer had 20 chickens, raised 15 more, and bought 10. How many chickens had he then?

25. A housekeeper bought in market some beef for 35 cents, some veal for 20 cents, and some mutton for 18 cents. What did all cost?

26. What amount of money is needed to pay 50 dollars for a horse, 25 dollars for a cart, and 15 dollars for a set of harness?

27. In June are 30 days, in July are 31 days, and in August are 31 days. How many days are in the summer months?

28. A father gave to one son 45 cents, and to another 35 cents. If he should give 9 cents more to each, how many would both have?

29. What is the sum of the ones in 7, 8, and 9? Of the tens in 10, 20, and 30? Of the ones and the tens in 9, 10, and 22?

30. How many tens and ones are in 10, 20, and 30? In 10, 12, and 25? In 10, 25, and 30?

From the preceding exercises are deduced the following

76. Principles of Addition.

I. *Only similar numbers can be added.*

II. *Only like orders of units in different numbers can be added.*

III. *The sum is a number similar to the parts added.*

CASE I.

Written Exercises.

When the Sum of the Units of each Order is less than Ten.

77. *Example.*—What is the sum of 312, 253, and 4232?

SOLUTION.

312 }
253 } *Parts.*
4232 }

4797 *Sum.*

EXPLANATION. — Since only like orders of units in different numbers can be added, write the parts to be added so that the figures expressing *ones* shall stand in a column, the figures expressing *tens* in the next, *hundreds* in the next, and *thousands* in the next; and draw a line under all.

Begin with ones, and add each column separately.

2 ones and 3 ones and 2 ones are 7 ones; which write under the column of *ones*.

3 tens and 5 tens and 1 ten are 9 tens; which write under the column of *tens*.

2 hundreds and 2 hundreds and 3 hundreds are 7 hundreds; which write under the column of *hundreds*.

Write 4 thousands in the *thousands'* place.

The sum is 4 thousands 7 hundreds 9 tens 7 ones, or *4797*.

Or, briefly, begin with ones, and name results only.

Two, five, seven; write 7 ones under the column of *ones*.

Three, eight, nine; write 9 tens under the column of *tens*.

Two, four, seven; write 7 hundreds under the column of *hundreds*.

Write 4 thousands in the *thousands'* place.

The sum is *4797*.

78. *Problem.* — A drover paid $1300 for cattle, $525 for sheep, $4150 for horses, and $1014 for mules. How much did he pay for all?

Note. — The character, $, is called the *dollar sign*, and a number having this sign before it expresses *dollars:* as, $1 expresses and is read *one dollar;* $200 expresses *200 dollars*.

SOLUTION.

$1300
$525
$4150
$1014
} *Parts.*

$6989 *Sum.*

EXPLANATION. — Since the drover paid 1300 dollars for cattle, 525 dollars for sheep, 4150 dollars for horses, and 1014 dollars for mules, he paid for all the sum of 1300 dollars, 525 dollars, 4150 dollars, and 1014 dollars.

Since only like orders of units in different numbers can be added, write the parts to be added, etc. (77).

Begin with ones, and add each column separately.

Four, nine; write 9 under the column of ones.

One, six, eight; write 8 under the column of tens.

One, six, nine; write 9 under the column of hundreds.

One, five, six; write 6 under the column of thousands.

The sum is *6989* dollars, the amount paid for all.

Problems.

Find the sum of the numbers to be added in each of the following problems; read each part and each amount: —

(1)	(2)	(3)	(4)	(5)	(6)	(7)
345	632	26	435	345	324	345
623	104	540	31	421	501	402
20	231	212	302	23	130	22
311	32	210	210	500	34	230

(8)	(9)	(10)	(11)	(12)	(13)
2032	23	1421	3224	3232	2322
3413	432	34	322	4203	213
202	3212	40320	32	320	6032
30	6020	4003	70300	33	320
121	202	93211	2021	2201	12

(14)	(15)	(16)	(17)
232 miles.	82033 yards.	42021 feet.	20 inches.
303 miles.	702 yards.	3603 feet.	212 inches.
23 miles.	4021 yards.	230 feet.	4221 inches.
210 miles.	51120 yards.	24 feet.	53303 inches.
6011 miles.	112 yards.	100 feet.	1121 inches.
220 miles.	11 yards.	94010 feet.	90021 inches.

Find the sum of —

18. 3123 + 41322 + 230 + 50000 + 31203.

19. 712106 + 121 + 7021210 + 4122 + 32440.

20. 5350 pounds + 1000 pounds + 2225 pounds + 20220 pounds + 30000 pounds.

21. 3000 acres + 20100 acres + 400 acres + 225 acres + 5050 acres + 222 acres.

22. Find the sum of 70231, 4013, 212, and 4022.

23. 2000 miles + 20200 miles + 300 miles + 125 miles + 5040 miles + 224 miles = how many miles?

24. What is the amount of 320, 50503, 7034, and 12101?

25. How many dollars are 5000 dollars, 920200 dollars, 75 dollars, 50500 dollars, and 4020 dollars?

26. Find the number of gallons in 3000000 gallons, 50000 gallons, 550 gallons, 30200 gallons, and 225 gallons.

27. How many ones, tens, hundreds, and thousands are in the sum of 3025, 50, 4, and 6720?

28. Add together three hundred two thousand fifty, three million fifty thousand five, twenty thousand seven hundred, and five thousand twenty.

29. Find the sum of one hundred thousand, two hundred fifty, seventy thousand five hundred, five million twenty-five, five thousand two, and nine million.

30. What is the amount of two thousand thirty, forty thousand five hundred, two hundred five thousand six, nine million fifty thousand fifty, and three hundred thousand two hundred?

CASE II.

When the Sum of the Units of any Order is Ten or more.

79. *Example.*—Find the sum of 19, 4989, 687, and 6875?

SOLUTION.

19	
4989	*Parts.*
687	
6875	
12570	*Sum.*

EXPLANATION. — Since only like orders of units in different numbers can be added, write the parts so that the figures expressing ones shall stand in a column, tens in the next, hundreds in the next, and thousands in the next; and draw a line under all.

Begin with ones, and add each column separately.

5 ones and 7 ones and 9 ones and 9 ones are 30 ones, or 3 tens 0 ones; write 0 ones under the column of ones, and add 3 tens with the column of tens (55, I).

3 tens and 7 tens and 8 tens and 8 tens and 1 ten are 27 tens, or 2 hundreds 7 tens; write 7 tens under the column of tens, and add 2 hundreds with the column of hundreds.

2 hundreds and 8 hundreds and 6 hundreds and 9 hundreds are 25 hundreds, or 2 thousands 5 hundreds; write 5 hundreds under the column of hundreds, and add 2 thousands with the column of thousands.

2 thousands and 6 thousands and 4 thousands are 12 thousands, or 1 ten-thousand 2 thousands; write 2 thousands under the column of thousands, and write 1 ten-thousand in the ten-thousands' place.

The sum is 1 ten-thousand 2 thousands 5 hundreds 7 tens 0 ones, or *12570.*

Or, briefly, begin with ones, and name the results only.

Five, twelve, twenty-one, thirty; write 0 under the column of ones, and add 3 with the column of tens.

Three, ten, eighteen, twenty-six, twenty-seven; write 7 under the column of tens, and add 2 with the column of hundreds.

Two, ten, twenty-five; write 5 under the column of hundreds, and add 2 with the column of thousands.

Two, eight, twelve; write 2 under the column of thousands, and write 1 ten-thousand in the ten-thousands' place.

The sum, or amount, is *12570*.

80. *Problem.* — A carpenter bought 2375 feet of white-pine boards, 1250 feet of yellow-pine, 625 feet of cherry, and 575 feet of walnut. How many feet of boards did he buy altogether?

SOLUTION.

2375 feet.
1250 feet.
625 feet.
575 feet.

4825 feet.

EXPLANATION. — If a carpenter bought 2375 feet of white-pine boards, 1250 feet of yellow-pine, 625 feet of cherry, and 575 feet of walnut, he bought altogether the sum of 2375 feet, 1250 feet, 625 feet, and 575 feet.

Since only like orders of units in different numbers, etc. (79).

Begin with ones, and add each column separately.

Five, ten, fifteen; write 5 under the column of ones, etc.

The sum is *4825* feet, the number of feet bought altogether.

From the preceding examples and explanations is deduced the following

81. Rule for Addition.

I. *Write the numbers so that units of the same order shall stand in the same column.*

II. *Begin with ones, and add each column of figures separately.*

III. *Write the sum of each column, if less than ten, under the column added. But, if the sum of any column is ten or more, write the ones of that order under the column added, and add the tens with the column of the next higher order.*

IV. *Write the whole sum of the column of the highest order.*

Problems.

Find the sum of the numbers to be added in each of the following problems; read each part, and each amount: —

(31)	(32)	(33)	(34)	(35)	(36)
9	23	234	765	876	654
81	4	56	456	54	7
654	567	7	78	8	987
321	891	898	9	987	89

(37)	(38)	(39)	(40)	(41)	(42)
96	89	7689	987	7	987
3579	8675	68	8	89	65
864	986	9	9865	7897	9
5678	8	9876	97	654	5678
9	6879	989	869	5845	789

(43)	(44)	(45)	(46)	(47)
90830	78700	90509	38009	94949
68540	69800	30908	49008	85858
57650	56700	80407	68007	76767
86960	67800	40806	57006	67676
75670	78900	70305	76005	58585

(48)	(49)	(50)	(51)
70650 acres.	90250 bales.	30876 tons.	90876 cords.
7075 acres.	9087 bales.	9075 tons.	67 cords.
970 acres.	60543 bales.	87 tons.	6789 cords.
90875 acres.	98 bales.	56789 tons.	87654 cords.
25 acres.	456 bales.	870 tons.	567 cords.
9876 acres.	7089 bales.	6789 tons.	98765 cords.

Find the sum of—

52. 25 + 369 + 75078 + 90705 + 8007 + 708.
53. 90807 + 6054 + 543 + 4679 + 50906 + 37896.
54. 98765 + 91829 + 8273 + 7346 + 475897 + 9889.
55. 79087 + 89080 + 607 + 98 + 98765 + 9 + 579.
56. 7495 + 789 + 65768 + 91827 + 89 + 9823.
57. 495867 + 7 + 78 + 728 + 79687 + 98596.
58. 800 feet + 8095 feet + 7084 feet + 80750 feet.

59. 9 yards + 785 yards + 2987 yards + 90876 yards.

60. 705 miles + 8075 miles + 90876 miles + 78548 miles.

61. 800 bushels + 90876 bushels + 9876 bushels + 809 bushels + 5678 bushels + 58697 bushels.

62. 90875 quarts + 875 quarts + 7478 quarts + 807086 quarts + 5768 quarts + 76879 quarts.

63. 40705 tons + 4907 tons + 65975 tons + 80075 tons + 80 tons + 9080 tons + 60809 tons.

64. What is the sum of 378, 4680, 35797, 98, and 97531? Of 1470, 5968, 48372, 9270, 7, 89, and 78694?

65. Tell the amount of 357, 9753, 24680, 86420, and 9786. Of 87654, 7895, 987654, 4689, 86, and 79876.

66. Add together 505 pounds, 70605 pounds, 8095 pounds, 70523 pounds, 9087 pounds, and 70609 pounds.

67. How many acres are 52000 acres, 75275 acres, 8725 acres, 875 acres, 8809870 acres, 89 acres, and 7025 acres?

68. 80750 cords + 825 cords + 70275 cords + 9875 cords + 42778 cords + 598 cords are how many cords?

69. How many dozen are 1800 dozen, 285 dozen, 47625 dozen, 84752 dozen, 9 dozen, 39 dozen?

70. 265 men + 7850 men + 10725 men + 1256 men + 7525 men + 875000 men are how many men?

71. Add together seventy thousand eighty-nine, nine thousand eighty-two, eight, ninety-seven, fifty thousand eighty-eight.

72. What is the amount of eight thousand seventy, nine hundred six, fifty thousand ninety-eight, eighty-five, and eight thousand nine?

73. Find the sum of four hundred thousand nine, ninety thousand eighty, eight thousand sixty-five, twenty thousand four hundred, and nine thousand six.

74. Add together twenty thousand three hundred four, eight hundred, nine thousand two hundred, eight hundred ninety-seven, seven thousand eight hundred nine, and ninety thousand.

(75)	(76)	(77)	(78)	(79)
709870	908765	908077	234567	200888
605430	345678	605046	654321	900876
405670	987654	506075	234567	700654
908760	567898	809084	876543	400789
506780	765432	706053	456789	500876
807650	345678	607082	876543	800789

(80)	(81)	(82)	(83)
89	79823	92837 46	7865
7876	87	39485	6789786
987654	9283764	89796756	768987
19283746	4758	738495	65764398
495867	837465	6978	59687
8273	49586	98729384	75864738
64	66574839	75647	9287

(84)	(85)	(86)	(87)
198765 cords.	175250 tons.	987654	4938574
78968 cords.	985 tons.	394857	3847567
7895 cords.	78695 tons.	837465	5867584
983746 cords.	8476 tons.	485766	9384756
985 cords.	89 tons.	746587	3456789
98278 cords.	982736 tons.	596875	9876543
8625 cords.	93847 tons.	693847	3948576
		738495	6978534
		926476	3849657

(88)	(89)	(86)	(87)
728376 bales.	989 feet.	987654	7586943
84756 bales.	9786 feet.	394857	3849576
8976 bales.	79685 feet.	925689	6385749
98 bales.	928374 feet.	435465	9786534
9876 bales.	86429 feet.	786756	7894563
79685 bales.	9753 feet.	986787	3456987
827364 bales.	789 feet.	654345	6543456
		789876	7898765

Find the amount of —

90. 78654 + 987567 + 579 + 769765 + 928374 + 789.

91. 579 + 852963 + 39584 + 987 + 80907 + 9807089; 9876 + 87654 + 7654 + 678 + 9876 + 78987.

92. 7586974 + 7485 + 907 + 80709 + 6578 + 78 + 9; 8076 + 9080 + 876 + 9765 + 579 + 8642 + 3579.

93. 378654 + 9384 + 876 + 6789 + 378094 + 374659 + 95867 + 384957 + 8564 + 79 + 87654.

94. 6374 + 84756 + 394857 + 8394756 + 984 + 7394 + 83798 + 3765 + 98 + 9 + 738 + 97531.

95. Find the sum of 3789, 83947, 328, 765678, 9283745, 394857, 7395, 3070408, and 7654321.

96. How many are 709304608 + 908765403 + 8798394 + 76985 + 295887 + 38756742 + 394857?

97. 785 sheep + 9755 sheep + 3875 sheep + 38755 sheep + 7865 sheep + 39485 sheep = how many sheep?

98. What is the amount of 725 dollars + 8250 dollars + 98 dollars + 75250 dollars + 126798 dollars + 596875 dollars + 785 dollars + 9753 dollars?

99. Add together 2070908 yards, 879456 yards, 34550 yards, 738275 yards, 29375 yards, 82756 yards, 378565 yards, 93765 yards, and 3579 yards.

100. How many bushels are 2075000 bushels + 750225 bushels + 5000000 bushels + 30728 bushels + 3948 bushels + 765 bushels?

101. How many dollars are 25750 dollars + 375225 dollars + 987753 dollars + 3746598 dollars + 9182736 dollars + 275 dollars?

102. What is the sum of nine million eighty thousand seven, seven hundred thousand eight hundred twenty, nine thousand eighty-nine, eight hundred seven, ninety thousand eight hundred eight?

103. Find the amount of seven hundred thousand ninety-five, seven thousand eighty-four, ninety thousand nine hundred nine.

104. Add together six million six thousand six hundred, seventy-five million twenty thousand five, eight hundred thousand eight hundred eight, seventy thousand seventy, seven thousand ninety-nine.

105. How many are seven hundred million seven hundred thousand seven hundred, eight million eighty thousand eighty, five million five thousand five, ninety-nine million ninety-nine thousand ninety-nine?

106. Benjamin Franklin was born in the year 1706, and lived to be 84 years old. When did he die?

107. In what year did a person born in the year 1814 become 57 years old?

108. By selling a house for 7225 dollars, a man lost 555 dollars. How much did the house cost?

109. A dealer bought a house for 8275 dollars, and sold it for 750 dollars more than he paid. What did he get for it?

110. How many feet will it take to go around a piece of ground which is 1789 feet long and 985 feet wide?

111. At a large foundry 9875 pounds of iron were cast each day for five days. How many pounds were cast in all?

112. A builder counted the lumber to be used in a barn to be worth 1750 dollars, the carpenter-work 275 dollars, the hardware 116 dollars, and the painting 195 dollars. What was the whole calculated cost?

113. A flour-dealer shipped 3275 barrels of flour one month, 5129 the second, 2570 the third, and 1975 the fourth. How many barrels did he ship in all?

114. John Smith bought a farm for $4525, built on it a house for $2275, and sold his property at a gain of $775. What was the selling price?

115. In building my house I used 4500 pressed bricks, 7500 dark stretchers, 5625 light stretchers, and 4275 salmon. How many bricks were used?

116. One week an engine drew a train of cars 989 miles, the next week 1765 miles, the next 2375 miles, and the next 1926 miles. How far did it run in the four weeks?

117. My farm cost me $7250; I built on it a house for $3575, a barn for $1786, and out-houses for $976. What was the entire cost?

118. How many yards of muslin were turned out in four weeks by a factory which made 15278 yards one week, 9789 yards the second week, 18275 yards the third, and 13968 yards the fourth?

119. I bought a store for $3775, paid $975 for enlarging it, $765 for repairs, and then sold it so as to gain $575. For how much did I sell it?

120. How many times does a clock strike which strikes the hours from noon till midnight?

121. Commencing with 767, find the sum of all the numbers below 776. To 781. To 793. To 800.

122. What is the sum of all the numbers that you name in counting from 175 to 200? From 985 to 1015?

123. I bought three city lots for $11785, and sold them so as to gain $225 on each lot. For how much did I sell them?

124. Five ships arrived in port and landed 568 bales of cotton, 7257 bales, 3598 bales, 8765 bales, and 6675 bales. What was the whole number of bales?

125. A dry-goods dealer bought 1250 yards of silk for $1375, 890 yards of cloth for $2280, 785 yards of calico for $75, and 1500 yards of flannel for $137. How many yards did he buy, and what did he pay?

82. The following arrangement of the nine digits affords an infinite variety of combinations for drill in addition.

For drill in addition by single figures, take line ***a***, or any other line, or any column, or any diagonal, and let each pupil name the sum produced by adding the figure last named.

Thus, 1 and 2? and 3? and 4? and 5? etc.

By commencing with any other figure in the same line or column an entirely new combination is produced.

For drill in the addition of figures in lines, take line ***a***, or any other line, and add the figures in order from left to right, or from right to left.

Thus, 1 + 2 + 3, etc.; or, line ***p***, 2 + 1 + 2 + 3, etc.

For drill in the addition of numbers, take any number of columns regarded as orders of units.

Thus, take columns, or orders, 1, 2, and 3, and add the numbers so formed from ***a*** to ***f***; to ***l***; to ***q***. From ***e*** to ***m***, etc. (See Key.)

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
a —	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1 — ***a***
b —	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	2 — ***b***
c —	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	2	3 — ***c***
d —	4	5	6	7	8	9	8	7	6	5	4	3	2	1	2	3	4 — ***d***
e —	5	6	7	8	9	8	7	6	5	4	3	2	1	2	3	4	5 — ***e***
f —	6	7	8	9	8	7	6	5	4	3	2	1	2	3	4	5	6 — ***f***
g —	7	8	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7 — ***g***
h —	8	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8 — ***h***
i —	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9 — ***i***
j —	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	8 — ***j***
k —	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	8	7 — ***k***
l —	6	5	4	3	2	1	2	3	4	5	6	7	8	9	8	7	6 — ***l***
m —	5	4	3	2	1	2	3	4	5	6	7	8	9	8	7	6	5 — ***m***
n —	4	3	2	1	2	3	4	5	6	7	8	9	8	7	6	5	4 — ***n***
o —	3	2	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3 — ***o***
p —	2	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2 — ***p***
q —	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1 — ***q***
17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	

Review.

1. What is ***Addition?*** What is the sign of addition? What is it named? How is it read? What does it show? What are the numbers to be added called?

2. Define ***Sum,*** or ***Amount.*** How many units must the sum contain? What is the sign of equality? What does it show? Tell whether or not 9 dollars and 16 dollars can be united in one sum, and why. 9 dollars and 16 pints, and why.

3. What are ***Similar Numbers?*** Name five similar numbers. What are numbers whose units are not alike called? Name five dissimilar numbers. What kind of numbers only can be added? What orders of units in different numbers only can be added? To what is the sum similar? Repeat the three principles of addition.

4. How are numbers to be added together written? With which column do you commence to add? What is done with the sum of each column, if less than ten? If the sum of any column is ten or more, what is to be done with the sum? How is the sum of the column of the highest order written? Repeat the rule for addition.

SECTION IV.

SUBTRACTION.

1. What number of pounds must be added to 5 pounds to make 8 pounds?

2. A cask can contain 12 gallons, and there are now in it 7 gallons. How many gallons must be added to fill it?

3. If Ann has 8 cents, how many more cents must her father give her that she may have 15 cents?

4. 16 quarts are how many quarts more than 9 quarts? 2 quarts are how many less than 17 quarts?

5. How many less are in a flock of 10 birds than in a flock of 18 birds? How many are 18 less 8?

6. How many are 5 less than 14? 7 less than 15? 9 less than 17? 19 less 12? 21 less 15?

7. From a pile of 22 bricks were taken 16 bricks. How many bricks remain? 17 taken from 24 leave how many?

8. 6 ones taken from 11 ones leave how many ones? 8 ones from 13 ones? 15 ones less 9 ones?

9. What number remains after taking the ones in 6 from the ones in 9? After taking 7 from 13?

10. How many remain after taking 17 from 23? 18 from 25? 19 from 27? 20 from 28?

11. Frank is 9 years old, and his brother is 16 years old. What is the difference in their ages?

12. What is the unit of 17 inches? Of 8 inches? Are the units of 17 inches and 8 inches like, or unlike?

13. Can 8 inches be taken from 17 inches? What is the difference between 17 inches and 8 inches?

14. What is the unit of 18 feet? Of 10 cents? Are the units of 18 feet and 10 cents like, or unlike? Can 10 cents be taken from 18 feet? Why?

15. Between what kind of numbers only can the difference be found? Between what kind of units?

In each of the preceding questions there are given two numbers to find the difference between them, or to find the remainder of the larger after taking away the smaller.

Definitions.

83. ***Subtraction*** is the process of taking one of two numbers from the other to find their difference.

84. The ***Difference,*** or ***Remainder,*** is the number obtained by subtracting one of two numbers from the other.

85. The ***Minuend*** is that one of two numbers from which the other is taken.

86. The ***Subtrahend*** is that one of two numbers which is taken from the other.

Thus, if 9 dollars are taken from 35 dollars, 35 dollars is the *minuend*, 9 is the *subtrahend*, and 26 dollars is the *difference*, or *remainder*.

87. The ***Sign of Subtraction*** is a short straight line, —. It is named *minus*, and means *less*.

When the sign of subtraction is placed between two numbers, it shows that the number placed after it is to be taken from the number placed before it.

Thus, 26 feet — 9 feet = 17 feet, is read 26 feet *minus* 9 feet *equal* 17 feet; and shows that 9 feet are to be taken from 26 feet, and that the difference between them is 17 feet.

88. Subtraction Tables.

1 from	*2* from	*3* from	*4* from	*5* from
1 leaves *0*	*2* leaves *0*	*3* leaves *0*	*4* leaves *0*	*5* leaves *0*
2 " *1*	*3* " *1*	*4* " *1*	*5* " *1*	*6* " *1*
3 " *2*	*4* " *2*	*5* " *2*	*6* " *2*	*7* " *2*
4 " *3*	*5* " *3*	*6* " *3*	*7* " *3*	*8* " *3*
5 " *4*	*6* " *4*	*7* " *4*	*8* " *4*	*9* " *4*
6 " *5*	*7* " *5*	*8* " *5*	*9* " *5*	*10* " *5*
7 " *6*	*8* " *6*	*9* " *6*	*10* " *6*	*11* " *6*
8 " *7*	*9* " *7*	*10* " *7*	*11* " *7*	*12* " *7*
9 " *8*	*10* " *8*	*11* " *8*	*12* " *8*	*13* " *8*
10 " *9*	*11* " *9*	*12* " *9*	*13* " *9*	*14* " *9*
11 " *10*	*12* " *10*	*13* " *10*	*14* " *10*	*15* " *10*

6 from	*7* from	*8* from	*9* from	*10* from
6 leaves *0*	*7* leaves *0*	*8* leaves *0*	*9* leaves *0*	*10* leaves *0*
7 " *1*	*8* " *1*	*9* " *1*	*10* " *1*	*11* " *1*
8 " *2*	*9* " *2*	*10* " *2*	*11* " *2*	*12* " *2*
9 " *3*	*10* " *3*	*11* " *3*	*12* " *3*	*13* " *3*
10 " *4*	*11* " *4*	*12* " *4*	*13* " *4*	*14* " *4*
11 " *5*	*12* " *5*	*13* " *5*	*14* " *5*	*15* " *5*
12 " *6*	*13* " *6*	*14* " *6*	*15* " *6*	*16* " *6*
13 " *7*	*14* " *7*	*15* " *7*	*16* " *7*	*17* " *7*
14 " *8*	*15* " *8*	*16* " *8*	*17* " *8*	*18* " *8*
15 " *9*	*16* " *9*	*17* " *9*	*18* " *9*	*19* " *9*
16 " *10*	*17* " *10*	*18* " *10*	*19* " *10*	*20* " *10*

Exercises.

1. How many are 9 less 8? 19 less 8? 29 less 8? 39 less 8? 49 less 8? 8 less 6? 18 less 6? 28 less 6? 38 less 6? 48 less 6? 58 less 4? 68 less 4? 78 less 4? 88 less 4? 98 less 4?

2. What is the difference between 12 and 7? 22 and 7? 32 and 7? 42 and 7? 52 and 7? 13 and 6? 23 and 6? 33 and 6? 43 and 6? 53 and 6? 64 and 5? 74 and 5? 84 and 5? 94 and 5?

3. What is the remainder of 17 less 8? 37 less 8? 57 less 8? 77 less 8? 97 less 8? 16 less 9? 26 less 9? 36 less 9? 46 less 9? 55 less 7? 65 less 7? 75 less 7? 85 less 7? 95 less 7?

4. Count back by 2's from 100 to 90. From 90 to 80. From 80 to 60. From 60 to 40. From 40 to 20, and then to 0. From 97 to 85. From 83 to 67. From 61 to 41. From 36 to 18, and then to 0.

5. Count back by 3's from 100 to 85. From 90 to 72. From 80 to 59. From 70 to 46. From 60 to 30, and then to 0. From 99 to 84. From 77 to 56. From 64 to 40. From 57 to 24, and then to 3.

6. Take 4 from every fourth number from 100 to 84. From 90 to 70. From 80 to 56. From 70 to 38. From 60 to 32, and then to 0. From 94 to 78. From 85 to 65. From 71 to 47. From 59 to 27, and then to 3.

7. Subtract 5 from every fifth number from 100 to 85. From 90 to 70. From 85 to 60. From 80 to 50. From 40 to 0. From 96 to 81. From 78 to 58. From 63 to 38. From 57 to 27, and then to 2.

8. Count back by 6's from 100 to 82. From 90 to 66. From 70 to 40. From 91 to 61. From 78 to 42. From 63 to 39, and then to 3. By 7's from 99 to 78. From 86 to 58. From 71 to 36. From 60 to 32. From 56 to 35, and then to 0.

9. Count back by 8's from 99 to 83. From 88 to 64. From 79 to 47. From 60 to 20. From 56 to 0. By 9's from 93 to 75. From 89 to 62. From 86 to 50. From 77 to 32. From 68 to 14. From 63 to 0. By 10's from 100 to 70. From 90 to 60. From 86 to 46. From 78 to 28. From 67 to 17. From 93 to 43, and then to 3.

Mental Exercises.

1. A hardware dealer bought 10 saws, and sold 6 of them. How many saws had he remaining?

89. Model. — If a hardware dealer bought 10 saws and sold 6 of them, he had remaining the difference between 10 saws and 6 saws, which is 4 saws.

2. A train of 20 cars was made up of coal-cars and cattle-cars. If 9 were coal-cars, how many were cattle-cars?

3. I paid 25 cents for a knife, and sold it for 20 cents? How much less was the selling price than the buying price?

4. Harry had a half-dollar in his purse, and offered it in payment of 40 cents' worth of candy. How much change did he receive?

5. What number is that which, if added to 8, equals 15? Equals 17? 19? 21? 23?

6. What number is that which, if taken from 19, leaves a remainder of 13? Of 15?

7. The sum of two numbers is 22, and the less is 9. Which is the greater? If the greater is 17, what is the less?

8. The sum of two parts is 23. If one part is 5, what is the other? If one part is 15?

9. If the minuend is 23 years and the subtrahend is 8 years, what is the remainder?

10. The minuend is 25 months and the remainder is 16 months. What is the subtrahend?

11. What is the minuend, if the subtrahend is 18 days and the remainder is 9 days?

12. What is the remainder of the minuend 28 dollars after 9 dollars are taken away?

13. What is the excess of the minuend 31 tons over the subtrahend 7 tons?

14. A drayman handled a cask containing 32 gallons so as to cause a leakage of 7 gallons. How many gallons remained?

15. By paying 36 cents for a knife, Thomas gave for it 9 cents more than its value. How much was it worth?

16. A boy took into a village 42 quarts of berries, and took home again 8 quarts. How many had he sold?

17. A tea-dealer bought 45 chests of tea on Monday, and 6 chests less on Tuesday. How many did he buy on Tuesday?

18. One morning a news-boy started with 10 cents, and in the evening he returned with 55 cents. What were his profits?

19. A man having 62 dollars in bank, drew out 20 dollars. How many dollars had he remaining in bank?

20. A grocer sold 19 pounds of butter from a firkin containing 32 pounds. How many pounds remained?

90. Model. — If a grocer sold 19 pounds of butter from a firkin containing 32 pounds, there remained the difference between 32 pounds and 19 pounds.

19 pounds equal 10 pounds and 9 pounds; 10 pounds from 32 pounds leave 22 pounds, and 9 pounds from 22 pounds leave 13 pounds. Therefore, there remained 13 pounds.

21. If I had 16 dollars more in my pocket-book, I should then have 42 dollars. How many dollars have I?

22. John says if he live 18 years longer, he will then be 45 years old. What is his age now?

23. Mr. Grant is 47 years old, and his son is 23 years old. What is the difference in their ages?

24. A young man started to walk to a city 52 miles distant. After walking 27 miles, how far had he to go?

25. There were 61 passengers on a train of cars. After 32 had left at a station, how many remained?

26. I bought a horse for 80 dollars, and sold him at a loss of 28 dollars. How many dollars did I receive for him?

27. If you exchange a watch worth 68 dollars for a set of harness worth 93 dollars, how much would you make?

28. A farmer sold a bushel of potatoes for 75 cents, and received in payment a dollar note. What change did he give?

29. What is the difference between the ones in 9 and in 7? The tens in 30 and in 20? The tens and ones in 45 and in 30?

30. How many tens and ones in 25 less 8? In 35 less 20? In 40 less 25? In 45 less 32? In 65 less 49?

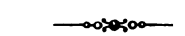
From the preceding questions and explanations are deduced the following

91. Principles of Subtraction.

I. *Subtraction is the reverse operation of addition.*

II. *Only similar numbers, and only like orders of units can be taken one from another.*

III. *The difference, or remainder, is a number similar to the minuend and the subtrahend.*



CASE I.

Written Exercises.

When the Units of the Subtrahend are not greater than those of the same Order in the Minuend.

92. *Example.* — From 17289 take 4058.

SOLUTION.

17289 ***Minuend.***
4058 ***Subtrahend.***
13231 ***Difference.***

EXPLANATION.—Since only like orders of units in different numbers can be taken one from the other, write the less number under the greater so that figures expressing *ones* shall be in the first column, the figures expressing *tens* in the next, *hundreds* in the next, *thousands* in the next, and *ten-thousands* in the next; and draw a line under all.

Begin with ones, and subtract each column separately.

8 ones from 9 ones leave 1 one; which write under the column of *ones*.

5 tens from 8 tens leave 3 tens; which write under the column of *tens*.

0 hundreds from 2 hundreds leave 2 hundreds; which write under the column of *hundreds*.

4 thousands from 7 thousands leave 3 thousands; which write under the column of *thousands*.

Write 1 ten-thousand in the *ten-thousands'* place.

The difference is 1 ten-thousand 3 thousands 2 hundreds 3 tens 1 one, or ***13231***.

Or, briefly, begin with ones, and name the results only.

8 from 9 leaves 1; write 1 under the column of *ones*.

5 from 8 leaves 3; write 3 tens under the column of *tens*.

0 from 2 leaves 2; write 2 hundreds under the column of *hundreds*.

4 from 7 leaves 3; write 3 thousands under the column of *thousands*.

Write 1 ten-thousand in the *ten-thousands'* place.

The difference is ***13231***.

93. *Problem.* — A farmer's sales one year amounted to $3765, and his expenses were $2162. How much did he clear?

SOLUTION.

$3765 ***Minuend.***
$2162 ***Subtrahend.***
$1603 ***Difference.***

EXPLANATION.—If a farmer's sales one year amounted to $3765, and his expenses were $2162, he cleared the difference between $3765 and $2162.

Since only like orders of units in different numbers can be taken, etc. (92).

Begin with ones, and subtract each column separately.

Briefly, 2 from 5 leaves 3; write 3 ones under the column of ones. 6 from 6 leaves 0; write 0 tens under the column of tens, etc.

The difference is $1603, the sum cleared.

Problems.

Find the difference between the numbers in the following problems, and read each term and each remainder: —

(1)	(2)	(3)	(4)	(5)	(6)	(7)
374	746	485	635	596	524	687
60	342	71	203	82	324	56

(8)	(9)	(10)	(11)	(12)	(13)
9283	7465	4657	6875	8697	9876
7132	402	25	54	572	8273

(14)	(15)	(16)	(17)	(18)
1928	74654	63829	38476	9786
705	2334	13025	7253	634

(19)	(20)	(21)	(22)
5325 gallons.	76750 quarts.	68500 pints.	8765 gills.
2320 gallons.	6750 quarts.	2400 pints.	450 gills.

Find the remainder of—

23. 9284 — 223; 7364 — 52; 5536 — 5025.

24. 4765 — 754; 9392 — 372; 73829 — 3526.

25. 7364 bales — 52 bales; 3718 cords — 703 cords.

26. 55647 feet — 5043 feet; 94756 tons — 4056 tons.

27. From 90723 take 601; and from 75987 take 735.

28. Find the remainder of 837465 less 702060.

29. 5968720 — 234500 = how many?

30. From 9624853 yards subtract 3403750 yards; and from 3928175 feet take 28065 feet.

31. How many gallons are 176847 gallons less 100326 gallons? 560705 gallons — 60700 gallons?

32. How many ones, tens, hundreds, and thousands are in the difference between 9076 and 2035?

33. Find the difference between seven million eighty-eight thousand five hundred seven and two million forty thousand four hundred four.

34. From nine hundred eight million eight hundred seventy thousand sixty-five take seven million fifty thousand thirty.

35. How many are left after taking seventy million four hundred forty thousand four hundred from ninety-eight million nine hundred ninety thousand nine hundred?

CASE II.

Written Exercises.

When the Units of the Subtrahend are greater than those of the same Order in the Minuend.

94. *Example* 1. — Find the difference between 45 and 9.

SOLUTION.

45 ***Minuend.***
9 ***Subtrahend.***
36 Difference.

EXPLANATION.—Since only like orders of units in different numbers, etc.

Begin with ones, and subtract each column separately.

Since 9 ones cannot be taken from 5 ones, take 1 ten, or 10 ones, from the 4 tens of the minuend, leaving 3 tens, and add the 10 ones borrowed to the 5 ones, making the minuend the same as 3 tens 15 ones.

9 ones from 15 ones are 6 ones; which write under the column of *ones*.

Write the 3 tens in the tens' place.

The difference is 3 tens 6 ones, or ***36***.

95. *Example* 2. — From 2345 take 987.

SOLUTION.

2345 ***Minuend.***
987 ***Subtrahend.***
1358 Difference.

EXPLANATION.—Since only like orders of units in different numbers, etc.

Begin with ones, and subtract each column separately.

Since 7 ones cannot be taken from 5 ones, take 1 ten from the 4 tens in the minuend, leaving 3 tens, and add the 10 ones borrowed to the 5 ones, making 15 ones. 7 ones from 15 ones are 8 ones; which write under the column of *ones*.

Since 8 tens cannot be taken from 3 tens, take 1 hundred from the 3 hundreds in the minuend, leaving 2 hundreds, and add the 10 tens borrowed to the 3 tens, making 13 tens. 8 tens from 13 tens are 5 tens; which write under the column of *tens*.

Since 9 hundreds cannot be taken from 2 hundreds, take 1 thousand from the 2 thousands in the minuend, and add the 10 hundreds borrowed to the 2 hundreds, making 12 hundreds. 9 hundreds from 12 hundreds are 3 hundreds; which write under the column of *hundreds*.

Write 1 thousand in the *thousands'* place.

The difference is 1 thousand 3 hundreds 5 tens 8 ones, or *1358*.

Or, briefly, 7 cannot be taken from 5; borrow 1 ten, or 10 ones, from 4 tens, and add the 10 ones to 5 ones. 7 from 15 leaves 8; which write under the column of *ones*.

8 cannot be taken from 3; borrow 1 hundred, or 10 tens, from 3 hundreds, and add the 10 tens to 3 tens. 8 from 13 leaves 5; which write under the column of *tens*.

9 cannot be taken from 2; borrow 1 thousand, or 10 hundreds, from 2 thousand, and add the 10 hundreds to 2 hundreds. 9 from 12 leaves 3; which write under the column of *hundreds*.

Write 1 thousand in the *thousands'* place.

The difference is *1358*.

96. *Problem.* — A man, born in the year 1853, died in the year 1901. What was his age at the time of his death?

SOLUTION.

1901 years.
1853 years.
48 years.

EXPLANATION. — If a man, born in the year 1853, died in the year 1901, his age was the number of years equal to the difference between 1901 years and 1853 years.

Since only like orders of units in different numbers, etc. (95).

The difference is *48 years;* therefore, his age, at the time of his death, was 48 years.

From the preceding exercises and explanations is deduced the following

97. Rule for Subtraction.

I. *Write the subtrahend under the minuend so that units of the same order shall stand in the same column.*

II. *Begin with ones, and subtract each column separately.*

III. *If the units of any order of the subtrahend are less than the units of the same order in the minuend, write the difference under the order subtracted.*

If the units of any order of the subtrahend are greater than the units of the same order in the minuend, take one unit from the next higher order of the minuend, and add ten, or the

value of the unit borrowed, to the units of the minuend, and subtract; then consider the units of that higher order of the minuend as one less, and proceed as before.

Problems.

Find the difference between the numbers in the following problems; read each term and each remainder: —

(36)	(37)	(38)	(39)	(40)	(41)
9185	7294	6370	5937	4826	3715
2736	3625	2579	1938	2390	1806

(42)	(43)	(44)	(45)	(46)
12934	56432	23470	30456	76543
987	8345	876	8377	789

(47)	(48)	(49)	(50)	(51)
758653	192830	648510	383740	536475
268564	183746	268717	283647	394857

(52)	(53)	(54)	(55)
307406 tons.	9012087 cords.	3405067 acres.	$807065
34785 tons.	706504 cords.	9868 acres.	6273

Find the remainder of —

56. 123045 — 7365; 516072 — 3456; 908375 — 8076.
57. 17320560 — 9238276; 27364051 — 8297329.
58. 20903080 — 2030405; 30725809 — 10349072.
59. 134504762 — 94393763; 230450670 — 37378589.
60. 342356457 — 52365598; 293847561 — 94758607.
61. 109028374 — 90209325; 203456765 — 32540775.
62. 135056067 — 84545859; 701027341 — 90200732.
63. $230560 — $123559 are how many dollars?
64. From 325765253 acres take 39876054 acres.
65. How many feet are 350725 feet — 57275 feet?
66. Take 123783732 bushels from 143287890 bushels.

67. 307040654 quarts — 9073473 quarts = how many?

68. From 150723045 pounds take 98765432 pounds.

69. How many are 2098034 barrels — 909725 barrels?

70. Subtract 253702654 tons from 918374657 tons.

71. 340760560 yards — 234567468 yards = how many yards? 123040506 feet less 9080708 feet?

72. From three hundred million five hundred six thousand twenty-four take ninety million fifty thousand nine.

73. Subtract ninety million eight hundred thousand six hundred nine from seven hundred three million fifty-five.

74. Find the difference between three hundred million fifty-four thousand thirty and ninety million five thousand five.

75. What is the remainder of seventy million sixty thousand forty less nine million fifty-nine thousand thirty-nine?

98. *Example.* — From 72006 take 508.

SOLUTION.

72006 ***Minuend.***
508 ***Subtrahend.***
71498 ***Difference.***

EXPLANATION. — Write the numbers, etc. (95).

Begin with ones, and subtract each column separately.

Since 8 ones cannot be taken from 6 ones, and as there are no tens and no hundreds to add to the ones, take 1 thousand from 2 thousands, leaving 1 thousand.

This 1 thousand equals 10 hundreds; take 1 hundred from 10 hundreds, leaving 9 hundreds. This 1 hundred equals 10 tens; take 1 ten from 10 tens, leaving 9 tens. This 1 ten equals 10 ones, which added to 6 ones equals 16 ones.

8 from 16 leaves 8; which write under the column of ones.

0 from 9 leaves 9; which write under the column of tens.

5 from 9 leaves 4; which write under the column of hundreds.

Write 1 thousand in the thousands' place, and 7 ten-thousands in the ten-thousands' place.

The difference is *71498*.

(76)	(77)	(78)	(79)	(80)
20908	*38004*	*40065*	*50006*	*70000*
2079	*705*	*9087*	*907*	*1009*

(81)	(82)	(83)	(84)	(85)
100902	*910080*	*200035*	*730600*	*300007*
90903	*10092*	*987*	*40009*	*70008*

(86)	(87)	(88)	(89)
607005 tons.	*4900058* cords.	*5000705* feet.	*308000* men.
90087 tons.	*969* cords.	*30908* feet.	*9001* men.

Find the difference and read the remainder of—

90. 405006 — 7094; 205080 — 6709; 73075 — 4348.
91. 324354 — 5907; 400732 — 974; 407327 — 9049.
92. 4307651 — 9863; 3050707 — 908; 4030004 — 96.
93. 4000040 — 909; 50050050 — 80807; 600600 — 707.
94. 600040 — 605; 1000000 — 1001; 20000000 — 2002.
95. Find the difference between 3009 and 3000303.
96. From 50050005 take 60606; and 505 from 40404.
97. How many are 3000000 less 30003? 4000000 less 5?
98. Subtract 25 from 25000000, and 750 from 150000000.
99. How many remain after taking 30000 from 404004000?
100. Find the difference between 605073 feet and 9645 feet.
101. 6336000 inches less 31680 inches are how many?
102. What is the remainder of $25000750 — $300875?
103. How many are 30050235 quarts — 60546 quarts?
104. 32525007 yards — 75000 yards are how many yards?
105. From 700420500 inches subtract 525750 inches.
106. How many tons are 200050000 tons less 725005 tons?

107. Take six million sixty-six thousand sixty-six from seventy million fifty thousand thirty.

108. Subtract four million fifty thousand fifty from seven hundred million six hundred thousand five hundred.

109. The minuend is eight hundred million eight hundred thousand eight hundred, and the subtrahend is seven million seven thousand seven. What is the difference?

110. If the substrahend is sixty million six, and the minuend seven hundred million seventy thousand seventy, what is the remainder?

111. What is the remainder, if the minuend is three hundred thirty million thirty thousand thirty and the subtrahend is four million forty thousand forty four?

112. What number increased by 23075 will become 10000000? What number increased by 50005?

113. What number must be taken from 7300705 to give a remainder of 97636?

114. What number must be added to 30075 to make 30075000? To make 500000005?

115. The sum of two numbers is 20050700, and the less number is 78205. What is the greater?

116. The sum of two numbers is 90050700, and the greater number is 70505075. What is the less?

117. The greater of two numbers is 17005320, and the difference between them is 7061725. What is the less?

118. America was discovered by Columbus in 1492. How many years have passed since that time?

119. The Revolutionary War commenced in 1775, and ended in 1783. How many years did it last?

120. Our Independence was declared on the Fourth of July, 1776. How many years ago was that on last Fourth of July?

121. Washington was born in 1732, and became president in 1789, and died in 1799. How old was he when he became president, and how old when he died?

122. If you subtract 203040 ten times from 5000000, what will each remainder be?

123. One year a cotton-factory turned out goods that were worth $225000, and the expenses of making them were $192275. What were the profits?

124. The earnings of a railroad one year were $125324, and the expenses were $119075. How much was cleared?

125. A builder agreed to put up a row of houses for $125000. The materials and work cost him $116250. How much did he make?

126. A man owns property worth $17000, and he owes $12750. If his debts were paid, what would he be worth?

127. A farmer bought a farm for $12500, and paid on it $7525 cash. How much did he still owe?

128. Into a water-basin that could hold 5000000 gallons of water, there ran 3010075 gallons. How nearly full was it?

129. The distance from the earth to the moon is about 240000 miles, and from the earth to the sun about 93000000 miles. How much further is it to the sun than to the moon?

130. A man put $13230 in two banks. If he put $9225 in one bank, how many dollars did he put in the other?

131. A store with the goods in it was worth $250000, and the building alone was worth $120325. How much were the goods worth?

132. At an election one man received 47234 votes, and the other received 43975. What majority had the first?

133. If a bushel of timothy-seed contains 41823360 grains, and a bushel of clover-seed 16400960, how many more grains of timothy-seed than of clover-seed are in a bushel?

134. If a city which contained 832507 inhabitants ten years ago, now contains 935765, how many has it increased?

135. A man bought a house for $7500, and paid for it by giving $1500 a year until the house was clear. How many years did it take to pay off the debt?

Review.

1. What is ***Subtraction?*** What is the sign of subtraction? What is it named? What does it show? What does the word *minus* mean?

2. Define ***Difference,*** or ***Remainder.*** How is the difference between two numbers found? What is the ***Minuend?*** What is the number taken from the minuend called?

3. What are ***Like Orders of Units?*** Name two like orders of units. Tell whether or not tens in one number can be taken from tens in another number. Tens in one number from thousands in another number.

4. Tell whether or not 9 dollars can be taken from 16 dollars, and why? 9 dollars from 16 pints, and why? What kind of numbers only can be taken one from the other? What orders of units in different numbers? Repeat the three principles of subtraction.

5. How are numbers to be taken one from the other written? With which order do you commence to subtract? If the units of any order in the subtrahend are less than the units of the same order in the minuend, what is done? If the units of any order in the subtrahend are greater than those of the same order in the minuend, what is done? Repeat the rule for subtraction.

EXERCISES COMBINING ADDITION AND SUBTRACTION.

Mental Exercises.

1. James had 15 cents, and spent 8 cents for a ball and 3 cents for an orange. How many cents had he remaining?

Model. — Since James spent 8 cents for a ball and 3 cents for an orange, he spent for both the sum of 8 cents and 3 cents, which is 11 cents.

Since James had 15 cents and spent 11 cents, he had remaining the difference between 15 cents and 11 cents, which is 4 cents.

2. A milkman filled an 8-quart can and a 12-quart can. After selling 9 quarts, how many quarts had he left?

3. Harry's father gave him 25 cents. He spent 15 cents for a bat and 8 cents for a ball. How much remained?

4. A grocer bought 8 dozen eggs from one farmer, and 18 dozen from another. After selling 9 dozen, how many dozen had he? After selling 19 dozen?

5. A street-car started with 12 passengers. 9 passengers soon left it, and afterwards 13 got in. How many passengers were then aboard? How many more than at first?

6. A carpenter earned 15 dollars one week, and 14 dollars the next week. After paying his expenses, amounting to 19 dollars, how many dollars had he left?

7. I had 18 dollars, spent 12 dollars, and then earned enough to make my money equal 25 dollars. What did I earn? How much less had I at first than at last?

8. Helen bought some needles for 15 cents, and some thread for 20 cents. How much change did she get from a half-dollar offered in payment?

9. George lost 8 cents, and found 18 cents. If he then had 45 cents, how many had he at first?

10. In a day are 24 hours. If a boy spends 10 hours in sleep, 3 in work, and 6 in study, how many hours are left for play? During how many hours is he awake?

11. If from 45 you take 35 less 7, how many will remain?

12. If to 24 less 5 you add 35, what will be the sum?

13. What number must be added to 25 to make a sum equal to 48 plus 8?

14. William has 25 dollars in his purse and 15 dollars in his pocket. After spending 8 dollars from each, how many dollars would he have?

15. A drover put in one field 35 sheep, and 25 in another. If 9 sheep should jump from the first field into the second, how many sheep more or less would there be in the first field than in the second?

Written Exercises.

Problem. — A man has a yearly income of $2500. If he pays $750 for rent and $1400 for living expenses, how much does he save?

SOLUTION.

$750 rent.
$1400 living expenses.
$2150 total expenses.

$2500 income.
$2150 expenses.
$350 amount saved.

EXPLANATION. — If a man pays $750 for rent and $1400 for living expenses, he spends altogether the sum of $750 and $1400.

Write the numbers so that units, etc. (77).

The sum is $2150, the amount expended.

Since the man's yearly income is $2500, and he expends $2150, he saves the difference between $2500 and $2150.

Write the numbers so that units of the same order, etc. (92).

The difference is *$350*, the amount saved.

Note. — Numbers connected by the sign + are to be united in one sum before subtracting the numbers connected or preceded by the sign —.

Problems.

Find and read the result of —

1. 7856 + 2783 + 468 — 8125.
2. 3948 + 9283 — 4857 + 6374.
3. 12075 — 7281 + 1573 + 2198.

4. From the sum of 17395 and 4938 take 9378.

5. To the difference between 10724 and 7859 add 7294.

6. How much is the sum of 90807 and 909 diminished by 89017? Of 357919 + 909 — 99099?

7. What is the result of 10278 — 7289 added to 9827 + 10278? Of 90807 — 908 + 40608 — 709?

8. If from 7250 yards you take 1875 yards less 976 yards, how many yards will remain?

9. What number of feet of boards must be taken from 1725 feet to leave 500 feet more than 928 feet?

10. How many dollars must be added to $2750 to make a sum of money $275 less than $3265?

11. From the sum of 6789 and 9876 take their difference.

12. To the difference between 9753 yards and 3579 yards add their sum. From their sum take their difference.

13. The sum of two numbers is 25250, and one of them is 18785. What is the other number?

14. The sum of three numbers is 8765 miles, and two of them are 2783 miles and 5296 miles. What is the third number?

15. If the sum of four numbers is $38765, and three of them are $12678, $9273, and $15987, what is the fourth?

16. A man was 21 years old in 1860, and he lived until he was 62 years old. In what year did he die?

17. In what year did a man, who was 72 years old in 1870, become of age?

18. A drover bought 750 sheep from one stock-farmer and 375 sheep from another, and then sold 467 sheep. How many had he remaining?

19. Three men bought a piece of ground for $8500. If the first paid $2725, and the second paid $2875, what remainder was there for the third to pay?

20. A merchant began business with $18450. In the first year he gained $1875, and in the second year he lost $3225. What was he then worth?

21. If a merchant began business with $10000, and during the first year lost $3725, and during the second year gained $5637, how much was he then worth?

22. A cistern receives 1525 gallons by one pipe and 785 gallons by another, and discharges 1278 gallons by a third. How much does it then contain?

23. A real-estate agent bought a farm for $7500, put up fences that cost $275, and buildings that cost $1228, and sold it for $11524. How much did he gain?

24. An agent bought a piece of ground for $5000, and put on it improvements that cost $875, and sold it at a loss of $487. What was the selling price?

25. A grain-dealer had 20015 bushels of grain after selling 8124 bushels and buying 9283 bushels. How many bushels had he at first?

26. Two partners bought goods to the amount of $17250, and sold them so as to gain each $928. What was the selling price? How much more than the cost?

27. A span of horses cost $1300, a carriage $450, and a set of harness $175. How much less than the horses did the carriage cost? The harness? The carriage and the harness? What was the entire cost?

28. If a man gave to his eldest son $7250, to the second son $775 less than to the eldest, and to the third $928 less than to the second, how much did each and all receive?

29. A merchant deposited in a bank at one time $1200, at another $755, and at another $525. If he then drew out $1778, how much remained on deposit?

30. A flour-dealer bought 500 barrels of flour for $3125,

and sold 217 barrels for $1725. How many barrels were left, and how much are they worth?

99. Further drill in addition and subtraction may be had from the use of the following table given by De Morgan:

For addition, the numbers in any column, or in any line, or diagonal, or any part of such numbers may be taken.

Thus, find the sum of the numbers in column 1, or in column 1 to *H*, or in column 8 to *e*, or from *d* to *D*, or from 6 to *f*, etc.

For subtraction, any two numbers in any column, line, or diagonal may be taken.

Thus, find the difference between the numbers *c* and *a* in column 10, or between *C* in column 2 and *D* in column 1, etc.

For addition and subtraction, the sum of any two or more numbers in any column, line, or diagonal, and any other number or sum indicated may be taken.

Thus, from the sum of the numbers in column 3 take *I* in column 2, or the sum of the numbers to *I*, etc. (See Key Page 21.)

	11	10	9	8	7	6	5	4	3	2	1	
A	2016	4212	1656	3852	1296	3492	936	3132	576	2772	216	*a*
B	252	2052	4248	1692	3888	1332	3528	972	3168	612	2412	*b*
C	2448	288	2088	4284	1728	3924	1368	3564	1008	2808	648	*c*
D	684	2484	324	2124	4320	1764	3960	1404	3204	1044	2844	*d*
E	2880	720	2520	360	2160	4356	1800	3600	1440	3240	1080	*e*
F	1116	2916	756	2556	396	2196	3996	1836	3636	1476	3276	*f*
G	3312	1152	2952	792	2592	36	2232	4032	1872	3672	1512	*g*
H	1548	3348	1188	2988	432	2628	72	2268	4068	1908	3708	*h*
I	3744	1584	3384	828	3024	468	2664	108	2304	4104	1944	*i*
J	1980	3780	1224	3420	864	3060	504	2700	144	2340	4140	*j*
K	4176	1620	3816	1260	3456	900	3096	540	2736	180	2376	*k*
	11	10	9	8	7	6	5	4	3	2	1	

SECTION V.

MULTIPLICATION.

1. How many cents are 4 cents and 4 cents? 4 cents added to 4 cents? 4 cents + 4 cents?

2. If Henry spends 5 cents at one time and 5 cents at another time, how many times does he spend 5 cents?

3. How many cents are 6 cents and 6 cents and 6 cents? 6 cents + 6 cents + 6 cents? 3 times 6 cents?

4. On each one of 5 branches are 4 apples. How many times 4 apples are there? How many apples?

5. How many blocks are 6 blocks taken or repeated 4 times? 4 balls repeated 6 times?

6. What number of dollars is produced by taking or repeating 6 dollars 5 times?

7. How many ones are in 6? How many quarts are 7 quarts repeated as many times as there are ones in 6?

8. What number is produced by repeating 6 as many times as there are ones in 7? In 8? In 9?

9. If 8 be placed 7 times in a column and the column be then added, what will be the sum?

10. John has 8 blocks, and his brother has 8 times as many. How many blocks has his brother?

11. How many are 3 times 4? 5 times 6? 6 times 7? 7 times 8? 8 times 9?

12. If you spend 8 dollars 6 times, do you spend more dollars or fewer dollars than if you spend 6 dollars 8 times?

13. If you buy 5 melons at 10 cents each, to find the cost should you take 10 cents 5 times, or take 5 ten times?

14. What is the unit of 6 feet? Of 5? Of the number produced by taking 6 feet 5 times?

15. Is the unit of the number produced by repeating any given number like the unit of the number repeated, or is it different?

In each of the preceding questions it has been required to find the result of taking one of two numbers as many times as there are ones in the other number.

Definitions.

100. Multiplication is the process of repeating one of two numbers as many times as there are ones in the other.

101. The ***Product*** is the number obtained by multiplying one of two numbers by the other.

102. The ***Multiplicand*** is that one of two numbers which is repeated or multiplied.

103. The ***Multiplier*** is that one of two numbers by which the other is multiplied.

104. The multiplicand and the multiplier are called the ***Factors*** of the product.

Thus, *3* and *4*, whose product is 12, are the *factors* of 12; *4* and *5* are *factors* of 20.

The product of three or more factors is called the *Continued Product.*

Thus, 60 is the *continued product* of 3 times 4 times 5.

105. The ***Sign of Multiplication*** is a short inclined cross, $\times$. It is read *multiplied by.*

When the sign of multiplication is placed between two numbers, it shows that the number placed before it is to be multiplied by the number placed after it.

Thus, 10 cents $\times$ 5 = 50 cents, is read 10 cents *multiplied by* 5 equals 50 cents. 10 cents is the *multiplicand*, 5 is the *multiplier*, and 50 cents is the *product.*

106. A ***Concrete Number*** is a number whose units are named.

Thus, *one man, five cents, 10 books, 25 cattle* are *concrete numbers.*

107. An ***Abstract Number*** is a number whose units are not named.

Thus, *one, five, 10,* and *25* are *abstract numbers.*

108. Multiplication Tables.

1 time	*2 times*	*3 times*	*4 times*	*5 times*
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50

6 times	*7 times*	*8 times*	*9 times*	*10 times*
1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 " 12	2 " 14	2 " 16	2 " 18	2 " 20
3 " 18	3 " 21	3 " 24	3 " 27	3 " 30
4 " 24	4 " 28	4 " 32	4 " 36	4 " 40
5 " 30	5 " 35	5 " 40	5 " 45	5 " 50
6 " 36	6 " 42	6 " 48	6 " 54	6 " 60
7 " 42	7 " 49	7 " 56	7 " 63	7 " 70
8 " 48	8 " 56	8 " 64	8 " 72	8 " 80
9 " 54	9 " 63	9 " 72	9 " 81	9 " 90
10 " 60	10 " 70	10 " 80	10 " 90	10 "100

Exercises.

1. Add by 2's from 0 to 8. From 4 to 12. From 6 to 16. From 0 to 20. How many times is 2 added? By 3's from 0 to 9. From 6 to 15. From 9 to 27. From 0 to 30. How many times has 3 been added?

Multiply from 2 times 2 to 5 times 2. From 4 times 2 to 7 times 2. From 1 time 2 to 10 times 2 and reverse. From 1 time 3 to 5 times 3. From 3 times 3 to 6 times 3. From 1 time 3 to 10 times 3 and reverse.

2. Add by 4's from 0 to 16. From 8 to 32. From 12 to 36. From 0 to 40. By 5's from 0 to 20. From 10 to 40. From 15 to 35. From 20 to 45. From 0 to 50.

Multiply from 2 times 4 to 4 times 4. From 4 times 4 to 8 times 4. From 1 time 4 to 10 times 4 and reverse. From 2 times 5 to 5 times 5. From 3 times 5 to 7 times 5. From 1 time 5 to 10 times 5 and reverse.

3. Add by 6's from 0 to 18. From 12 to 36. From 18 to 48. From 24 to 54. From 0 to 60. By 7's from 0 to 21. From 14 to 49. From 21 to 56. From 28 to 63. From 0 to 70.

Multiply from 1 time 6 to 6 times 6. From 3 times 6 to 9 times 6. From 1 time 6 to 10 times 6 and reverse. From 2 times 7 to 7 times 7. From 3 times 7 to 9 times 7. From 1 time 7 to 10 times 7 and reverse.

4. Add by 8's from 0 to 24. From 24 to 48. From 32 to 64. From 40 to 72. From 0 to 80. By 9's from 9 to 36. From 18 to 54. From 27 to 72. From 36 to 81. From 0 to 90.

Multiply from 1 time 8 to 5 times 8. From 3 times 8 to 8 times 8. From 1 time 8 to 10 times 8 and reverse. From 1 time 9 to 4 times 9. From 3 times 9 to 8 times 9. From 1 time 9 to 10 times 9 and reverse.

Mental Exercises.

1. If a pint of milk costs 5 cents, and a quart costs 2 times as much, what is the cost of a quart of milk?

109. Model. — If a pint of milk costs 5 cents, and a quart costs 2 times as much, the cost of a quart of milk is 2 times 5 cents, which are 10 cents.

2. Emma took 4 cents at a time from her pocket to pay for each of 5 oranges. How many times 4 cents did she pay?

3. John wrote 6 seven times in a column, and then added the numbers together. How many are 7 times 6?

4. What number is produced by repeating the number 6 as many times as there are ones in 8?

5. What is the cost of 5 tons of coal at 8 dollars a ton? What is the cost of 7 tons? Of 9 tons?

6. If a shoemaker can make 7 pair of shoes in a week, how many pair can he make in 8 weeks?

7. There are 5 school-days in a week. How many school-days in 5 weeks? In 7 weeks? In 9 weeks?

8. If 6 pipes fill a cistern in 9 hours, how many hours will it take one pipe to fill it?

9. If 8 men do a piece of work in 9 days, how many men would it take to do the same work in one day?

10. I sold 7 silk umbrellas at 10 dollars each. What was the amount of the bill?

11. If in one year there are 12 months, how many months are in 6 years? In 8 years? In 10 years?

12. If a joiner can make 15 doors in a week, how many doors can he make in 7 weeks?

110. Model. — Since a joiner can make 15 doors in one week, in 7 weeks he can make 7 times 15 doors.

15 doors equal 10 doors and 5 doors. 7 times 10 doors are 70 doors, and 7 times 5 doors are 35 doors; 70 doors and 30 doors are 100 doors, and 5 doors are 105 doors.

13. In an orchard are 3 rows of trees, and each row has in it 16 trees. How many trees are in the orchard?

14. A housekeeper bought 4 pounds of sirloin-steak at 20 cents a pound. What did it cost her?

15. A grown person breathes 18 times in a minute. How many breaths does he draw in 5 minutes?

16. If the railroad fare between two cities is 21 dollars, how much must 6 passengers pay for their tickets?

17. A clerk rents a cottage at 25 dollars a month. What amount does he pay for 6 months' rent?

18. If 6 plated-spoons make a set, how many spoons are in 10 sets? In 12 sets?

19. What is the cost of 13 felt-hats at 4 dollars each?

111. Model. — Since 4 dollars is the price of one felt-hat, the cost of 13 felt-hats is 13 times 4 dollars.

13 equals 10 and 3. 10 times 4 dollars are 40 dollars, and 3 times 4 dollars are 12 dollars; 40 dollars and 10 dollars are 50 dollars, and 2 dollars are 52 dollars.

20. If a butcher bought 15 calves at 5 dollars each, what amount did he pay for them? At 6 dollars each?

21. I laid in 16 tons of coal for winter use. How much would it cost me at 6 dollars a ton? At 7 dollars?

22. A fruit-dealer sold 18 oranges at 7 cents each. How much did he get for them? How much for 19 oranges?

23. I drew from bank 20 five-dollar notes to pay for a horse. What was the price of the horse?

24. If 7 men can frame a house in 4 weeks of 6 working days each, how many days would it take one man to frame it?

25. At $2 a bushel, what is the value of 25 sacks of wheat each containing 4 bushels? Of 35 sacks?

26. What is the product of 12 pounds multiplied by 9? The product of 8 pounds multiplied by 18?

27. The multiplicand is 10 feet, and the multiplier is 10. What is the product?

28. What is the unit of 10 feet? Of 10? Of the product of 10 feet multiplied by 10?

29. What is the unit of the multiplicand 12 cents, of the multiplier 6, and of the product of 12 cents multiplied by 6?

30. Is the multiplicand 12 cents a concrete, or an abstract number? The multiplier 6? Is the product similar to the multiplicand, or to the multiplier?

From the preceding exercises are deduced the following

112. Principles of Multiplication.

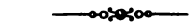
I. *Multiplication is a short method of performing addition.*

II. *The multiplicand may be either a concrete or an abstract number.*

III. *The multiplier is always regarded as an abstract number.*

IV. *The product is always a number similar to the multiplicand.*

Note. — In the solution of problems either factor may, for convenience, be regarded as the multiplier, although the product must be a number similar to the true multiplicand.



CASE I.

Written Exercises.

When the Multiplier is one Figure.

113. *Example.* — How many are 4 times 567?

FIRST SOLUTION.

567
567
567
567 } *Parts.*

2268 *Sum.*

EXPLANATION. — Since the sum of the units in 567 added or repeated 4 times is required, the number may be found by writing 567 four times as parts to be added, and then finding their sum.

Write the numbers so that units, etc. (79).

Begin with ones, and add each column separately.

The sum, or amount, is *2268*.

SECOND SOLUTION.

567 *Multiplicand.*
4 *Multiplier.*

2268 *Product.*

EXPLANATION. — Since 567 is to be repeated as many times as there are ones in 4, the result may be obtained by the process of multiplication, in which 567 is the *multiplicand* and 4 is the *multiplier*.

Write the multiplier under the multiplicand so that units of the same order shall stand in the same column.

Begin with ones, and multiply the units of each order of the multiplicand separately.

4 times 7 ones are 28 ones, or 2 tens 8 ones; write 8 ones under *ones*, and carry the 2 tens to add to the product of the tens.

4 times 6 tens are 24 tens; 24 tens and 2 tens are 26 tens, or 2 hundreds 6 tens; write 6 tens under *tens*, and carry the 2 hundreds to add to the product of the hundreds.

4 times 5 hundreds are 20 hundreds; 20 hundreds and 2 hundreds are 22 hundreds, or 2 thousands 2 hundreds; write 2 hundreds under *hundreds*, and write 2 thousands in the thousands' place.

The product is 2 thousands 2 hundreds 6 tens 8 ones, or *2268*.

Or, briefly, begin with ones, and omit names of orders.

Four times seven are twenty-eight; write 8 under the ones, and add 2 to the product of the tens.

Four times six are twenty-four, and two are twenty-six; write 6 under the tens, and add 2 to the product of the hundreds.

Four times five are twenty, and two are twenty-two; write 2 under the hundreds, and write 2 in the thousands' place.

The product is *2268*.

114. *Problem.* — If 2075 tons of coal are shipped from a mine in one week, how many tons at the same rate are shipped in 4 weeks?

SOLUTION.

2075 tons.
4
8300 tons.

EXPLANATION. — Since 2075 tons of coal are shipped from a mine in one week, at the same rate, in 4 weeks there are shipped 4 times 2075 tons (112, II, III).

Write the multiplier under the multiplicand, etc. (113).

Begin with ones, and multiply the units of each order, etc.

The product is *8300* tons, the number of tons shipped.

Problems.

Find the product of the numbers in each of the following problems; read each number and each product; —

(1)	(2)	(3)	(4)	(5)	(6)
2345 × 3	3456 × 4	4567 × 5	5678 × 6	6789 × 7	7800 × 8

(7)	(8)	(9)	(10)	(11)	(12)
2938 × 7	38475 × 8	47566 × 9	56657 × 9	65748 × 8	7480 × 7

(13)	(14)	(15)	(16)	(17)
30764 × 9	840055 × 8	500067 × 7	654003 × 6	43021 × 5

(18)	(19)	(20)	(21)
392805 miles. × 6	579064 yards. × 8	240678 feet. × 9	903765 inches. × 7

Find the product of—

22. 4532 × 9; 90765 × 8; 203405 × 7; 307596 × 6.

23. 50708 × 5; 43078 × 7; 567898 × 6; 987654 × 8.

24. 607089 × 6; 708987 × 7; 890765 × 8; 903456 × 9.

25. 708076 × 7; 707777 × 6; 7876797 × 8; 97068 × 9.

26. 3976456 × 3; 9; 4; 8; 5; 7; 6; 2; and 1.

27. Find the product of 356789 × 6; of 1719283 × 7.

28. What is the product of 23984576 multiplied by 9.

29. If the multiplicand is 98273645, and the multiplier is 8, what is the product? If the multiplier is 7? 9?

30. Multiply 375375 bushels by 9; 54320 feet by 7.

31. What is the product of 7345630 dollars multiplied by 5? By 7? By 9? By 4? By 6? By 8?

32. Multiply four hundred five thousand ninety-six by six; by eight; by five; by seven; by nine.

33. What is the product of six hundred twenty thousand forty-four multiplied by six? By eight? By five? By seven?

34. The multiplicand is nine million seventy-four thousand ninety, and the multiplier is nine. What is the product?

35. Multiply seventy-one million forty-seven thousand five hundred sixty-six by nine; by eight; by six.

36. Find the product of three hundred seven million fifty thousand sixty-four multiplied by eight.

37. In one day there are 1440 minutes. How many minutes are in a week, or 7 days?

38. A builder owns a block of 7 small houses worth $2750 each. What is the value of all the houses?

39. A long ton of coal weighs 2240 pounds. How many pounds do I get in 8 tons? In 7 tons? In 9 tons?

40. If $15250 were paid for grading and making a mile of railroad, what would 7 miles cost? 9 miles?

41. If six men build a factory, each man giving 10350 dollars, what is the cost? If each gives 15225 dollars?

42. The distance around the earth is 25000 miles. How far does a vessel go in sailing around it 4 times?

43. Light moves 192000 miles in a second of time. How far does it move in 9 seconds? In 5 seconds?

44. In one mile are 63360 inches. How many inches are in 5 miles? In 7 miles? In 9 miles? In 8 miles?

45. To what does the President's salary amount in 4 years at $50000 a year? In 8 years?

CASE II.

Written Exercises.

When the Multiplier is a number of Tens, Hundreds, Thousands, etc.

115. *Example* 1. — Multiply 789 by 10.

SOLUTION.

789 ***Multiplicand.***
10 ***Multiplier.***
7890 ***Product.***

EXPLANATION.—Write the multiplier, etc.

Begin with ones, and multiply, etc.

10 times 9 ones are 90 ones, or 9 tens 0 ones; write 0 ones, etc.

10 times 8 tens are 80 tens; 80 tens and 9 tens are 89 tens, etc.

10 times 7 hundreds are 70 hundreds; 70 hundreds, etc.

The product is *7890*.

116. *Example* 2. — Multiply 789 by 100.

SOLUTION.

789
100
78900

EXPLANATION. — Write the multiplier under the multiplicand so that the ciphers shall stand to the right of the ones of the multiplicand.

Begin with ones of the multiplicand, and multiply each order by 1.

Annex the ciphers of the multiplier on the right of the partial product, *789*.

The product is *78900*, the same figures as are in the multiplicand, with two ciphers annexed.

Since 1 unit of any order is equal to 10 of the next lower order, the writing of a cipher on the right of a number removes each figure one place to the left, and makes the value expressed by each figure 10 times as great as it was before. Hence,

I. *A number is multiplied by* 10 *by annexing one cipher to the right.*

II. *A number is multiplied by* 100 *by annexing two ciphers to the right. Etc.*

117. *Example* 3. — Multiply 789 by 300.

SOLUTION.

789
300
236700

EXPLANATION. — The multiplier, 300, equals 3 times 100, or 100×3; and 789×300 must equal the product of $789 \times 3 \times 100$.

Write the multiplier, etc.

Multiply each order of the multiplicand by 3, giving *2367* as a partial product.

Multiply the partial product 2367 by 100, by annexing two ciphers.

The product is *236700*.

From the preceding examples and explanations is deduced the following

Principle.

The value of any figure is increased tenfold for every place it is removed from the right toward the left.

Problems.

Find the product of the numbers in each of the following problems; read each number, and each product:—

(46)	(47)	(48)	(49)	(50)	(51)
2389	5478	6745	5656	4567	3478
10	20	100	300	400	50

(52)	(53)	(54)	(55)	(56)
4056 feet.	9807 yards.	5670 tons.	6700 cords.	7000 miles.
500	6000	7000	8000	900

Find the product of—

57. 372604 × 500; 4835 × 600; 5940 × 700.

58. 3546 × 8000; 80765 × 9000; 39040 × 6000.

59. What is the product of 53745 multiplied by 500?

60. Multiply 5280 feet by 90, by 300, and by 8000.

61. How many barrels are 5000 times 30650 barrels?

62. The multiplicand is 901080 feet, and the multiplier is 800. What is the product?

63. The multiplicand is seven thousand seven pounds, and the multiplier is six thousand. Find the product.

64. How many are forty thousand forty multiplied by forty? By four hundred? By four thousand?

65. Multiply three hundred seven thousand three hundred twenty by three hundred.

66. How much will 2000 barrels of flour cost at 8 dollars a barrel? How much will 20000 barrels cost?

67. If a barrel of pork weighs 200 pounds, how many pounds does a packer need to fill 10000 barrels?

68. If there are 1010 men in a regiment of soldiers, how many soldiers are in an army of 500 regiments?

69. If a ton of railroad-iron costs $250 how much do 2000 tons cost? 4000 tons? 6000 tons? 10000 tons?

70. At 2 dollars per acre, how much are 1000000 acres of government land worth? At 4 dollars? 5 dollars?

When the Multiplier is two or more Figures.

118. *Example* 1. — Multiply 234 by 45.

FIRST SOLUTION.

234	*234*
5	*40*
1170	*9360*

1170 } ***Partial***
9360 } ***Products.***

10530 ***Product.***

EXPLANATION. — 45 is equal to 4 tens 5 ones, or 40 + 5; and 45 times 234 is equal to the sum of 40 times 234 and 5 times 234, or the sum of 5 times 234 and 40 times 234.

The product of 234 multiplied by 5 is *1170*, which is the first partial product.

The product of 234 multiplied by 4 tens, or 40, is *9360*, which is the second partial product.

Since 45 times 234 is the sum of 5 times 234 and 40 times 234, the sum of the partial products, *1170* and *9360*, must be the entire product, which is *10530*.

SECOND SOLUTION.

234 ***Multiplicand.***
45 ***Multiplier.***

1170 } ***Partial***
9360 } ***Products.***

10530 ***Entire product.***

SECOND EXPLANATION. — Write the multiplier under the multiplicand so that figures expressing units of the same order shall stand in the same column.

Begin with ones of the multiplier, and multiply the units of each order in the multiplicand separately.

The product of 234 multiplied by 5 ones, or 5, is *1170*, which is the first partial product.

The product of 234 multiplied by 40 is *9360*, which is the second partial product.

The sum of the product of 234 by 5, and the product by 40, must be the product by 45, or *10530*.

THIRD SOLUTION.

234
45

1170 } ***Partial***
936 } ***Products.***

10530 ***Product.***

EXPLANATION. — Write the multiplier under the multiplicand, etc.

Begin with the ones, etc.

5 times 4 ones are 20 ones, or 2 tens 0 ones, etc.

5 times 3 tens are 15 tens; 15 tens and 2 tens are 17 tens, etc.

5 times 2 hundreds are 10 hundreds; 10 hundreds and 1 hundred, etc.

The first partial product is *1170*.

40 times 4 ones, or 4 tens times 4, are 16 tens, or 1 hundred 6 tens; write 6 tens under the column of *tens*, and carry 1 hundred to add with the hundreds of the next product.

40 times 3 tens, or 4 tens times 3 tens, are 12 hundreds; 12 hundred and 1 hundred are 13 hundreds, etc.

40 times 2 hundreds, or 4 tens times 2 hundreds, are 8 thousands 8 thousands and 1 thousand are 9 thousands, etc.

The second partial product is *9360*, or 936 tens.

The sum of the two partial products, or the entire product, is *10530*.

119. *Example* 2. — Find the product of 456 multiplied by 304.

SOLUTION.

456
304

1824 = *product by* 4.
1368 = *product by 300.*

138624 = *product by 304.*

EXPLANATION.—Write the multiplier, etc.

Begin with the ones, etc.

Briefly, 4 times 6 are 24, etc.

The first partial product is *1824*.

The product of 456 multiplied by nothing, or 0, is *0*, which may be omitted from the partial products.

3 hundreds times 6 are 18 hundreds, or 1 thousand 8 hundreds, etc.

3 hundreds times 5 tens are 15 thousands; 15 thousand and 1 thousand are 16 thousands, etc.

3 hundreds times 4 hundreds are 12 ten-thousands, etc.

The third partial product is *136800*, or 1368 hundreds.

The sum of the partial products, or the entire product, is *138624*.

From the preceding examples and explanations are deduced the following

120. Rules for Multiplication.

I. When the multiplier is a single figure,

Write the multiplier under the ones of the multiplicand.

Multiply the units of each order in the multiplicand, beginning with ones, by the multiplier.

Place the right-hand figure of each product under the order multiplied, and add the left-hand figure, if any, to the product of the next higher order.

II. When the multiplier is a number of tens, hundreds, etc.,

Multiply the units of each order in the multiplicand by the significant figures of the multiplier, and to the right of this partial product annex as many ciphers as are in the multiplier.

III. When the multiplier is two or more figures,

Write the multiplier under the multiplicand so that units of the same order shall stand in the same column.

Multiply the units of each order in the multiplicand by each figure of the multiplier, except ciphers, beginning with ones.

Place the right-hand figure of each partial product under the figure of the multiplier used to obtain that product.

Add the partial products for the whole, or entire, product.

Problems.

(71)	(72)	(73)	(74)	(75)	(76)	(77)
234	345	456	567	765	654	543
23	345	45	567	78	807	98

(78)	(79)	(80)	(81)	(82)
12345	98765	23456	87654	34567
980	230	870	340	7760

(83)	(84)	(85)	(86)	(87)
97530	86420	13519	24600	123987
102	8090	2030	7080	304

(88)	(89)	(90)	(91)
35790 bales.	*24806* barrels.	*97530* bushels.	*86425* rods.
1234	*90870*	*23450*	*8007*

Find the product of—

92. 4075 × 135; 8064 × 531; 7503 × 357.

93. 5003 × 579; 4002 × 957; 3010 × 791.

94. 19170 × 305; 28260 × 907; 37350 × 507.

95. 24068 × 2304; 35709 × 4032; 57901 × 3507.

96. 91827 × 2345; 82736 × 5432; 73645 × 3456.

97. What is the product of 8175 multiplied by 706?

98. Find the product of 37260 multiplied by 9876.

99. How many are 192837 multiplied by 789?

100. 9761 × 456 = how many? 456 × 9167?

101. 785 times 5876 are how many? 1324 times 67?

102. How many inches are 30000 inches × 345?

103. What is the product of 16500 feet × 1928?

104. Multiply 1440 minutes by 365; by 1095; by 1000.

105. The multiplicand is 91870, and the multiplier is 706. What is the product?

106. What is the product if the multiplier is 2376, and the multiplicand is 75 dollars?

107. Multiply thirty thousand ninety-nine by four hundred fourteen; and by four thousand.

108. Find the product of three thousand times nine thousand eighty-seven.

109. The multiplicand is seven thousand ninety-eight feet, and the multipler is three hundred five. What is the product?

When there are ciphers on the right of both factors.

121. *Example.*—Multiply 89000 by 6700.

SOLUTION.

89000
6700
———
623
534
———
596300000

EXPLANATION.— To shorten the process of multiplication, write the significant figures of the multiplier under the significant figures of the multiplicand, and omit the ciphers from the partial products.

The sum of the partial products by the significant figures is *5963*.

Since the multiplicand is not 89, but 89 thousands, or 89 × 1000, annex three ciphers for the true product of 89000 × 67; also,

Since the multiplier is not 67, but 67 hundreds, or 67 × 100, annex two ciphers for the true product by 6700. Hence,

When there are ciphers on the right of both factors,

Find the product of the significant figures of the multiplicand and the multiplier, and to the right of the partial product annex as many ciphers as are on the right of both factors.

(110)	(111)	(112)	(113)	(114)
345	*4560*	*56700*	*67000*	*70900*
300	*400*	*5000*	*6000*	*6080*

(115)	(116)	(117)	(118)	(119)
365 days.	*4000 years.*	*4500 feet.*	*3040 rods.*	*5280 feet.*
2500	*1040*	*2700*	*7500*	*5600*

120. Two factors are 405 and 7020. What is their product?

121. What is the product of the two factors 45 and 670?

122. If the earth moves around the sun at the rate of 1092 miles in a minute, how far does it move in 1440 minutes, or one day? In 7 days?

123. In a square mile are 640 acres. How many acres are in Pennsylvania, which contains 46000 square miles?

124. What would it cost to build 375 miles of a telegraph line, at $1725 per mile? At $2025 per mile?

125. How many yards of carpet are turned out in a year of 309 working days by a factory in which a daily average of 175 yards is made? How many yards in 458 days?

126. A farm of 146 acres was sold at auction at $175 an acre. What was the selling price of the farm?

127. If a quantity of provisions will supply 960 soldiers 92 days, how many soldiers would it supply for one day?

128. A railroad company engaged to have 55 palace-cars built, each $18275. What was the amount of the contract?

129. Find the product of the three factors $23 \times 45 \times 67$.

130. If a ferry-boat carries on an average 68 passengers each trip, how many passengers does it carry in 6 days, making 15 trips a day? In 12 days? 18 days?

131. A farmer had 46 acres of wheat which yielded 28 bushels an acre. What was the crop worth at $2 a bushel?

132. How much money is needed to pay a regiment of 960 men for 12 months, if each man receives $16 a month?

133. What is the product of 325 multiplied by itself?

134. What is the product of 23 times 45 times 678?

135. How many words are in a book of 228 pages, if it averages 32 lines on each page, and 12 words in each line?

136. If sound moves at the rate of 1142 feet in a second, how far will it move in 5 minutes of 60 seconds each?

137. If a cubic foot of white oak weighs 43 pounds, what is the weight of 15 logs, each containing 50 cubic feet?

138. If a horse requires 15 pounds of hay per day, how many pounds of hay are required to feed 6 horses during the 30 days of April? The 31 days of December?

139. A freight-train consists of 20 cars; each car contains 75 barrels of flour, and each barrel weighs 196 pounds. What weight of flour is in the cargo?

140. Multiply 34567 by 1234; by 2345; by 3456.

141. Multiply 405678 by 4506; by 5607; by 6708.

142. Multiply 5607089 by 5067; by 56078; by 70898.

143. Multiply 60078009 by 60708; by 70809; by 80907.

144. Multiply 70809080 by 34560; by 46890; by 57930.

145. Multiply 35798700 by 40600; by 57900; by 46800.

146. Multiply 24689000 by 357900; by 9876000.

147. The multiplicand is 97536900, and the multiplier is 349856. What is the product?

148. What is the product if the multiplier is 987654, and the multiplicand is $24685900?

149. Find the product of the two factors 987654321 and 123456789.

Review.

1. What is ***Multiplication?*** What is the ***Sign of Multiplication?*** How is it read? What does it show?

2. Define ***Product.*** How is the product of two numbers found? What is the ***Multiplicand?*** What is the number which shows how many times the multiplicand is repeated called?

3. What are the ***Factors*** of the product? What is the product of more than two factors called? Define ***Concrete Number.*** What numbers are called ***Abstract Numbers?*** Name five abstract and five concrete numbers.

4. To find the value of 5 melons at 10 cents each, which is the multiplicand and which is the multiplier? Is the multiplicand always concrete, or always abstract, or may it be either? What is the multiplier always? To which term is the product always similar? Repeat the four principles of multiplication.

5. How is the multiplier written in performing an operation in multiplication? Where do you begin to multiply? When the multiplier is one figure, how do you multiply? When it is two or more figures, where is the first figure of each partial product written? How is the complete product found? How do you multiply by 10, 100, etc.? What is the principle? How do you multiply when there are ciphers on the right of both factors? Repeat the rules for multiplication.

EXERCISES IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

Mental Exercises.

1. Henry is 7 years old, and his father is 5 years more than 4 times as old. What is his father's age?

122. Model. — Since Henry is 7 years old, his father's age is 5 years more than 4 times 7 years.

4 times 7 years are 28 years, and 5 years are 33 years.

2. Emma is 9 years old, and her mother is 5 years less than 5 times as old. How old is her mother?

3. If a boy earns 10 dollars a month, and spends 8 dollars a month, how much can he save in 12 months?

4. A man earns $12 a week, and his son 3 dollars. How much do they both earn in a month?

5. Kate bought 9 oranges at 5 cents each. What change did she get from a half-dollar offered in payment?

6. If apples are worth 4 cents each, and oranges 6 cents each, what will 5 apples and 6 oranges cost?

7. How much must I pay for 8 tons of coal at 6 dollars a ton, and 4 cords of wood at 5 dollars a cord?

8. A farmer bought 4 plows at $10 each. If he gave in payment 8 five-dollar bills, how much change was due him?

9. If oranges are 6 cents each, and lemons are 4 cents each, how much less will 12 lemons cost than 12 oranges?

10. A grocer bought 12 barrels of apples at $4 a barrel, and sold them so as to gain $12. What did he get for all?

11. By selling some maps at $10 a set, an agent lost $2 on each set. What was the cost of his stock of 12 sets?

12. A farmer had 16 bushels of wheat, and sold 9 bushels at $2 a bushel, and the balance at 3 dollars a bushel. What did he get for his wheat?

13. Thomas has 10 cents, and Fanny has 3 times as many less 5 cents, and Charles has as many as both of them together. How many cents has Charles?

14. If two men start from the same place and go in the same direction, one at the rate of 8 miles an hour, and the other at 6 miles an hour, how far apart will they be in 5 hours?

15. If two men start from the same place and go in opposite directions, one at the rate of 8 miles an hour, and the other at 6 miles an hour, how far apart will they be in 5 hours?

Written Exercises.

Problem. — A drover bought 3 horses at $165 each, and gave in payment five 100-dollar bills. How much change did he receive?

SOLUTION.

$165	*$100*	*$500 cost of horses.*
3	*5*	*$495 value of bills.*
$495 cost of horses.	*$500 value of bills.*	*$5 amount of change.*

EXPLANATION. — If a drover bought 3 horses at $165 each, they cost 3 times $165, which by multiplication are found to be $495.

If one bill is worth $100, the value of 5 bills is 5 times $100, which are $500.

If the cost of the horses was $495, and the value of the bills was $500, the drover received as change the difference between $500 and $495, which is $5.

Note. — Numbers connected by the sign, $\times$, are to be multiplied first; then numbers or results connected by $+$ are to be added; and, lastly, numbers or results preceded by $-$ are to be subtracted.

1. What is the sum of 57 times 123 and 68 times 234?

2. What is the difference between 78 times 345 and 89 times 456?

3. What is the result of $642 \times 68 - 246 \times 86$?

4. Find the sum of $357 \times 57 + 579 \times 75$. The result of $6325 + 1875 - 125 \times 25$.

5. Multiply the sum of 305 and 970 by the difference between 1005 and 705.

6. What is the product obtained by multiplying the difference between \$2050 and \$905 by the sum of 725 and 280? The product of \$2050 + \$905 multiplied by 725 — 280?

7. The sum of three numbers is 900; the least of them being 100, and the greatest 500. What is the product of the three numbers multiplied together?

8. Harry earned in a year 25 times 25 dollars less 275 dollars. What amount did he earn?

9. If a lady-teacher receives \$40 a month, and pays \$32 a month for her expenses, how much can she save in a year, or 12 months? How much in 9 years?

10. An engineer earns \$60 a month, and his expenses are \$625 a year. How much can he save in 2 years? How much in 15 years?

11. A pork-packer put up 800 barrels of pork that cost him \$12 a barrel, and sold them all at a gain of \$1725. What was the selling price?

12. Three partners commenced business; A put in \$750, B put in twice as much, and C put in 5 times as much as both. How much did each and all of them put in?

13. A man paid \$2500 for the building of his house, and for his farm he paid 4 times as much, lacking \$750. What was the cost of his property?

14. A cotton-merchant bought 436 bales of cotton at \$125 a bale, and sold them at \$144 a bale. How much did he gain?

15. If a drover who has 175 cattle should buy 87 more, and then sell them all at \$45 each, how much would he receive for them?

16. A tea-dealer bought 175 chests of tea, each 72 pounds, and sold 39 chests to one man, and 52 chests to another. How many chests and how many pounds of tea had he left?

17. The President of the United States receives a salary of $50000 a year. If his expenses are $100 a day, how much does he save in 365 days, or a year?

18. A company built 425 miles of railroad at $15275 a mile. After having paid $5000000, how much did the company still owe?

19. If a man has $10000, and should pay 125 dollars an acre for a 75-acre farm, how much would he have left?

20. If a flour-dealer who buys 1500 barrels of flour at 9 dollars a barrel, should pay cash $9000 and give his note for the balance, what would be the value of the note?

21. A banker has an income of $12750 a year, and pays $1500 for house-rent, and 5 times as much for family expenses. How much does he save annually?

22. A drover bought 125 cattle at $35 a head, and it cost him $6 a head to get them to market. If he sold them at $45 a head, how much did he gain or lose?

23. Two persons start from the same place and walk in opposite directions, one going 35 miles a day, and the other 30 miles a day. How far apart are they at the end of the 15th day? At the end of the 25th day?

24. Two persons are 1200 miles apart, and walk towards each other, one going 35 miles a day, and the other 30 miles a day. How far apart will they be at the end of 15 days? At the end of 18 days?

25. A cleared in business one year $750, B 3 times as much, lacking $575, and C twice as much as A and B together. How much did B clear? How much did C clear? How much did all clear?

SECTION VI.

DIVISION.

1. How many cents are 6 cents less 6 cents? 6 cents less 3 cents less 3 cents? 6 cents less 2 cents less 2 cents less 2 cents?

2. How many times can 6 cents be taken from 6 cents? 3 cents from 6 cents, how often? 2 cents from 6 cents?

3. A can holds 8 pints of milk. If 4 pints are taken out, how many pints remain? If 4 pints more are taken out?

4. How many times 4 pints are 8 pints? 8 pints are how many times 4 pints? 8 pints contain 4 pints how often?

5. How often can the contents of a 4-quart can be put in a 12-quart can? 12 contains 4 how often?

6. 10 dollars are how many times 5 dollars? Prove it by your fingers. 15 is how many times 5?

7. 15 pears are how many times 5 pears? Prove it by pencil-strokes? 20 is how many times 5?

8. How many times are 6 pints contained in 18 pints? Prove it by counters? $6 in $30? 6 in 36?

9. How many times 5 bushels make 20 bushels? 20 bushels contain 5 bushels how often?

10. How many times can 6 pears be taken from a basket that contains 24 pears?

11. At 7 cents each, how many oranges can be bought for 35 cents? 35 cents are how many times 7 cents?

12. If I have 40 dollars, and take 10 dollars at a time to pay for a plow, how many times can I take 10 dollars? How many plows can I buy?

13. How many times 6 make 24? How many times 8? How many times is 6 contained in 24? 8 in 24?

14. How many 4's in 16? 5's in 20? 6's in 36? How many 7's in 28? In 35? In 42?

15. How many times 9 feet make 45 feet? How many times can 9 feet be taken from 45 feet? How many times do 45 feet contain 9 feet?

In each of the preceding questions it has been required to find how many times one of two numbers can be taken from the other, or to find how many times one of two numbers is contained in the other.

Definitions.

123. Division is the process of finding how many times one of two numbers is contained in the other.

124. The ***Quotient*** is the number obtained by dividing one of two numbers by the other.

125. The ***Dividend*** is that one of two numbers which is separated or divided.

126. The ***Divisor*** is that one of two numbers by which the other is divided.

127. The ***Sign of Division*** is a short straight line with a dot above and a dot below the middle of it, $\div$. It is read *divided by.*

When the sign of division is placed between two numbers, it shows that the number placed before it is to be divided by the number placed after it.

Thus, 24 quarts $\div$ 4 quarts $=$ 6, is read 24 quarts divided by 4 quarts equal 6. 24 quarts is the *dividend*, 4 quarts is the *divisor*, and 6 is the *quotient* or the *number of times* 24 quarts can contain 4 quarts, or that 4 quarts can be contained in 24 quarts.

Division is also indicated by placing the dividend above and the divisor below a short horizontal line.

Thus, $\frac{24}{8}$ is read 24 *divided by* 8, indicates that 24 is to be divided by 8.

128. Division Tables.

1 in		*2 in*		*3 in*		*4 in*		*5 in*	
1	*1* time.	*2*	*1* time.	*3*	*1* time.	*4*	*1* time.	*5*	*1* time.
2	*2* times	*4*	*2* times	*6*	*2* times	*8*	*2* times	*10*	*2* times
3	*3* "	*6*	*3* "	*9*	*3* "	*12*	*3* "	*15*	*3* "
4	*4* "	*8*	*4* "	*12*	*4* "	*16*	*4* "	*20*	*4* "
5	*5* "	*10*	*5* "	*15*	*5* "	*20*	*5* "	*25*	*5* "
6	*6* "	*12*	*6* "	*18*	*6* "	*24*	*6* "	*30*	*6* "
7	*7* "	*14*	*7* "	*21*	*7* "	*28*	*7* "	*35*	*7* "
8	*8* "	*16*	*8* "	*24*	*8* "	*32*	*8* "	*40*	*8* "
9	*9* "	*18*	*9* "	*27*	*9* "	*36*	*9* "	*45*	*9* "
10	*10* "	*20*	*10* "	*30*	*10* "	*40*	*10* "	*50*	*10* "

6 in		*7 in*		*8 in*		*9 in*		*10 in*	
6	*1* time.	*7*	*1* time.	*8*	*1* time.	*9*	*1* time.	*10*	*1* time.
12	*2* times	*14*	*2* times	*16*	*2* times	*18*	*2* times	*20*	*2* times
18	*3* "	*21*	*3* "	*24*	*3* "	*27*	*3* "	*30*	*3* "
24	*4* "	*28*	*4* "	*32*	*4* "	*36*	*4* "	*40*	*4* "
30	*5* "	*35*	*5* "	*40*	*5* "	*45*	*5* "	*50*	*5* "
36	*6* "	*42*	*6* "	*48*	*6* "	*54*	*6* "	*60*	*6* "
42	*7* "	*49*	*7* "	*56*	*7* "	*63*	*7* "	*70*	*7* "
48	*8* "	*56*	*8* "	*64*	*8* "	*72*	*8* "	*80*	*8* "
54	*9* "	*63*	*9* "	*72*	*9* "	*81*	*9* "	*90*	*9* "
60	*10* "	*70*	*10* "	*80*	*10* "	*90*	*10* "	*100*	*10*

Exercises.

1. Subtract by 2's from every second number from 6 to 2; from 12 to 2; 20 to 2. By 3's from every third number from 9 to 3. From 15 to 6. From 27 to 9. From 30 to 3. How often is each subtracted?

Divide every second number by 2 from 2 in 6 to 2 in 14. From 2 in 8 to 2 in 18. From 2 in 20 to 2 in 2. Every third number by 3 from 3 in 3 to 3 in 15. From 3 in 12 to 3 in 27. From 3 in 30 to 3 in 3.

2. Subtract by 4's from every fourth number from 16 to 4. From 32 to 8. From 40 to 4. By 5's from 20 to 5. From 35 to 10. From 50 to 5. How many times is each subtracted?

Divide by 4 from 4 in 4 to 4 in 16. From 4 in 12 to 4 in 28. From 4 in 40 to 4 in 4 and reverse. By 5 from 5 in 5 to 5 in 15. From 5 in 10 to 5 in 35. From 5 in 50 to 5 in 5 and reverse.

3. Subtract by 6's from every sixth number from 18 to 6. From 42 to 12. From 54 to 24. From 60 to 6. By 7's from 21 to 7. From 42 to 14. From 56 to 21. From 70 to 7.

Divide by 6 from 6 in 6 to 6 in 30. From 6 in 18 to 6 in 48. From 6 in 60 to 6 in 6 and reverse. By 7 from 7 in 7 to 7 in 28. From 7 in 21 to 7 in 56. From 7 in 70 to 7 in 7 and reverse.

4. Subtract by 8's from every eighth number from 32 to 8. From 56 to 16. From 72 to 24. From 80 to 8. By 9's from 36 to 9. From 63 to 18. From 91 to 36. From 90 to 9.

Divide by 8 from 8 in 8 to 8 in 32. From 8 in 24 to 8 in 56. From 8 in 80 to 8 in 8 and reverse. By 9 from 9 in 9 to 9 in 45. From 9 in 27 to 9 in 63. From 9 in 90 to 9 in 9 and reverse.

Mental Exercises.

1. How many times can 3 inches be subtracted from 6 inches? From 12 inches? From 15 inches?

2. How many times can 4 be taken from 12? 5 from 20? 6 from 30? 7 from 42? 8 from 56?

3. How many times 5 feet make 15 feet? How many times 4 gallons make 20 gallons?

4. What is the product of 6 apples multiplied by 4? Of 7 men multiplied by 5? Of 6 times 8?

5. 48 buttons are how many times 6 buttons?

129. Model. — 48 buttons are 8 times 6 buttons, because 6 buttons can be taken from 48 buttons 8 times. Or, 48 buttons are 8 times 6 buttons, because 8 times 6 buttons are 48 buttons.

6. 8 quarts are how many times 4 quarts? Why? 8 quarts contain 4 quarts how many times? Why?

7. How many times will the contents of a 12-quart can fill a 4-quart can? How many times do 12 quarts contain 4 quarts?

8. How many times can the contents of a 4-quart can be put in a 12-quart can? 4 quarts are contained in 12 quarts how many times?

9. There are 4 quarts in a gallon. How many gallons are in 20 quarts? How many times must 4 quarts be taken to equal 20 quarts?

10. How many times do 36 bushels contain 4 bushels? 6 bushels? 9 bushels?

11. The product of two factors is 42 days, and one of the factors is 7 days. What is the other factor?

130. Model. — If the product of two factors is 42 days, and one of the factors is 7 days, the other factor is the number of times 7 days must be taken to equal 42 days, which is 6 times. Therefore, the other factor is 6.

12. If the product of two factors is 48 cents, and one of the factors is 6 cents, what is the other? If one of them is 8 cents, what is the other?

13. The dividend is 54, and the divisor is 9, what is the quotient? If the dividend is 63, and the quotient is 7, what is the divisor?

14. How many sheets of drawing-paper can be bought for 50 cents, if one sheet costs 10 cents?

131. Model. — If one sheet of drawing-paper costs 10 cents, for 50 cents there can be bought as many sheets as the number of times 10 cents are contained in 50 cents, which is 5 times. Therefore, 5 sheets of drawing-paper can be bought for 50 cents.

15. How many 5-dollar bills must a butcher pay for a steer that costs him $50? For one that costs $40?

16. A furniture-dealer received $35 for rocking-chairs sold at $7 each. How many chairs did he sell?

17. Into how many fields of 8 acres each can a farm of 64 acres be divided? A farm of 48 acres?

18. How many shoes can a blacksmith set with 48 nails, using 8 nails for each shoe? With 64 nails?

19. John exchanged a half-dollar for 10-cent pieces. How many 10-cent pieces did he receive?

20. If 4 pounds of coffee cost a dollar, how much will 36 pounds cost? What will 40 pounds cost?

21. If a mechanic can earn $90 in 9 weeks, what are his weekly wages?

132. Model. — If a mechanic can earn $90 in 9 weeks, his weekly wages are as many dollars as the number of times 9 is contained in 90, which is 10 times. Therefore, the mechanic's weekly wages are $10.

22. A farm containing 54 acres was fenced off into 6 equal fields. How many acres were in each field?

23. If one man can do a piece of work in 63 days, in how many days can 7 men do it? 9 men?

24. If a coach-painter receives $72 for painting 8 wagons, how much apiece does he get? If he receives $80?

25. A stove-dealer bought 9 cook-stoves for $81. What was the price of each? If he had bought them for $72?

26. How many times can 10 cents be spent from a quarter-dollar? How many cents will remain?

27. How many times are 7 gallons contained in 30 gallons? How many gallons remain?

28. How often is 5 contained in 2 tens, or 20? In 2 tens 5 ones, or 20 and 5, or 25? In 2 hundreds 3 tens 5 ones, or 200 and 30 and 5, or 235?

29. If 6 pine-apples cost 72 cents, what is the price of one pine-apple?

133. Model. — If 6 pine-apples cost 72 cents, the price of one pine-apple is as many cents as the number of times 6 is contained in 72.

72 equals 60 and 12; 6 is contained in 60 10 times, and 6 in 12 2 times; 10 times and 2 times are 12 times. Therefore, the price of one pine-apple is 12 cents.

30. What is the value of one centre-table if $75 are paid for 5 centre-tables?

134. The ***Object of Division*** is twofold: —

First. To find how often one of two numbers is contained in the other.

Thus, if it is desired to know how many pears at 4 cents each can be bought for 8 cents, the object is to find how many times 4 cents are contained in 8 cents, and

Since 4 taken 2 times = 8, 4 is contained in 8 2 times. Hence, 2 pears can be bought for 8 cents.

Second. To separate one of two numbers into as many equal parts as there are ones in the other.

Thus, if it is desired to divide 8 pears equally between two boys, the object is to separate 8 pears into 2 equal parts; and

Since 2 times 4 = 8, or 4 + 4 = 8, one of the 2 equal parts of 8 is 4. Hence, each boy receives 4 pears.

135. The names of the equal parts of a thing or number vary according to the number of those parts.

Thus, one of two equal parts is named *one-half;* one of four equal parts, *one-fourth*, etc.

So, also, are obtained the names *fifths, sixths, fifteenths, twentieths*, etc.

136. The equal parts of a thing or number may be expressed by writing the number denoting the *name* of the parts as a *divisor below* a short horizontal line, and the number denoting the *number* of parts taken or used as a *dividend above* the line.

Thus, *one-half*, or 1 of 2 equal parts, is written $\frac{1}{2}$.

Three-fourths, or 3 of 4 equal parts, is written $\frac{3}{4}$.

So, also, *nine-tenths*, or 9 divided by 10, is written $\frac{9}{10}$; and 13 divided by 15 is written $\frac{13}{15}$, and named *thirteen-fifteenths* (127).

31. What is one of two equal parts of 4?

137. Model. — One of the equal parts of 4 is 2, because 2 is contained in 4 2 times.

32. Find one of the three equal parts of 9; of 15; of 21. One of the four equal parts of 20; of 28; of 36. One of the five equal parts of 30; of 40; of 50.

33. What is one of the 6 equal parts of 12? 24? 30? One of the 7 equal parts of 28? 35? 42? One of the 8 equal parts of 56? 64? One of the 9 equal parts of 63? 81?

34. How is one of 2 equal parts of a number found? One of 3 equal parts? One of 4? Of 5? Of 6? Of 7? Of 8? Of 9?

35. If any number or thing is separated into 2 equal parts, what is each part called?

138. Model. — If any number or thing is separated into 2 equal parts, each part is called *one-half* of that number or thing.

36. If any number or thing is separated into 3 equal parts, what is each part called? If divided into 4 equal parts? If into 5 equal parts?

37. What is one of the 6 equal parts of a number called? One of 7 equal parts? Of 8 equal parts? Of 9 equal parts?

38. How many dollars is one-half of 6 dollars?

139. Model. — One-half of 6 dollars is as many dollars as the number of times 2 is contained in 6, which is 3 times. Therefore, one-half of 6 dollars is 3 dollars.

39. What is one-third of 9 years? Of 12 months? One-fourth of 20 houses? Of 24 minutes? One-fifth of $35? Of $40? One-sixth of 54? Of 60?

40. How many is one-seventh of 28 hogsheads? Of 42 barrels? One-eighth of 40 quarts? Of 56 pints? One-ninth of 63? Of 81?

41. How is one-half of any number or thing found? One-third? One-fourth? One-fifth? One-sixth? One-seventh? One-eighth? One-ninth?

42. If 18 cents be shared equally among 3 girls, what part of 18 cents will each girl have? If among 6 girls? If among 9 girls?

43. If 20 blocks are separated into 10 equal groups, what part of 20 blocks is in each group? How many blocks are in each part?

44. If a party of 5 boys pay 40 cents for the use of a boat, how much does each boy pay?

140. Model. — If 5 boys pay 40 cents for the use of a boat, each boy pays one-fifth of 40 cents, which is 8 cents.

45. If a wire 24 feet long be cut into 3 equal parts, how long will each part be? If into 4 equal parts?

46. If a drayman takes 21 hogsheads of sugar in 7 loads, how many hogsheads does he take at each load?

47. A laboring-man paid $90 for 9 months' rent of his house. What monthly rent did he pay?

48. When you find how many times 9 dollars are contained in 90 dollars, is the result a concrete, or an abstract number?

49. When you find one of the nine equal parts of 90 dollars, is the result a concrete, or an abstract number?

50. When you divide 90 dollars among 9 men, to find how many dollars each man gets, do you find how often 9 men are contained in 90 dollars, or do you find one of the 9 equal parts of 90 dollars?

Definitions.

141. An ***Integer*** is a number whose units are whole or undivided.

An integer is also called a *Whole Number.*

Thus, 2 blocks, 4 dollars, 6 men, 8, 30, or 500 is an integer.

142. The ***Remainder*** is the part of the dividend left after division.

Thus, if 20 dollars are divided among 6 boys, each boy receives 3 dollars, and there is a remainder of *2 dollars.*

143. The equal parts into which a thing or number is divided are called ***Fractions.***

The remainder in division, and the divisor are generally

written in the form of a fraction, and placed at the right of the quotient.

Thus, 27 ÷ 7 equals 3, and a remainder of 6, which is written as a dividend above the line, and the divisor below the line; and the fraction is then written as a part of the quotient, giving $3\frac{6}{7}$.

From the preceding exercises and explanations are deduced the following

144. Principles of Division.

I. *Division is a short method of performing subtraction, and is the reverse of multiplication.*

II. *The dividend is the product of the divisor multiplied by the quotient.*

III. *The divisor and the dividend are similar numbers.*

IV. *The quotient is always an abstract number.*

Note. — In the solution of problems, to find one of the equal parts of a number, the divisor is regarded as an abstract number, and the quotient as a number similar to the dividend.

CASE I.

Written Exercises.

When the Divisor is one Figure.

145. *Example.* — Divide 975 by 3.

FIRST SOLUTION.

	Dividend.	
Divisor 3)*975*	*300*	*1st partial quotient.*
900		
75	*20*	*2d partial quotient.*
60		
15	*5*	*3d partial quotient.*
15		
—	*325*	*Entire quotient.*

EXPLANATION. — The dividend 975 is equal to 900 + 70 + 5; and 975 divided by 3 is equal to 900 divided by 3 + 70 divided by 3 + 5 divided by 3.

Write the divisor at the left of the dividend, and separate them by a curved line; and draw a line at the right of the dividend to separate it from the partial quotients.

Begin with the highest order of units, and divide the units of each order separately.

3 is contained in the partial dividend, 900, three hundred times (144, II); write 300 as the first partial quotient. 300 times 3 are 900, which write under 975; subtract 900 from 975, leaving a remainder, 75, or 70 + 5, as a new partial dividend.

3 is contained in 70 20 times; write 20 as the second partial quotient. 20 times 3 are 60, which write under 75; subtract 60 from 75, leaving 15 as a new partial dividend.

3 is contained in 15 5 times; write 5 as the third partial quotient. 5 times 3 are 15, which write under 15, and subtract, leaving nothing.

As 900 + 70 + 5, or 975, the entire dividend, has been divided, the sum of the partial quotients, 300, 20, and 5, must be the entire quotient, which is *325*.

SECOND SOLUTION.

Dividend.

Divisor 3)*975*(*325* ***Quotient.***

9

7

6

15

15

EXPLANATION. — Write the divisor at the left of the dividend, etc. (145).

Begin at the highest order, etc.

3 is contained in 9 hundreds 3 hundreds times; write 3 as the hundreds' figure of the quotient; 3 times 3 hundreds are 9 hundreds, which write under 9 hundreds; subtract 9 hundreds from 9 hundreds, leaving nothing. Use 7 tens as the next partial dividend.

3 is contained in 7 tens 2 tens times; write 2 as the tens' figure of the quotient. 3 times 2 tens are 6 tens, which write under 7 tens; subtract 6 tens from 7 tens, leaving 1 ten, or 10 ones, and to it add 5 ones, the next figure of the dividend, for a new partial dividend.

3 is contained in 15 ones 5 times; write 5 as the ones' figure of the quotient. 3 times 5 ones are 15 ones, which write under 15 ones; subtract 15 ones from 15 ones, leaving nothing.

The quotient is 3 hundreds 2 tens 5 ones, or *325*.

THIRD SOLUTION.

Divisor 3)*975* ***Dividend.***

325 ***Quotient.***

EXPLANATION. — Write the divisor at the left of the dividend, and draw a line under the dividend to separate it from the quotient.

Begin with the highest order of the dividend, etc.

3 is contained in 9 hundreds 3 hundreds times; write 3 as the hundreds' figure of the quotient. 3 times 3 hundreds are 9 hundreds, and 9 hundreds from 9 hundreds leave nothing. Use 7 tens as the next partial dividend.

3 is contained in 7 tens 2 tens times; write 2 as the tens' figure of the quotient. 3 times 2 tens are 6 tens, and 6 tens from 7 tens leave 1 ten, or 10 ones; add to it 5 ones, the next figure of the dividend, for a new partial dividend.

3 is contained in 15 ones 5 ones times; write 5 as the ones' figure of the quotient. 3 times 5 ones are 15 ones, and 15 ones from 15 ones leave nothing.

The quotient is 3 hundreds 2 tens 5 ones, or *325*.

Note 1. — For convenience, it may be best for beginners, before commencing to divide, to form a table of the products of the divisor multiplied by each of the first nine numbers.

By comparing each partial dividend with the several products placed in the table, the quotient figure may be easily determined.

$3 \times 1 = 3$
$3 \times 2 = 6$
$3 \times 3 = 9$
$3 \times 4 = 12$
$3 \times 5 = 15$
$3 \times 6 = 18$
$3 \times 7 = 21$
$3 \times 8 = 24$
$3 \times 9 = 27$

Note 2. — The same principles are applied in the solutions given, and the same steps are taken. The second solution is shortened by omitting the ciphers in the partial quotients, as the places in which the quotient figures are written determine their value without the use of the ciphers; and the third solution differs from the first, and from the first and the second, only in not having the results so fully written.

146. Short Division is the method of dividing generally used when the divisor is one figure only.

In the process of short division, the quotient only is written, and the result of every other step in the operation is carried in the mind.

Problems.

Read the dividend, the divisor, and the quotient in each of the following problems: —

(1)	(2)	(3)	(4)	(5)
4)5868	*5)6835*	*6)7968*	*5)8765*	*4)9732*

(6) 5)98765 (7) 6)87672 (8) 7)76545 (9) 6)67890 (10) 5)56785

(11) 6)918684 (12) 7)785673 (13) 7)867587 (14) 6)687948

(15) 7)8988 *years* (16) 8)98688 *months* (17) 8)99464 *days* (18) 7)8764 *hours*

Find the quotient of—

19. 4896 ÷ 4; 6785 ÷ 5; 8766 ÷ 6; 9961 ÷ 7.

20. 98765 ÷ 5; 87696 ÷ 6; 78981 ÷ 7; 91824 ÷ 8.

21. 78984 ÷ 6; 89873 ÷ 7; 98760 ÷ 8; 10908 ÷ 9.

22. 798854 ÷ 7; 234568 ÷ 8; 1901988 ÷ 9.

23. 9064 reams ÷ 8; 8298 quires ÷ 9; 9872 sheets ÷ 8.

24. 128979 bushels ÷ 9; 985768 quarts ÷ 8.

25. 106584 square yards ÷ 8; 679761 square feet ÷ 9.

26. Divide 255150 by 3, by 5, by 7, and by 9.

27. Divide 376320 by 4, by 6, by 7, and by 8.

28. What is the quotient of 290439 feet ÷ 9?

29. The dividend is 292530, and the divisor is 7. What is the quotient?

30. What is the quotient, if the dividend is 256896 gallons, and the divisor is 8?

147. *Example.* — Divide 15296 by 5.

SOLUTION.

5)15296

3059 + 1 ***Remainder.***

Or $3059\frac{1}{5}$ ***Quotient.***

EXPLANATION. — Write the divisor, etc.

Begin with the highest order, etc.

5 is contained in 15 3 times, with nothing remaining; write 3 as the thousands' figure of the quotient.

5 is contained in 2 0 times; write 0 as the hundreds' figure of the quotient, and with 2 unite 9, the next figure of the dividend.

5 is contained in 29 5 times, with 4 remaining; write 5, etc.

5 is contained in 46 9 times, with 1 remaining; write 9, etc.

The quotient is *3059*, and the remainder is *1;* or the remainder may be written over the divisor, and added as a part of the quotient, giving the true quotient $3059\frac{1}{5}$.

148. *Problem.* — Six men engaged in a business clearing $12500, which was equally divided among them. What amount did each receive?

SOLUTION.

$6)\$12500$

$\$2083\frac{2}{6}$

EXPLANATION. — If 6 men in business cleared $12500, which was equally divided among them, each received one-sixth of $12500 (134).

Write the divisor at the left of the dividend, etc.

Begin with the highest order of the dividend, etc.

One-sixth of 12 thousands is 2 thousands; write 2, etc.

One-sixth of 5 hundreds is 0 hundreds; write 0, etc.

One-sixth of the sum cleared is $\$2083\frac{2}{6}$, the amount that each received (144, Note).

Problems.

Find the quotient in each of the following problems, and read each term and each result: —

(31)	(32)	(33)	(34)	(35)
$5)3579$	$6)24680$	$7)57913$	$6)46802$	$5)7913$

(36)	(37)	(38)	(39)
$6)579135$	$7)682468$	$7)791357$	$6)824680$

(40)	(41)	(42)
$7)627384$ pounds.	$8)593715$ shillings.	$9)286439$ pence.

Find by both methods the quotient of —

43. 3579 ÷ 7; 24680 ÷ 8; 97531 ÷ 8; 4680 ÷ 7.

44. 193750 ÷ 8; 204860 ÷ 9; 395710 ÷ 8.

45. 2947 cases ÷ 6; 38560 chests ÷ 7; 8642 sacks ÷ 8.

46. 37250 bales ÷ 7; 92720 pair ÷ 8; 46358 tons ÷ 9.

47. Divide 7050301 by 5, by 7, by 9, by 6, and by 8.

48. Divide 3151719 feet by 6, by 8, by 7, and by 9.

49. 783256 is 6 times what number? 8 times what?

50. 7 times a certain number is 1020304. What is the number? 1020304 is 8 times what number?

What is

51. One-sixth of 237500?
52. One-eighth of 125725?
53. One-seventh of 37265?
54. One-ninth of 928650?
55. One-fourth of $12572?
56. One-sixth of 27852 rods?
57. One-fifth of 92837 feet?
58. One-ninth of 30502 tons?

59. If the dividend is 102750 feet, and the divisor is 8, what is the quotient? If the divisor is 9?

60. Find the divisor if the dividend is $237284, and the quotient is 7. If the quotient is 9? 8?

61. The product of two numbers is 2350275 tons, and the multiplier is 6. What is the multiplicand?

62. Find the quotient of seven million twenty thousand nineteen divided by seven; by six; and by nine.

63. Divide ten million fifty thousand four hundred by three; by five; by seven; and by nine.

Find one of the

64. 4 equal parts of 12505.
65. 6 equal parts of 23457.
66. 5 equal parts of 35790.
67. 7 equal parts of 46790.
68. 6 equal parts of 98765.
69. 8 equal parts of 56789.
70. 7 equal parts of 97531.
71. 9 equal parts of 13579.

72. The sum of the eight equal parts of a number is 237650 dollars. What is one of the parts?

73. An estate of $250000 was divided equally among a wife and 6 children. What did each receive?

74. If 8 miles of railroad cost \$135000, what was the average cost per mile? If they cost \$250000?

75. A contractor paid \$75250 for some bricks at \$9 a thousand. How many thousand did he buy?

76. How many barrels of flour can be made from 2570250 bushels of wheat, if 5 bushels make one barrel?

77. A grain-dealer shipped 125000 bushels of wheat in 4-bushel sacks. How many sacks did he fill?

78. Mr. James is one-ninth owner of a vessel valued at \$175000. What is his share worth?

79. When sugar is worth \$9 a hundred, how many hundred pounds can be bought for \$135135?

80. The national debt of the United States was at one time \$2300000000. How much would have been required each year to have paid it in 7 equal yearly payments?

CASE II.

When the Divisor is more than one Figure.

149. *Example* 1. — Divide 9765 by 30.

SOLUTION.

Dividend.

Divisor 30)9765(325$\frac{15}{30}$ ***Quotient.***

90

76

60

165

150

15 ***Rem.***

EXPLANATION.—Write the divisor, etc. (145).

Begin with the highest, etc. 30 is not contained in 9 thousands any thousands times.

With the 9 thousands, or 90 hundreds, unite 7 hundreds, the next figure of the dividend, making 97 hundreds for the first partial dividend.

30 is contained in 97 hundreds 3 hundreds times; write 3 as the hundreds' figure of the

quotient. 3 hundreds times 30 are 90 hundreds, which write under 97 hundreds of the dividend, and subtract, leaving a remainder of 7 hundreds. With 7 hundreds, or 70 tens, unite 6 tens, the next figure of the dividend, making 76 tens for the next partial dividend.

30 is contained in 76 tens 2 tens times; write 2, etc.

30 is contained in 165 ones 5 ones times, or 5 times; write 5, etc.

The quotient is *325;* the remainder is *15*, which is written with the divisor under it as a part of the quotient, making the entire quotient $325\frac{15}{30}$.

150. *Example* 2. — Divide 97654 by 39.

SOLUTION.

39)97654(3
117
---

39)97654(1
39

58

39)97654(2503 $\frac{37}{39}$
78

196
195

154
117

37

EXPLANATION. — Write the divisor, etc.

Begin with the highest order, etc.

39 is not contained in 9 ten-thousands any ten-thousands times. With the 9 ten-thousands, or 90 thousands, unite, etc. (149).

Suppose 39 is contained in 97 thousands 3 thousands times; write 3 as the thousands' figure of the quotient; 3 thousands times 39 are 117 thousands, which write under 97 thousands of the dividend. 117 thousands are more than 97 thousands. Therefore, 39 is not contained in 97 so many as 3 times.

Suppose 39 is contained in 97 thousands 1 thousand times. Write 1, etc. 1 thousand times 39 are 39 thousands, which write under 97 thousands, and subtract. The remainder, 58, is greater than the divisor, 39. Therefore, 39 is contained in 97 thousands more than 1 thousand times.

39 is contained in 97 2 times; write 2 as the thousands' figure, etc. (145).

39 is contained in 196 5 times; write 5, etc.

39 is contained in 15 0 times; write 0 as the tens' figure of the quotient, and with 15 unite 4, the next figure of the dividend, etc.

39 is contained in 154 3 times; write, etc.

The quotient is *2503;* the remainder is *37*, or the entire quotient, $2503\frac{37}{39}$.

Note. — For convenience, and to prevent the trial of two or more quotient figures, it may be best for beginners, before commencing to divide, to form a table of the products of the divisor multiplied by each of the first nine numbers.

By comparing each partial dividend with the several products placed in the table, the quotient figure may be easily determined.

$39 \times 1 = 39$
$39 \times 2 = 78$
$39 \times 3 = 117$
$39 \times 4 = 156$
$39 \times 5 = 195$
$39 \times 6 = 234$
$39 \times 7 = 273$
$39 \times 8 = 312$
$39 \times 9 = 351$

From the preceding examples it appears that —

I. *If a product of the divisor by the quotient figure is greater than the partial dividend, the quotient figure is too great.*

II. *If a remainder is greater than the divisor, the quotient figure is too small.*

III. *If a partial dividend is less than the divisor,* 0 *is placed as the quotient figure, and the next figure of the dividend is used to form a new partial dividend.*

151. Long Division is the method of dividing generally used when the divisor is more than one figure.

In the process of long division, the result of every step in the operation is written.

From the preceding examples and explanations is deduced the following

152. Rule for Division.

I. *Write the divisor at the left of the dividend.*

II. *Find how many times the divisor is contained in the least number of the left-hand orders of the dividend that will contain it; and place the result as the first figure of the quotient, at the right hand of the dividend in long division, or, in short division, under the right-hand order of the partial dividend used.*

III. *Multiply the divisor by this quotient figure; subtract the product from the partial dividend used; with the remainder unite the next lower order of the dividend, for a new partial dividend.*

IV. *Proceed in the same manner until all the orders of the dividend have been used.*

V. *Write the last remainder, if any, with the divisor under it, as a part of the quotient.*

From the preceding, it appears that there are five principal steps in division: —

1st. *Write the numbers.*	3d. *Multiply.*
2d. *Find how many times.*	4th. *Subtract.*

5th. *Bring down the next figure.*

Problems.

(81) *12)23456* (82) *14)34567* (83) *16)45678* (84) *18)56789*

(85) *20)987654* (86) *32)876543* (87) *44)765432* (88) *56)654321*

(89) *203)9753* (90) *304)24680* (91) *405)75310* (92) *506)3579*

(93) *450)192837 feet* (94) *560)$864200* (95) *670)283746 tons*

Find the quotient of —

96. 80706 ÷ 24; 50607 ÷ 26; 60504 ÷ 28; 40506 ÷ 30.
97. 97005 ÷ 33; 75003 ÷ 35; 53001 ÷ 37; 31009 ÷ 39.
98. 200345 ÷ 405; 300456 ÷ 506; 400567 ÷ 607.
99. 908060 ÷ 708; 807060 ÷ 809; 706050 ÷ 908.
100. 4200 cords ÷ 40; 6400 tons ÷ 44; $86000 ÷ 48.
101. 50000 yards ÷ 53; 70000 acres ÷ 57; $9000 ÷ 63.
102. 200304 pounds ÷ 234; 500790 barrels ÷ 567.
103. 305006 miles ÷ 456; 406335 cattle ÷ 789.

How many times is

104. 68 contained in 6985? In 60985? In 160985?

105. 608 contained in 123456? In 1020304?

106. 75 contained in 2345? In 20345? In 220345?

107. 705 contained in 204060? In 3050709?

108. Find the quotient of 40500 divided by 89; by 809.

109. What remains after dividing $304005 by 876?

110. 4005006 square feet ÷ 890 gives what remainder?

111. Divide 608040 by 56; by 67; by 78; by 89.

112. Divide 6008004 by 506; by 607; by 708.

113. Find the quotient of thirty-two thousand ninety-eight divided by nine hundred eighty-seven.

114. How many times does four million four hundred thousand contain one thousand fourteen?

115. What is the quotient of thirty million thirty thousand thirty divided by two thousand thirty-four.

116. How many remain after dividing sixty million sixty thousand six hundred by three thousand fifty-six.

CASE III.

When there are Ciphers at the right of the Divisor.

153. *Example* 1. — Divide 560 by 10.

FIRST SOLUTION.

10)560(56 ***Quotient.***
50
60
60

EXPLANATION. — Write the dividend, etc.

10 is contained in 50 5 times, etc.

10 is contained in 60 6 times, etc.

The quotient is *56*, — the same figures as are in the dividend, with the right-hand figure omitted.

SECOND SOLUTION.

10)56,0

56 ***Quotient.***

In the second solution 560 is divided by 10, by cutting off the cipher in the divisor, and the right-hand figure of the dividend, and using the figures remaining in each.

The quotient is the same, for the reason that cutting off the right-hand figure removes each of the remaining figures one place further to the right, and, therefore, the value of each is decreased 10 times, or is divided by 10 (49, 2).

154. *Example* 2. — Divide 5678 by 100.

SOLUTION.

$1\overset{\wedge}{0}0)56\overset{\wedge}{7}8$

$56 + 78$ ***Remainder.***

Or $56\frac{78}{100}$ ***Quotient.***

EXPLANATION. — Cut off the ciphers in the divisor, and as many figures from the right of the dividend as there are ciphers in the divisor. The quotient of the remaining figures of the dividend by the remaining figures of the divisor is 56, and the remainder is 78, — the two right-hand figures of the dividend. Hence,

I. *A number is divided by* 10 *by removing one figure from the right.*

II. *A number is divided by* 100 *by removing two figures from the right. Etc.*

III. *The figures removed form the remainder.*

155. *Example* 3. — Divide 5678 by 340.

SOLUTION.

$34\overset{\wedge}{0})567\overset{\wedge}{8}(16\frac{238}{340}$ ***Quotient.***

34

227

204

238 ***Remainder.***

EXPLANATION. — Write the divisor, etc.

The divisor 340 = 34 times 10.

Divide first by 10 by cutting off the right-hand figure of the dividend and the cipher of the divisor, and the result is a partial quotient of 567 and a remainder of 8 *ones*.

Divide next the remaining figures of the dividend by the remaining figures of the divisor.

34 is contained in 56 1 time, etc.

The quotient is 16; the second remainder is 23 *tens*, which united with 8 *ones* cut off from the dividend, give the true remainder 238.

Hence, the quotient required is $16\frac{238}{340}$.

The preceding examples and explanations are based upon the following

156. Principle.

The value of any figure is decreased tenfold for every place it is removed from the left toward the right.

157. Rule.

When the divisor is 10, 100, 1000, etc.,

From the right hand of the dividend cut off as many figures as there are ciphers in the divisor.

The figures remaining are the quotient, and the figures cut off are the remainder.

When the divisor has ciphers at the right,

I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor.*

III. *To the last remainder annex the figures cut off from the dividend, for the true remainder.*

Problems.

(117) *10)124680* (118) *20)135791* (119) *40)192837* (120) *60)283746*

(121) *30)82736* (122) *500)39485* (123) *70)71625* (124) *900)48576*

(125) *350)90800 pints* (126) *5700)$200304* (127) *790)80605 feet*

Find the quotient of—

128. 20405 ÷ 200; 90087 ÷ 400; 30560 ÷ 600.

129. 80076 ÷ 230; 45006 ÷ 340; 76050 ÷ 560.

130. 357000 pounds ÷ 2000; 1357090 tons ÷ 9000.

131. 246809 divided by 30, by 450, by 506, by 6700.

132. What is one-tenth of 375? One-hundredth of 7965?

133. What number is equal to 79500 ÷ 4000?

134. A contractor paid $7680 for lumber at $30 per thousand feet. How many thousand feet did he buy?

135. A carriage-maker received $15000 for light carriages at $200 each. How many carriages did he sell?

136. There are 480 sheets in a ream. How many reams will 259200 sheets make? 242880 sheets?

137. The stock of a turnpike company is $250000, in shares of 500 dollars each. How many shares are there?

138. 640 acres make a square mile. How many square miles in Pennsylvania, which contains about 29440000 acres?

139. How many regiments averaging 750 men will make an army of 45000 men? An army of 64125 men?

140. At $1750 per car, how many cars can a railroad company buy with $52500? With $78750?

141. How many city lots at $2500 each can be bought for $227500? How many for $267500?

142. A builder received $395000 for some brick houses at an average price of $5000. How many houses did he sell?

143. A cord of wood contains 128 solid feet. How many cords are in a pile containing 12750 solid feet?

144. The earth moves round the sun at the rate of about 11491200 miles in 168 hours. What is the rate per hour?

145. The average daily receipts of a ferry-boat are $275. In how many days will the receipts amount to $50000?

146. If 245 bushels of wheat weigh 14945 pounds, what is the average weight per bushel?

147. The distance from New York to Canton by way of the Cape of Good Hope is 19400 miles. How long would it take a steamer to make the voyage, if it moved at the rate of 185 miles a day?

148. A railroad 355 miles long cost $2000000. What was the average cost per mile?

149. Mount Everest in Asia is about 29000 feet high. How many miles high is it, each mile being 5280 feet?

150. Divide 30507090 by 4056; by 5067; by 6078.

151. Divide 80060042 by 1234; by 2345; by 3456.

152. Divide 400070009 by 5607; by 6708; by 7809.

153. Divide 980007654 by 4567; by 5768; by 6879.

154. Divide 20030040 by 60708; by 70809; by 90705.

155. Divide 76000543 by 23456; by 34567; by 46857.

156. Divide 560780900 by 69584; by 72839; by 89076.

157. Divide 975312468 by 67898; by 98765; by 78506.

158. The product of two numbers is 99876540, and one of the numbers is 98765. What is the other?

159. The dividend is 920700309, and the divisor is 24689. What is the quotient?

160. The dividend is 8705003750, and the quotient is 96075. What is the divisor?

Review.

1. What is ***Division?*** What is the sign of division? How is it read? What does it show? In what other way may division be indicated?

2. Define ***Quotient.*** What does the quotient show? What is the ***Dividend?*** What is the number by which the dividend is divided called? Explain the difference between long division and short division.

3. What are the two ***Objects of Division?*** Illustrate each. How do the names of the equal parts of a number vary? How may the number of equal parts of a number be expressed? What are the equal parts of a number called? Define ***Integer.*** What is the ***Remainder?*** How are the remainder and the divisor generally written?

4. To find the number of pencils at 5 cents each that can be bought for 25 cents, which number is regarded as the dividend, and which the divisor? To find the value of one pencil, if 5 pencils cost 25 cents? Is the dividend always concrete, or always abstract, or may it be either? Are the dividend and the divisor always similar, or may they be dissimilar numbers? What is the quotient always? Repeat the four principles of division.

5. How is the divisor written in performing an operation in division? Where do you begin to divide? What is a partial dividend? When is 0 placed in the quotient? When is the quotient figure too large? When is it too small? How do you divide by 10, 100, etc.? What is the principle? How do you divide when there are ciphers on the right of the divisor? State the five steps in the operation of division. Repeat the rules for division.

EXERCISES IN ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

Mental Exercises.

1. To 4 add 8, subtract 3, multiply by 5, divide by 9, subtract 5. What remains?

2. From 14 take 5, add 2, multiply by 6, divide by 3, subtract 7, multiply by 6. What is the result?

3. Multiply 6 by 5, add 5, divide by 7, subtract 3, multiply by 9, add 7, divide by 5. Result?

4. Divide 49 by 7, multiply by 6, subtract 7, add 15, divide by 10, add 8. Result?

5. The quotient is 10, the remainder is 8, and the divisor is 12. What is the dividend?

6. A newsboy bought 10 papers for 35 cents, and sold them so as to gain 15 cents. How much apiece did he get?

7. A grocer bought 7 barrels of flour at $6 a barrel. For how much a barrel must he sell it to gain $14 on the lot?

8. If you earn 9 dollars a week, and pay 5 dollars for board, and 2 dollars for other expenses, how much will you lay up in 8 weeks? How much in 12 weeks?

9. A trader bought 12 pairs of shoes at $3 and 6 hats at $4, and paid for them in 5-dollar bills. How many did it take? How many 10-dollar bills would it have taken?

10. If 5 oranges are worth 25 cents, and 6 lemons are worth 42 cents, how much more is a lemon worth than an orange? How much are an orange and a lemon worth?

11. A furniture-dealer bought 8 tables for $40, and sold them at $6 each. How much did he clear?

12. How many pounds of rice at 10 cents a pound will cost as much as 6 pounds of sugar at 15 cents?

13. If 4 men can do a piece of work in 6 days, in what time can 3 men do the same work?

158. Model. — If 4 men can do a piece of work in 6 days, it will take one man 4 times 6 days, or 24 days, to do it.

Since it takes one man 24 days to do the work, 3 men can do it in one-third of 24 days, which is 8 days. Therefore, etc.

14. If 5 masons lay a wall in 12 days, how many men would be required to lay it in 3 days? In 4 days?

15. If a man earns $5 while his son earns $2, how many dollars will the man earn while his son is earning $20?

16. What is the cost of 18 quarts of milk, at the rate of 40 cents for 5 quarts? The cost of 25 quarts?

17. A boy paid 30 cents for some oranges at the rate of 10 cents for 5. How many oranges did he buy?

18. A miller bought 10 bushels of wheat for $20, and I bought 12 bushels at the same rate. How much did I pay?

19. If 5 barrels of flour will pay for 10 bushels of wheat at $3 a bushel, what is the flour worth per barrel?

20. How many 4-quart cans of milk can be filled from five 8-quart cans? From six 12-quart cans?

21. In how many days can 7 men earn as much as 5 men can earn in 14 days? As much as 4 men in 21 days?

22. If four 10-dollar bills are paid for 8 yards of cloth, what is the price per yard?

23. If a farmer can dig 6 bushels of potatoes in an hour, and his boy 4 bushels, how long will it take them to dig 20 barrels of 3 bushels each?

24. A butcher bought 2 calves at $9 each, and one calf for $12. What was the average cost?

25. A grocer bought 5 barrels of flour of different brands at $7, $8, $9, $10, and $11 a barrel. What was the average price?

Written Exercises.

159. A ***Parenthesis,*** ( ), is used to enclose two or more numbers, that are to be considered together and treated as a single number.

A ***Vinculum,*** or ***Bar,*** is used for the same purpose.

Thus, $(6 \times 5 + 10) \div 8$, or $\overline{6 \times 5 + 10} \div 8$, denotes that the value of $6 \times 5 + 10$, or $30 + 10$, or 40, is to be divided by 8.

Note. — Operations of multiplication and division must be performed before those of addition and subtraction, unless parentheses indicate otherwise (see Notes pp. 62 and 86).

Problems.

160. *Example.* — Find the value of $(2 + 3 \times 4) \div 7 - 2$.

SOLUTION.

$(2 + 3 \times 4) \div 7 - 2 =$
$(2 + 12) \div 7 - 2 =$
$14 \div 7 - 2 = 2 - 2 = 0$

EXPLANATION. — Since the operation of multiplication is to be performed first, find the product of 3×4, which is 12.

Since the numbers within the parenthesis are to be treated as a single number, find the sum of $2 + 12$, which is 14.

Since the operation of division is to be performed before that of subtraction, find the quotient of $14 \div 7$, which is 2.

Lastly, find the value of $2 - 2$, which is 0, the value required.

Find the value of—

1. $5 + 4 \times (9 - 6)$.
2. $(5 + 4) \times 8 - 5$.
3. $4 + 3 \times (8 - 5)$.
4. $4 + 3 \times 8 - 5$.
5. $7 + (19 - 3) \times (4 - 3)$.
6. $7 + \overline{19 - 3} \div 4 - 3$.
7. $7 + 19 - 3 \times 4 - 3$.
8. $\overline{7 + 19 - 3} \times 4 - 3$.
9. Find the sum of 65×76 and $6764 \div 89$.
10. From $7524 \div 76$ take $8765 - 8678$.
11. Multiply $7865 + 5678$ by $139725 \div 405$.
12. Divide 809×765 by $35 + 765 - 40$.
13. Find the value of $(75 + \overline{25 \times 25} - 50) \times 205$.
14. What is the result of $174225 \div 345 + 20 \times 75$?
15. Multiply the sum of 305 and 975 by the quotient of 105225 divided by 305.
16. Divide the product of 98 times 765 by the sum of 1865 and 2035 less 3543.
17. Divide 13230 by 98, subtract the quotient from 76 times 67, and to the remainder add one-sixth of 258.
18. From 7531 take 1357, multiply the remainder by 25, and divide the product by 725.
19. If 50625 be divided by 75, by what number must the quotient be multiplied to produce 46375?
20. The dividend is 7973, the quotient is 64, and the remainder is 27. Find the divisor.
21. The divisor is 16, the quotient is 108, and the remainder is 15. What is the dividend?
22. A man bought 75 acres of land at \$90 an acre, paid \$325 for improvements, and then sold it for \$8675. How much did he gain per acre?

23. A and B started on a journey, A traveling 29 miles a day, and B 35 miles. In how many days will they be 84 miles apart? How soon will they be 96 miles apart?

24. A and B started together and traveled in opposite directions, A traveling 29 miles a day, and B 35 miles. How far apart were they in 14 days?

25. A merchant bought 12 pieces of cloth, each 35 yards, for $1060, and sold it at $3 a yard. What did he gain?

26. If a mechanic earns $775 a year, and pays $240 for rent, and $390 for family expenses, how many years will it take him to save $2900?

27. A merchant who bought 12 pieces of cloth, each 35 yards, for $1260, sold it so as to gain $315. What was the selling price per yard?

28. When 36 acres of land cost $2160, how many acres can be bought for $5000? For $15000?

29. If rations for 500 soldiers will last 38 days, how long will they last 4 companies of 95 soldiers each?

30. How many casks, each holding 21 gallons, can be filled from 25 hogsheads of wine, containing 84 gallons each?

31. I bought a farm of 125 acres for $7500, and sold it at $75 an acre. How much did I gain?

32. A dealer bought 125 barrels of flour for $875, and sold 90 barrels of it at $9 a barrel, and the remainder at $7 a barrel. How much did he gain or lose?

33. If a mechanic earns $550 a year, and his expenses are $425, how many years will it take him to buy a farm of 50 acres at $75 an acre?

34. A coal-dealer bought 175 tons of coal by the long ton of 2240 pounds, and sold it by the short ton of 2000 pounds. How many short tons did he gain?

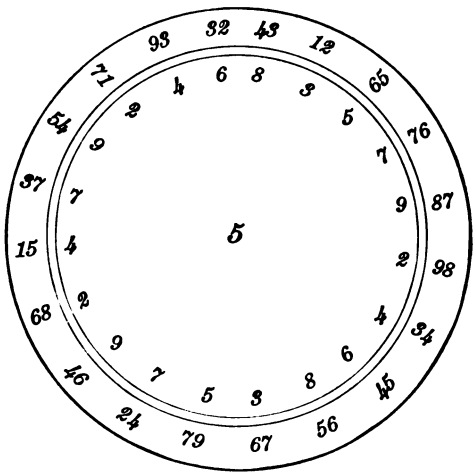
35. A dairy-man in packing 765 pounds of butter, found that it filled 18 tubs, with 9 pounds left over. How much did he put in each tub?

36. A clerk was engaged at $1824 a year. At the end of 10 months his health failed, and he left. How much should he have received?

37. If three steers weigh respectively 875 pounds, 944 pounds, and 1025 pounds, what is their average weight?

38. If a merchant's sales amount to $25225 in a week of 6 days, what are his average daily sales?

39. A farmer bought 25 acres at $80 an acre, and 30 acres at $75 an acre? What was the average price per acre?



161. Nothing in business transactions is of so much importance as the ability to calculate rapidly and accurately.

The little design introduced above, by using first the numbers indicated, and then others substituted for them, will furnish an infinite variety of combinations for *oral* exercises in all the fundamental rules; and, by using larger numbers, for *written* exercises also.

For *addition*, use as parts to be added, the central number and each number inside and each outside of the circle; then each number inside and the number directly outside; also, all the numbers inside; and two or more of the numbers outside.

Thus, find the sum of 5 and 7; 5 and 37; 7 and 37; 7 and 5 and 37, etc.; 37 and 54, etc.

For *subtraction*, use as terms the central number and each number inside and each outside the circle; then each number inside and the number directly outside.

Thus, find the difference between 5 and 7; 5 and 37, etc.; 7 and 37; 37 and 54, etc.

For *multiplication*, use as terms the central number and each number inside and each outside the circle; then each number inside and the number directly outside; also, any two or more inside, etc.

Thus, find the product of 5 times 7; 5 times 37; 7 times 37; 7 times 3 times 6, etc.

For *division*, use as terms the central number and each number outside; also, each number inside and directly outside.

Thus, find the quotient of 5 into 37; 7 into 37, etc.

Also, use in pairs the numbers inside and those directly outside to find their sum, difference, product, and quotient.

Thus, find the sum of 7 and 37; their difference; their product; their quotient.

The teacher should adapt the size of the numbers to the grade of his pupils; should require quick answers; and should never weary of *drill, drill, drill.*

General Review of Fundamental Rules.

162. Addition, Subtraction, Multiplication, and Division are called the four *Fundamental Rules of Arithmetic.*

1. By what process is the sum of two or more numbers of the same kind found? By what process is the difference between two numbers of the same kind found? How, then, do addition and subtraction differ from each other?

2. If the minuend and the subtrahend are given, how is the difference found? If the minuend and the remainder are given, how is the subtrahend found? If the subtrahend and the remainder are given, how is the minuend found?

3. If a number and one of two parts are given, how is the other found? If the sum of two numbers and one of the numbers are given, how is the other number found? If the greater of two numbers and the difference between them are given, how is the less number found? How is the greater of two numbers found, if the less number and the difference between them are given? If the sum of several numbers and all but one of the numbers are given, how is that one found?

4. By what process do you find the product of two or more numbers? By what process do you find the quotient of one number divided by another? How, then, do multiplication and division differ from each other?

5. If two or more factors are given, how do you find their product? How do you find the product of the multiplicand and the multiplier? If the product and the multiplicand are given, how is the multiplier found? How is the multiplicand found, if the product and the multiplier are given? If the product of two factors and one of them are given, how is the other factor found?

6. How is the quotient found, when the divisor and the dividend are given? If the divisor and the quotient are given, how is the dividend found? Tell how to find the divisor when the dividend and the quotient are given. When the divisor, the quotient, and the remainder are given, how do you find the dividend?

SECTION VII.

DENOMINATE NUMBERS.

163. Avoirdupois Weights are used in weighing all coarse and heavy articles; such as *groceries*, *iron*, *coal*, etc.

The units or denominations of avoirdupois weights are *ounce*, *pound*, *hundred-weight*, and *ton*.

Table.

16 ounces (oz.)	are	*1 pound,*	*lb.*
100 pounds	"	*1 hundred-weight,*	*cwt.*
20 hundred-weight	"	*1 ton,*	*T.*

In 1 lb. how many ounces? In 2 lbs.? etc.

Dry Measures.

164. Dry Measures are used in measuring dry substances; such as *grain, fruit, vegetables, lime, coal,* etc.

The units or denominations of dry measure are *pint, quart, peck,* and *bushel.*

Table.

2 pints (pt.)	are	*1 quart, qt.*
8 quarts	"	*1 peck, pk.*
4 pecks	"	*1 bushel, bu.*

How many pecks in 2 bushels? In 3 bushels?
How many quarts in 3 pecks? In 4 pecks?
How many pints in 3 quarts? In 4 quarts?
How many bushels in 8 pecks? In 12 pecks?

Liquid Measures.

165. Liquid Measures are used in measuring liquids; such as *water, milk, oil, liquors,* etc.

The units or denominations of liquid measure are *gill, pint, quart,* and *gallon.*

Table.

4 gills (gi.)	are	*1 pint, pt.*
2 pints	"	*1 quart, qt.*
4 quarts	"	*1 gallon, gal.*

How many quarts in 2 gallons? In 3 gallons?
How many pints in 4 quarts? In 5 quarts?
How many gills in 2 pints? In 4 pints?
How many gallons in 16 quarts? In 12 quarts?

Linear Measures.

166. Linear Measures are used in measuring lines and distances.

The units or denominations of linear measures are *inch, foot, yard, rod,* and *mile.*

Table.

12 inches (in.)	are	*1 foot, ft.*	1 *inch.*
3 feet	"	*1 yard, yd.*	
5½ yards, or *16½ feet*	"	*1 rod, rd.*	
320 rods	"	*1 mile, mi.*	

In 2 yards how many feet? In 3 yards?
In 3 feet how many inches? In 2 feet?
In 1 rod how many feet? In 2 rods?
How many yards in 12 feet? In 9 feet?

Measures of Time.

167. Measures of Time are used in measuring time.

The units or denominations of time are *second*, *minute*, *hour*, *day*, *week*, *month*, and *year*.

Table.

60 seconds (sec.)	are	*1 minute,*	*min.*
60 minutes	"	*1 hour,*	*hr.*
24 hours	"	*1 day,*	*da.*
365 days	"	*1 common year,*	*yr.*
366 days	"	*1 leap year,*	*yr.*
7 days	"	*1 week,*	*wk.*
52 weeks	"	*1 year,*	*yr.*
12 months	"	*1 year,*	
100 years	"	*1 century,*	c.

The names of the months and the number of days in each month are as follows:

		Days.			Days.
January,	*Jan.*,	31.	July,	*July*,	31.
February,	*Feb.*,	28 or 29.	August,	*Aug.*,	31.
March,	*Mar.*,	31.	September,	*Sept.*,	30.
April,	*Apr.*,	30.	October,	*Oct.*,	31.
May,	*May*,	31.	November,	*Nov.*,	30.
June,	*June*,	30.	December,	*Dec.*,	31.

How many days in 4 weeks? In 3 weeks?
How many weeks in 14 days? In 21 days?
How many hours in 2 days? In 3 days?
What months have 31 days? 30 days? 28 or 29 days?

United States Money.

168. United States Money is the legal money or currency of the United States.

The units or denominations of United States money are *mill*, *cent*, *dime*, *dollar*, and *eagle*.

Table.

10 mills (m.)	are	*1 cent*,	*ct.* or *c.*
10 cents	"	*1 dime*,	*d.*
10 dimes, or 100 cents	"	*1 dollar*,	$.
10 dollars	"	*1 eagle*,	*e.*

How many mills in 2 cents? In 3 cents?
How many cents in 3 dimes? In 4 dimes?
How many dimes in 2 dollars? In 3 dollars?
How many cents in 2 dollars? In 1 dollar?

The ***Coins*** of the United States are of gold, silver, nickel, and bronze.

The ***Gold Coins*** are the double-eagle, eagle, half-eagle, and the quarter-eagle.

The ***Silver Coins*** are the dollar, half-dollar, quarter-dollar, and dime.

The ***Nickel Coin*** is the five-cent piece.

The ***Bronze Coin*** is the cent.

169. The ***Dollar*** is the principal unit of United States money.

Notations and Numeration of United States Money.

170. Dollars are written as integers with the dollar sign ($) prefixed: thus 5 dollars are written $5, 25 dollars $25.

Dimes, cents, and mills are written to the right of the dollars, with a period between.

171. Directions.

Write the dollar mark first, then the number of dollars, then a period, then the dimes, cents, and mills.

In business the dimes and cents are expressed together as cents.

Read the following:

$.01 $.02 $.03 $.04 $.05 $.06 $.07 $.08 $.09
$.001 $.002 $.003 $.004 $.005 $.006 $.008
$.011 $.012 $.023 $.036 $.047 $.068 $.049 $1.25
$.625 $.105.

Exercises.

1. Express in figures, or write, *seven dollars twelve cents.*

172. Model.—Write seven dollars as an integer with the dollar sign prefixed, $7; then a period; then twelve cents as one dime and two cents, or twelve cents, giving $7.12.

Express in figures the following sums of United States money:

Two dollars fourteen cents; eight dollars twenty-five cents; twelve cents; seventy-four dollars seventy-five cents; ninety-eight dollars ten cents; twenty-five dollars seventeen cents; forty-eight dollars thirty-one cents.

2. Express in words, or numerate and read, *$12.375.*

173. Model.—Read first the integer as dollars, giving $12; then the first two figures to the right of the period as cents, giving 37 cents; then the third figure to the right of the period as mills, giving 12 dollars 37 cents 5 mills, the expression required.

Express in words the following sums of United States money:

1. *$12; $15; $18.*	4. *$60.07; $70.05.*	7. *$.007; $.005.*
2. *$21.10; $34.25.*	5. *$80.04; $92.08.*	8. *$2.005; $6.007.*
3. *$47.36; $58.70.*	6. *$27.125; $70.217.*	9. *$3.065; $9.014.*

Computations in United States Money.

174. Principle.

All operations upon United States currency are performed in the same manner as similar operations upon integers.

Written Exercises.

175. *Example* 1.—Find the sum of \$10.50, \$.37½, \$25.48, \$100.

SOLUTION.

\$10.50
.375
25.48
100.
\$136.355

EXPLANATION.—Since only like orders of integral units can be added, write the parts so that units of the same order and denomination shall stand under each other.

Begin at the right, and add as in integers, and place the period before cents in the sum, giving \$136.355, the sum required.

176. *Example* 2.—From \$20.08 subtract \$10.027.

SOLUTION.

\$20.08
10.027
\$10.053

EXPLANATION.—Since only like orders of integral units can be taken one from the other, write the parts so that units of the same denomination shall stand under each other.

Begin at the lowest order, and subtract as in integers.

177. *Example* 3.—Find the product of \$100.45 by 87.

SOLUTION.

\$100.45
87
70315
80360
\$8739.15

EXPLANATION.—Begin with the lowest order of the multiplier, multiply as in integers and separate two places from the right of the product for cents.

178. *Example* 4.—How much is \$302.375 ÷ 41?

SOLUTION.

41)302.375(7.375
287
153
123
307
287
205
205

EXPLANATION.—Begin with the highest order of the dividend, and divide as in integers, pointing off in the quotient three figures from the right.

179. Rule for Computations in United States Money.

Write the numbers, and add, subtract, multiply, and divide as in integers and separate from the right of the result a number of figures equal to the number of figures used to express cents or mills in the numbers added, subtracted, etc.

Problems.

Add together the following sums of money: —

(1)	(2)	(3)	(4)	(5)
$7.35	$12.075	$100.05	$75.50	$407.381
8.50	.625	1.005	.75	70.65
7.75	8.50	.07	4.007	5.006
8.62	.375	95.736	100.	.76
4.25	40.	140.068	.568	.078

6. $75.07 + $9.075 + $100 + $76.075 + 98.75.

7. $90.07 + $400.31 + $75.06 + $80 + $73.18.

8. What is the amount of $76.25, $800, $78.25, $98.625?

9. Add together $50, $400.31, $8, $40.50, and $78.37.

10. A merchant has in one bank $350, in another $306.50, and in a third $50.75; and has on hand $275.37. How much money has he in all?

Find the remainder of —

(11)	(12)	(13)	(14)	(15)
$375.50	$8.70	$.5625	$100.	$75.25
9.75	.378	.375	7.625	9.187

16. $8.50 — $1.75; $76.25 — $1.625; $90 — $.312.

17. $1 — .875; $50.50 — $.505; $100 — $76.25.

18. What is the difference between $400 and $376.375?

19. From $200 take $100.25; from $100 take $1.001.

20. An agent sold a farm for $1000, which was $250.62 more than he paid for it. What was its cost?

Find the product in each of the following:—

(21)	(22)	(23)	(24)	(25)	(26)
$150	*$8.37*	*$25.705*	*$100.005*	*$9.08*	*$700.50*
7	*8*	*95*	*105*	*32*	*325*

27. $.625 × 15; $10.005 × 25; $75.37 × 15.

28. $100 × 75; $250.25 × 375; $1200.25 × 625.

29. Find the product of $375.625 multiplied by 31.

30. How many dollars are 75 times $40.375?

31. How much would 33 yards of carpeting cost at $1.375 per yard? At $1.56? At $1.66? At $2.12?

Find the quotient of—

(32)	(33)	(34)	(35)
12)$.625	*25)$100.50*	*75)$20.50*	*25)$730*

36. $7.50 ÷ 5; $70.50 ÷ 55; $700.50 ÷ 25.

37. $9.875 ÷ 8; $17.565 ÷ 15; $500.375 ÷ 56

BILLS AND RECEIPTS.

A ***Bill of Goods*** is a written statement of goods sold.

The ***Footing*** of a bill is the entire cost of the items or articles.

The character @ signifies *at*. Thus, 6 yards of muslin @ $.12 means 6 yards of muslin at 12 cents per yard.

PHILADELPHIA, *July 1, 1904.*

1. *Mr. William Hunter,*

Bought of HUDSON BROS.

12 lb. Hyson Tea,	*@*	*$1.12*	*13*	*44*
5 " Java Coffee,	*@*	*.37*	*1*	*85*
10 " Crushed Sugar,	*@*	*.14*	*1*	*40*
			16	*69*

Received Payment,

Hudson Bros

2. Make out and receipt the following bills in the same form as the above: New York, July 30, 1904, Henry Fenton bought of Stuart & Co., 15 yards sheeting @ $.18; 10 yards muslin @ $.16; 8 yards cambric @ $.25.

3. Boston, March 30, 1904. John Williams bought of Edgar Peirson, 9 yards of calico @ $.12; 15 yards cambric @ $.25; 16 yards sheeting @ $.18; 5 dozen towels @ $3.75.

4. April 13, 1904, New York. J. Williams & Co. sold to James Garsed, 12 yards cassimere @ $1.12; 20 yards doeskin @ $.91; 18 yards flannel @ $.62; 10 yards alpaca @ $.87.

5. William Watson bought of Henry Stout, Richmond, May 2, 1904, 12 pounds Oolong tea @ $1.10; 8 pounds Young Hyson tea @ $.95; 15 pounds Rio Coffee @ $.37; 20 pounds sugar @ $.12.

6. Charles Foulke bought of T. P. Smith & Bro., Philadelphia, Nov. 15, 1904, 8 pair calf boots @ $5.62; 10 pair brogans @ $1.12; 12 pairs ladies' kid boots @ $1.66; 15 pair boys' gaiters @ $2.25.

Receipts.

PHILADELPHIA, *Jan. 15, 1904.*

Received from John Wilson, Fifty Dollars on account.

$50. *Samuel T. Hood.*

PHILADELPHIA, *Jan. 15, 1904.*

Received from James Kirby, One Hundred Twelve Dollars and Fifty Cents in full of all demands.

$112.\frac{50}{100}$ *Wm. Bonner.*

PHILADELPHIA, *Jan. 15, 1904.*

Received of James Stewart the sum of Fifteen Dollars in full for one month's rent of house, 1505 East Ontario St.

$15. *George Brown.*

REDUCTION OF DENOMINATE NUMBERS.

CASE I.

Reduction Descending.

Reduce 5 gallons, 3 quarts, 1 pint, to pints.

OPERATION.

gal.	*qts.*	*pt.*
5	*3*	*1*
4		
20 qts.		
3		
23 qts.		
2		
46 pts.		
1		
47 pts.		

EXPLANATION.—In one gallon there are 4 quarts, and in 5 gallons there are 5 times 4 quarts, or 20 quarts; 20 quarts and 3 quarts are 23 quarts. In 1 quart there are 2 pints, and in 23 quarts there are 23 times 2 pints, or 46 pints; 46 pints and 1 pint are 47 pints.

Reduce—

1. 12 yd. 2 ft. 8 in. to inches.
2. 2 T. 9 cwt. 16 lb. to pounds.
3. 7 cwt. 24 lb. 7 oz. to ounces.
4. 9 gal. 3 qt. 1 pt. to pints.
5. 8 bu. 3 pk. 7 qt. to quarts.
6. 12 pk. 6 qt. 1 pt. to pints.
7. 10 yr. 7 mo. 3 wk. to weeks.
8. 6 mo. 2 wk. 6 da. to days.
9. 3 wk. 5 da. 18 hr. to hours.
10. 5 hr. 24 min. 18 sec. to seconds.
11. 5 sq. yd. 7 sq. ft. 14 sq. in. to square inches.
12. 9 sq. yd. 6 sq. ft. to square inches.
13. 8 qt. 1 pt. 2 gi. to gills.
14. 9 da. 12 hr. 42 min. to minutes.
15. 14 yd. 1 ft. to inches.
16. 9 hr. 12 min. to seconds.
17. 2 wk. 3 da. to hours.
18. 7 mo. 1 wk. to days.
19. 9 yr. 5 mo. to weeks.
20. 5 T. 7 cwt. to pounds.

21. 7 gal. 2 qt. to pints.
22. 9 cwt. 14 lb. to ounces.
23. 12 bu. 2 pk. to quarts.
24. 3 qt. 1 pt. to gills.
25. 12 sq. yd. 5 sq. ft. to square inches.
26. 2 cu. yd. 18 cu. ft. 142 cu. in. to cubic inches.
27. 6 cu. yd. 21 cu. ft. to cubic inches.
28. 12 cu. yd. to cubic inches.
29. 8 cu. yd. 450 cu. in. to cubic inches.
30. 16 yd. 7 in. to inches.
31. 2 cen. 7 mo. to months.

CASE II.

Reduction Ascending.

Change 352 inches to yards.

EXPLANATION.—There are 12 inches in 1 foot, and in 352 inches there are as many feet as 12 is contained times into 352 inches, or 29 feet with 4 inches remaining. There are 3 feet in 1 yard, and in 29 feet there are as many yards as 3 is contained times into 29 feet, or 9 yards with 2 feet remaining. Therefore, 352 inches equal 9 yards, 2 feet, 4 inches.

OPERATION.

12)*352 inches.*
3)*29 feet* + 4 *inches.*
9 yards + 2 *feet.*

Ans.: *9 yds. 2 ft. 4 in.*

Reduce —

1. 756 in. to yards.
2. 136 pt. to pecks.
3. 123 gi. to quarts.
4. 321 qt. to bushels.
5. 451 pt. to pecks.
6. 127 pints to gallons.
7. 645 oz. to cwt.
8. 7542 lb. to tons.
9. 3642 sec. to hours.
10. 9761 min. to days.
11. 472 hr. to weeks.
12. 764 da. to months.
13. 873 da. to years.
14. 10,000 sq. in. to sq. yds.
15. 3764 sq. in to sq. yards.
16. 15,552 cu. in. to cu. feet.
17. 62,721 cu. in. to cu. yds.
18. 425 pt. to bushels.
19. 677 gi. to gallons.
20. 7427 lb. to tons.
21. 64,000 oz. to tons.
22. 7864 min. to days.
23. 2600 sq. in. to sq. yards.
24. 476 pt. to bushels.

SECTION VIII.

PROPERTIES OF NUMBERS.

The ***Factors*** of a number are those numbers which, multiplied together, produce that number.

Thus, *2* and *3* are factors of *6;* for, being multiplied together, they produce *6;* so, also, *2*, *3*, and *5* are the factors of *30.*

A ***Prime Number*** is a number that has no other factor than 1 and itself.

Thus, *5* is a *prime* number; so, also, are *2*, *3*, *7*, *11*, *13*, *17*, etc.

A ***Composite Number*** is a number that has other factors than 1 and itself.

Thus, *15* is a *composite* number, since $15 = 3 \times 5$; and *20*, since $20 = 2 \times 2 \times 5$, etc.

An ***Even Number*** is a number that can be divided by 2 without a remainder.

An ***Odd Number*** is a number that cannot be divided by 2 without a remainder.

A number can be exactly divided by 2 when it ends with 0, 2, 4, 6, or 8. A number can be exactly divided by 5 when it ends with 5 or 0.

Factoring.

Factoring is the process of separating a composite number into its prime factors.

Since 1 is a factor of every number, it is not regarded either in naming or in finding the prime factors of numbers.

A ***Prime Factor*** is a factor that is a prime number.

Thus, *5* is a prime factor of *15;* *7* of *42.*

A ***Composite Factor*** is a factor that is a composite number.

Thus, *6* is a composite factor of *12*; *9* of *18*.

What are the prime factors of 36?

OPERATION.	EXPLANATION.
3)*36* *3*)*12* *2*)*4* *2*	Dividing 36 by 3, the quotient is 12; dividing 12 by 3, the quotient is 4; dividing 4 by 2, the quotient is 2. Therefore 3, 3, 2 and 2 are the prime factors of 36, since they are the prime numbers which, multiplied together, produce 36.

Written Exercises.

Find the prime factors of—

1. 10, 12, 14, 16, 20, 21.
2. 15, 18, 27, 30, 32, 35.
3. 22, 24, 26, 28, 40, 49.
4. 42, 50, 56, 66, 75, 81.
5. 39, 44, 46, 48, 51, 52.
6. 68, 69, 70, 74, 76, 78.
7. 80, 88, 92, 98, 104, 110.
8. 86, 95, 99, 102, 114, 125.
9. 85, 96, 105, 112, 116, 124.
10. 126, 132, 140, 144, 152, 180.

11. Find which numbers between 50 and 100 are prime and which are composite.

Greatest Common Divisor.

A ***Divisor*** or ***Measure*** is a number that divides any given number without a remainder.

Thus, *6* is a *divisor*, or *measure*, of *18*, since *6* divides *18* without a remainder.

A ***Common Divisor*** is a number that divides each of two or more numbers without a remainder.

Thus, *3* is a *common divisor* of *18* and *24*, since *3* divides each of them without a remainder.

The ***Greatest Common Divisor*** is the greatest number that divides each of two or more given numbers without a remainder.

Thus, *6* is the *greatest common divisor* of *18* and *24*, since it is the greatest number that divides each of them without a remainder.

What is the greatest common divisor of 36, 63, and 81?

Operation.	Explanation.
$36 = 2 \times 2 \times 3 \times 3$	By separating 36, 63, and 81 into their prime factors, 3 and 3 are found to be the only prime factors common to these numbers.
$63 = 3 \times 3 \times 7$	
$81 = 3 \times 3 \times 3 \times 3$	
G. C. D. $3 \times 3 = 9$.	

Since the product of all the common prime factors of two or more numbers is their greatest common divisor, 9 or 3×3 is the greatest common divisor of 36, 63, and 81.

Written Exercises.

Find the greatest common divisor of—

1.	28 and 42.	11.	24, 36, 60.	21.	32, 64, 96.
2.	36 and 48.	12.	21, 45, 49.	22.	36, 72, 108.
3.	32 and 64.	13.	12, 21, 36.	23.	75, 90, 120.
4.	56 and 72.	14.	28, 36, 48.	24.	42, 84, 126.
5.	66 and 80.	15.	10, 30, 50.	25.	56, 70, 114.
6.	75 and 90.	16.	18, 36, 54.	26.	34, 68, 136.
7.	35 and 49.	17.	24, 42, 72.	27.	80, 112, 144.
8.	42 and 56.	18.	28, 49, 84.	28.	90, 126, 144.
9.	56 and 80.	19.	81 and 117.	29.	105, 126, 147.
10.	63 and 91.	20.	105 and 135.	30.	114, 133, 152.

31. Find the greatest number of feet that will exactly divide 48 feet, 72 feet, and 84 feet.

32. What is the greatest length of rails that can, without cutting, be used in fencing the road front of three lots measuring 42 feet, 70 feet, and 91 feet?

33. In a house are three rooms: the first is 16 feet wide, the second 20 feet, and the third 24 feet. What is greatest width of matting that will exactly fit each?

34. A farmer put 66 bushels of corn, 90 of wheat, and 120 of oats into the largest possible sacks of equal size. What did each sack contain?

Least Common Multiple.

A ***Multiple*** or ***Dividend*** is a number that can be divided by a given number without a remainder.

Thus, *12* is a *multiple* of *4*, since it can be divided by 4 without a remainder. 24 is a multiple of 6.

A ***Common Multiple*** is a number that can be divided by each of two or more numbers without a remainder.

Thus, *12* is a *common multiple* of *3* and *4*, since it can be divided by 3 and by 4 without a remainder. So, also, is 24.

The ***Least Common Multiple*** is the least number that can be divided by each of two or more numbers without a remainder.

Thus, *12* is the *least common multiple* of *3* and *4*, since it is the least number that can be divided by 3 and by 4 without a remainder.

Note.—The terms *Multiple*, or *Product*, and *Dividend* differ only in use; *multiple* suggesting the process of multiplication, and *dividend* the process of division.

Find the least common multiple of 10 and 12.

OPERATION.

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

EXPLANATION.

Separate the numbers into their prime factors. The least common multiple cannot be less than 12, the greater of the two numbers, since it must contain 12, and, therefore, all the prime factors of 12, which are 2, 2 and 3. The least common multiple of 10 and 12 must contain also the prime factors of 10. Of the factors of 10, 2 is common also to 12. It may be omitted as a factor of the least common multiple, and only 5, the

remaining factor, is retained. Hence 2, 2, 3 and 5 are all the prime factors of 10 and 12, and their product 60 is the least common multiple.

Written Exercises.

Find the least common multiple of—

1. 2, 4, and 6.
2. 3, 6, and 9.
3. 4, 6, and 8.
4. 6, 8, and 9.
5. 8, 10, and 15.
6. 9, 12, and 18.
7. 7, 21, and 28.
8. 8, 32, and 48.
9. 9, 27, and 63.
10. 15, 35, and 60.
11. 17, 100, and 68.
12. 24, 32, and 40.
13. 18, 24, and 36.
14. 24, 42, and 54.
15. 8, 24, 36, and 72.
16. 13, 78, 52, and 104.
17. 84, 7, 21, 63, and 6.
18. 10, 50, 25, and 100.
19. 28, 36, and 60.
20. 36, 45, and 81.
21. 21, 42, and 56.
22. 28, 35, and 63.
23. 16, 24, and 56.
24. 18, 27, and 63.
25. 36, 48, and 84.
26. 63, 90, and 108.
27. 12, 16, 32, and 54.
28. 18, 27, 49, and 63.
29. 14, 36, 63, and 117.
30. 20, 48, 72, and 144.

31. What is the least number that can be divided by each of the nine digits without a remainder?

32. What is the shortest piece of wire that can be divided into lengths of 5 feet, 125 feet, 15 feet, or 75 feet?

33. What is the least number of marbles that can be equally divided among 18, 30, or 36 boys?

34. What is the least number of cents with which you could buy an exact number of lemons at 6 cents each, oranges at 8 cents, bananas at 10 cents, or melons at 16 cents?

SECTION IX.

FRACTIONS.

A ***Fraction*** is one or more of the equal parts of a unit.

Thus, *1 third, 3 fifths, 5 sixths* are fractions.

Fractions are divided into two classes: *Common Fractions* and *Decimal Fractions.*

132

A ***Common Fraction*** is expressed in figures by two numbers, written one above the other, with a line between them.

The numbers used to express a fraction are called the *Denominator* and the *Numerator.*

The ***Denominator*** of a fraction is the number written *below* the line.

The denominator shows into *how many equal parts* the unit is divided, and *denominates,* or *gives name to,* the parts.

Thus, *10* is the denominator of the fraction $\frac{9}{10}$; it shows that the unit is divided into ten equal parts, which are named *tenths.*

The ***Numerator*** of a fraction is the number written *above* the line.

The numerator shows the *number* of the equal parts of the unit expressed by the fraction.

Thus, *9* is the numerator of the fraction $\frac{9}{10}$; it shows that 9 of 10 equal parts of a unit are expressed by the fraction.

The ***Terms*** of a fraction are its numerator and denominator.

Thus, *9* and *10* are the terms of the fraction $\frac{9}{10}$.

A ***Fractional Unit*** is one of the equal parts into which the unit is divided.

Thus, $\frac{1}{3}$ is the fractional unit of $\frac{2}{3}$; $\frac{1}{5}$ of $\frac{3}{5}$; $\$\frac{1}{4}$ of $\$\frac{3}{4}$.

According to *value*, fractions are divided into two kinds: *Proper Fractions* and *Improper Fractions*.

A ***Proper Fraction*** is a fraction whose value is less than 1.

The numerator of a proper fraction is always less than its denominator.

Thus, $\frac{2}{3}$, $\frac{4}{15}$, and $\frac{20}{25}$ are *proper fractions*.

An ***Improper Fraction*** is a fraction whose value is equal to, or greater than, 1.

The numerator of an improper fraction is always equal to, or greater than, its denominator.

Thus, $\frac{3}{3}$, $\frac{15}{4}$, and $\frac{25}{20}$ are *improper fractions*.

A ***Mixed Number*** is a number expressed by an integer and a fraction.

Thus, $2\frac{1}{2}$ is a *mixed number*, read *2 and* $\frac{1}{2}$; $17\frac{1}{6}$, read *17 and* $\frac{1}{6}$.

An ***integer*** may be expressed in the ***form of a fraction*** by writing the integer as the numerator, and 1 as the denominator.

Thus, 4 may be expressed by $\frac{4}{1}$, read 4 *ones*.

Exercises.

1. Analyze the fraction $\frac{7}{8}$.

Model 1.—$\frac{7}{8}$ is read *seven eighths;* its terms are 7 and 8; the *denominator* is 8, which shows that the unit is divided into 8 equal parts; the *numerator* is 7, which shows that 7 of these equal parts are expressed; it is a *proper fraction*, since its numerator is less than its denominator, and its value is less than 1.

2. Analyze the mixed number $7\frac{8}{9}$.

Model 2.—$7\frac{8}{9}$ is read *seven and eight ninths;* it is a *mixed number*, since it is a number expressed by a whole number and a fraction; the whole number is *7;* the fraction is $\frac{8}{9}$; the terms of the fraction are 8 and 9; etc. (Model 1).

Analyze the following fractions:

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$.
2. $\frac{6}{7}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{9}{6}$.
3. $\frac{7}{9}$; $\frac{9}{10}$; $\frac{8}{11}$; $\frac{10}{8}$.
4. $\frac{5}{3}$; $\frac{5}{9}$; $\frac{7}{4}$; $\frac{7}{12}$.
5. $\frac{4}{11}$; $\frac{9}{5}$; $\frac{5}{13}$; $\frac{15}{11}$.
6. $\frac{10}{7}$; $\frac{13}{15}$; $\frac{23}{16}$; $\frac{7}{17}$.
7. $\frac{53}{21}$; $\frac{32}{53}$; $\frac{97}{65}$; $\frac{9}{77}$.
8. $7\frac{2}{9}$; $9\frac{3}{11}$; $13\frac{7}{13}$.
9. $25\frac{13}{27}$; $34\frac{21}{35}$; $45\frac{32}{43}$.

SECTION X.

REDUCTION OF FRACTIONS.

THE ***Reduction of Fractions*** is the process of changing their *form* without changing their *value*.

CASE I.

To change a whole or a mixed number to an improper fraction.

Mental Exercises.

How many fourths in $5\frac{3}{4}$?

OPERATION.

$5\frac{3}{4}$

$\frac{4}{20}$

$\frac{3}{23}$

$\frac{23}{4}$

EXPLANATION.

In 1 there are 4 fourths, and in 5 there are 5 times 4 fourths, or 20 fourths; 20 fourths and 3 fourths equal 23 fourths. Therefore, $5\frac{3}{4} = \frac{23}{4}$.

1. How many thirds in $6\frac{1}{3}$? $5\frac{2}{3}$? $4\frac{1}{3}$? $2\frac{1}{3}$? $7\frac{2}{3}$? $8\frac{1}{3}$?
2. How many fourths in $5\frac{1}{4}$? $6\frac{2}{4}$? $8\frac{1}{4}$? $7\frac{3}{4}$? $4\frac{3}{4}$? $7\frac{1}{4}$?
3. How many fifths in $5\frac{1}{5}$? $4\frac{2}{5}$? $6\frac{3}{5}$? $7\frac{4}{5}$? $3\frac{1}{5}$? $8\frac{2}{5}$?
4. How many sixths in $3\frac{1}{6}$? $7\frac{3}{6}$? $4\frac{2}{6}$? $6\frac{5}{6}$? $2\frac{5}{6}$? $5\frac{3}{6}$?
5. How many sevenths in $3\frac{1}{7}$? $5\frac{2}{7}$? $8\frac{3}{7}$? $6\frac{4}{7}$? $4\frac{6}{7}$? $7\frac{5}{7}$?
6. How many eighths in $2\frac{1}{8}$? $4\frac{3}{8}$? $6\frac{5}{8}$? $5\frac{7}{8}$? $3\frac{6}{8}$? $7\frac{4}{8}$?
7. How many ninths in $3\frac{1}{9}$? $5\frac{2}{9}$? $4\frac{5}{9}$? $7\frac{8}{9}$? $2\frac{6}{9}$? $8\frac{7}{9}$?
8. How many tenths in $4\frac{1}{10}$? $5\frac{3}{10}$? $6\frac{7}{10}$? $8\frac{9}{10}$? $3\frac{2}{10}$?

9. How many elevenths in $3\frac{1}{11}$? $5\frac{8}{11}$? $2\frac{5}{11}$? $4\frac{2}{11}$?

10. How many twelfths in $2\frac{1}{12}$? $3\frac{5}{12}$? $5\frac{7}{12}$? $4\frac{11}{12}$?

Written Exercises.

Reduce to improper fractions —

11.	$12\frac{3}{4}$,	$14\frac{4}{7}$,	$15\frac{7}{8}$,	$16\frac{7}{9}$.
12.	$20\frac{2}{5}$,	$24\frac{5}{6}$,	$28\frac{7}{12}$,	$32\frac{6}{7}$.
13.	$38\frac{7}{10}$,	$42\frac{5}{8}$,	$45\frac{11}{12}$,	$48\frac{5}{18}$.
14.	$52\frac{9}{14}$,	$40\frac{7}{15}$,	$35\frac{8}{17}$,	$28\frac{5}{16}$.
15.	$25\frac{8}{9}$,	$14\frac{11}{18}$,	$33\frac{19}{20}$,	$47\frac{17}{24}$.
16.	$60\frac{3}{16}$,	$54\frac{5}{12}$,	$46\frac{8}{19}$,	$49\frac{6}{11}$.
17.	$20\frac{3}{20}$,	$18\frac{9}{12}$,	$17\frac{5}{19}$,	$23\frac{8}{15}$.
18.	$29\frac{7}{16}$,	$37\frac{9}{20}$,	$26\frac{7}{18}$,	$45\frac{5}{14}$.
19.	$27\frac{5}{9}$,	$39\frac{9}{12}$,	$49\frac{8}{11}$,	$36\frac{5}{18}$.
20.	$19\frac{4}{17}$,	$54\frac{7}{18}$,	$41\frac{14}{15}$,	$32\frac{11}{14}$.

CASE II.

To change an improper fraction to a whole or a mixed number.

Mental Exercises.

How many ones in $\frac{16}{3}$?

OPERATION.

$$\frac{16}{3)16}$$
$$5\frac{1}{3}$$

EXPLANATION.

Since in 1 there are 3 thirds, in 16 thirds there are as many ones as 3 thirds are contained into 16 thirds, or $5\frac{1}{3}$. Therefore, $\frac{16}{3} = 5\frac{1}{3}$.

How many ones in —

1.	$\frac{13}{4}$?	$\frac{15}{4}$?	$\frac{9}{2}$?	$\frac{8}{3}$?		6.	$\frac{11}{4}$?	$\frac{13}{6}$?	$\frac{15}{8}$?	$\frac{17}{10}$?
2.	$\frac{9}{5}$?	$\frac{14}{6}$?	$\frac{15}{7}$?	$\frac{17}{8}$?		7.	$\frac{22}{3}$?	$\frac{19}{5}$?	$\frac{18}{7}$?	$\frac{24}{9}$?
3.	$\frac{25}{10}$?	$\frac{24}{11}$?	$\frac{21}{9}$?	$\frac{25}{12}$?		8.	$\frac{25}{7}$?	$\frac{20}{11}$?	$\frac{19}{9}$?	$\frac{35}{12}$?
4.	$\frac{16}{5}$?	$\frac{25}{4}$?	$\frac{17}{6}$?	$\frac{27}{8}$?		9.	$\frac{19}{4}$?	$\frac{21}{6}$?	$\frac{23}{8}$?	$\frac{27}{7}$?
5.	$\frac{22}{7}$?	$\frac{17}{5}$?	$\frac{35}{9}$?	$\frac{32}{12}$?		10.	$\frac{37}{8}$?	$\frac{45}{4}$?	$\frac{37}{7}$?	$\frac{49}{12}$?

Written Exercises.

Change or reduce to whole or mixed numbers—

1. $\frac{21}{9}$; $\frac{82}{11}$; $\frac{43}{15}$; $\frac{54}{27}$; $\frac{98}{25}$; $\frac{100}{17}$; $\frac{125}{10}$; $\frac{137}{30}$; $\frac{185}{18}$; $\frac{190}{18}$.
2. $\frac{17}{5}$; $\frac{75}{32}$; $\frac{65}{16}$; $\frac{171}{17}$; $\frac{352}{21}$; $\frac{424}{25}$; $\frac{576}{38}$; $\frac{698}{45}$; $\frac{708}{50}$; $\frac{875}{75}$.
3. $\frac{29}{18}$; $\frac{800}{15}$; $\frac{401}{20}$; $\frac{503}{25}$; $\frac{717}{35}$; $\frac{829}{41}$; $\frac{65}{32}$; $\frac{101}{30}$; $\frac{201}{95}$; $\frac{89}{30}$.
4. $\frac{205}{27}$; $\frac{801}{89}$; $\frac{420}{41}$; $\frac{500}{73}$; $\frac{607}{18}$; $\frac{708}{94}$; $\frac{809}{82}$; $\frac{909}{31}$; $\frac{420}{52}$.

CASE III.

To change a fraction to higher terms or to a given denominator.

Mental Exercises.

Change $\frac{3}{4}$ to eighths.

ANALYSIS.—Since in *1* there are *8* eighths, in *1* fourth there are *1* fourth of *8* eighths or *2* eighths, and in *3* fourths there are *3* times *2* eighths or *6* eighths. Therefore, $\frac{3}{4} = \frac{6}{8}$.

1. Change $\frac{1}{2}$ to 4ths; to 6ths; to 8ths; to 10ths.
2. Change $\frac{2}{3}$ to 6ths; to 9ths; to 12ths; to 15ths.
3. Reduce $\frac{3}{4}$ to 8ths; to 12ths; to 16ths; to 20ths.
4. Change $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ to 16ths; to 24ths; to 40ths.
5. Reduce $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, and $\frac{5}{5}$ to 10ths; to 15ths; to 20ths.

Written Exercises.

Reduce or change—

6. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 12ths; to 24ths; to 36ths.
7. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$ to 20ths; to 40ths; to 60ths.
8. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$ to 24ths; to 48ths; to 72ds; to 96ths.
9. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{17}{18}$ to 36ths; to 72ds.
10. $\frac{5}{6}$, $\frac{6}{7}$, $\frac{13}{21}$ to 42ds; to 84ths.
11. $\frac{4}{5}$, $\frac{7}{9}$, $\frac{8}{15}$ to 45ths; to 90ths.
12. $\frac{5}{8}$, $\frac{15}{16}$, $\frac{23}{24}$ to 48ths; to 96ths.
13. $\frac{3}{4}$, $\frac{5}{7}$, $\frac{11}{14}$ to 28ths; to 56ths; to 84ths.
14. $\frac{3}{5}$ and $\frac{6}{7}$ to fractions having 35 as the denominator.
15. $\frac{11}{15}$ and $\frac{8}{25}$ to fractions whose denominator is 75.
16. $\frac{7}{8}$ to 16ths; 32ds; 48ths; 64ths; 80ths; 96ths.

CASE IV.

To change a fraction to lower terms.

Mental Exercises.

How many fourths in $\frac{6}{8}$?

ANALYSIS.—Since *1* equals *8* eighths, *1* fourth equals *1* fourth of *8* eighths or *2* eighths, and *6* eighths equal as many fourths as the number of times *2* eighths are contained into *6* eighths, which is *3* times. Therefore, $\frac{6}{8} = \frac{3}{4}$.

Change or reduce—

1.	$\frac{6}{9}$ to 3ds.	6.	$\frac{10}{12}$ to 6ths.	11.	$\frac{10}{15}$ to 3ds.
2.	$\frac{8}{12}$ to 3ds.	7.	$\frac{12}{14}$ to 7ths.	12.	$\frac{18}{24}$ to 4ths.
3.	$\frac{9}{12}$ to 4ths.	8.	$\frac{10}{16}$ to 8ths.	13.	$\frac{15}{20}$ to 5ths.
4.	$\frac{12}{16}$ to 4ths.	9.	$\frac{14}{18}$ to 9ths.	14.	$\frac{21}{24}$ to 8ths.
5.	$\frac{8}{10}$ to 5ths.	10.	$\frac{12}{18}$ to 6ths.	15.	$\frac{24}{36}$ to 9ths.

Written Exercises.

Change or reduce to their lowest terms—

16.	$\frac{18}{24}$; $\frac{18}{27}$; $\frac{16}{28}$; $\frac{14}{35}$; $\frac{24}{32}$.	19.	$\frac{36}{68}$; $\frac{46}{69}$; $\frac{42}{70}$; $\frac{68}{72}$; $\frac{45}{75}$.
17.	$\frac{26}{39}$; $\frac{18}{42}$; $\frac{24}{40}$; $\frac{27}{45}$; $\frac{42}{48}$.	20.	$\frac{57}{76}$; $\frac{65}{78}$; $\frac{36}{80}$; $\frac{60}{84}$; $\frac{75}{90}$.
18.	$\frac{42}{49}$; $\frac{13}{52}$; $\frac{45}{54}$; $\frac{40}{56}$; $\frac{48}{60}$.	21.	$\frac{32}{48}$; $\frac{40}{56}$; $\frac{48}{64}$; $\frac{54}{72}$; $\frac{76}{95}$.

CASE V.

To change dissimilar fractions to similar fractions.

Similar Fractions are fractions whose denominators are alike; as $\frac{3}{5}$, $\frac{5}{5}$, and $\frac{7}{5}$.

Dissimilar Fractions are fractions whose denominators are not alike; as $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{3}{8}$.

The ***Least Common Denominator*** is the least or smallest denominator common to two or more fractions.

NOTE.—The least common denominator of two or more fractions is the least common multiple of their denominators.

Written Exercises.

Change to similar fractions—

1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
2. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.
3. $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.
4. $\frac{3}{5}, \frac{3}{6}, \frac{3}{7}$; $\frac{2}{9}, \frac{1}{2}, \frac{5}{8}$.
5. $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$; $\frac{1}{3}, \frac{6}{7}, \frac{1}{4}$.
6. $\frac{4}{6}, \frac{4}{8}, \frac{4}{8}$; $\frac{7}{8}, \frac{2}{5}, \frac{6}{7}$.
7. $\frac{5}{7}, \frac{7}{9}, \frac{9}{11}$; $\frac{2}{8}, \frac{3}{10}, \frac{4}{5}, \frac{5}{2}$.
8. $\frac{5}{6}, \frac{7}{8}, \frac{9}{10}$; $\frac{3}{7}, \frac{5}{9}, \frac{7}{11}, \frac{9}{3}$.
9. $\frac{9}{10}, \frac{10}{11}, \frac{11}{12}$; $\frac{4}{9}, \frac{5}{8}, \frac{8}{7}, \frac{9}{20}$.

10. To what similar fractions are $\frac{3}{7}, \frac{5}{9}, \frac{7}{12}$ equal.

11. Change to equivalent fractions having a common denominator $\frac{5}{6}$ of a foot and $\frac{7}{8}$ of a foot; $\$\frac{3}{4}$ and $\$\frac{7}{8}$.

12. Change to similar fractions $\frac{1}{2}$ of an acre, $\frac{5}{6}$ of an acre, $\frac{9}{10}$ of an acre, $\frac{3}{4}$ of an acre.

Change to similar fractions having the least common denominator—

13. $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$; $\frac{1}{6}, \frac{3}{9}, \frac{5}{12}$.
14. $\frac{2}{5}, \frac{1}{6}, \frac{3}{9}$; $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}$.
15. $\frac{3}{5}, \frac{5}{8}, \frac{9}{16}$; $\frac{5}{6}, \frac{7}{9}, \frac{8}{15}$.
16. $\frac{7}{10}, \frac{5}{12}, \frac{8}{15}$.
17. $\frac{7}{12}, \frac{7}{15}, \frac{7}{18}$.
18. $\frac{9}{20}, \frac{7}{24}, \frac{7}{30}$.
19. $\frac{11}{36}, \frac{11}{42}, \frac{11}{48}$.
20. $\frac{12}{45}, \frac{13}{60}, \frac{14}{75}$.
21. $\frac{25}{48}, \frac{37}{60}, \frac{49}{96}$.

SECTION XI.

ADDITION OF FRACTIONS.

Written Exercises.

EXPLANATION.—To add fractions, change the fractions to a common denominator. Then add the numerators and write their sum over the common denominator.

OPERATION.

$$\frac{3}{4} = \frac{9}{12}$$
$$\frac{2}{3} = \frac{8}{12}$$
$$\frac{17}{12} = 1\frac{5}{12}, \text{ Sum.}$$

EXPLANATION.—In adding mixed numbers, add first the fractions and then the whole numbers. Unite their sums.

OPERATION.

$$2\tfrac{5}{6} = 2\tfrac{5}{6} \qquad \tfrac{5}{6} + \tfrac{4}{6} = \tfrac{9}{6} = 1\tfrac{3}{6} = 1\tfrac{1}{2}$$
$$4\tfrac{2}{3} = 4\tfrac{4}{6}$$
$$7\tfrac{1}{2}$$

Find the sum of—

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
2. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
3. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.
4. $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$.
5. $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$.
6. $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.
7. $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$.
8. $\frac{1}{3}$, $\frac{2}{5}$, $\frac{5}{6}$.
9. $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$.
10. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$.
11. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$.
12. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{8}$.
13. $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{12}$.
14. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
15. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$.
16. $\frac{6}{7}$, $\frac{5}{12}$, $\frac{4}{15}$.
17. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{5}{18}$.
18. $\frac{3}{4}$, $\frac{7}{8}$, $1\frac{1}{2}$.
19. $\frac{2}{5}$, $\frac{7}{10}$, $\frac{8}{15}$.
20. $\frac{5}{6}$, $\frac{8}{9}$, $\frac{5}{12}$.
21. $\frac{6}{7}$, $\frac{7}{9}$, $\frac{2}{8}$.
22. $2\frac{1}{4}$, $4\frac{2}{3}$, $7\frac{1}{2}$.
23. $6\frac{1}{5}$, $5\frac{1}{3}$, $2\frac{1}{2}$.
24. $4\frac{5}{6}$, $7\frac{3}{4}$, $8\frac{2}{8}$.
25. $9\frac{3}{7}$, $6\frac{3}{5}$, $12\frac{1}{4}$.
26. $6\frac{3}{8}$, $12\frac{5}{6}$, $9\frac{1}{3}$.
27. $10\frac{5}{8}$, $8\frac{3}{4}$, $16\frac{3}{7}$.
28. $12\frac{7}{8}$, $6\frac{2}{5}$, $7\frac{1}{2}$.
29. $15\frac{4}{9}$, $6\frac{2}{3}$, $7\frac{1}{6}$.
30. $12\frac{5}{9}$, $8\frac{3}{4}$, $12\frac{7}{8}$.

31. A butcher paid $\$1\frac{1}{2}$ for a sheep, $\$\frac{9}{4}$ for a calf, and $\$\frac{10}{8}$ for a lamb. What was the whole cost?

32. A owns $\frac{3}{7}$ of a vessel, B owns $\frac{2}{8}$ of it, and C owns $\frac{18}{56}$ of it. How much do the three own?

33. I sold $\frac{2}{5}$ of an acre to one man, $\frac{7}{15}$ of an acre to another, and $\frac{17}{16}$ of an acre to a third. How much did I sell to all?

34. If a man spent $\$7\frac{3}{8}$ on Monday, $\$\frac{7}{10}$ on Tuesday, and $\$8\frac{2}{5}$ on Wednesday, how much did he spend in all?

35. How many acres are in a farm which consists of $82\frac{7}{16}$ acres of cleared land, and $10\frac{5}{8}$ acres of woodland?

36. A grocer bought some flour for $18\frac{3}{4}$, sugar for $37\frac{1}{6}$, and tea for $12\frac{2}{3}$. What was the amount of his bill?

37. Richard walked $27\frac{3}{8}$ miles one day, 24 miles the next, and $30\frac{3}{4}$ miles the next. How far did he go?

38. How many yards are in three pieces of carpet that contain $52\frac{7}{12}$ yards, $69\frac{5}{8}$ yards, and $80\frac{2}{3}$ yards?

39. John has $7\frac{3}{4}$, Thomas has $3\frac{2}{5}$ more than John, and Henry has $9\frac{5}{8}$ more than both the others. How much has each, and how much have all of them?

SECTION XII.

SUBTRACTION OF FRACTIONS.

Written Exercises.

EXPLANATION.—To subtract fractions, change the fractions to a common denominator. Then subtract the numerators and write their difference over the common denominator.

OPERATION.

$$\frac{2}{3} = \frac{4}{6}$$
$$\frac{1}{2} = \frac{3}{6}$$
$$\frac{1}{6}, \textit{ Difference.}$$

EXPLANATION.—In subtracting mixed numbers, subtract first the fractions and then the whole numbers.

OPERATION.

$$4\frac{1}{5} = 4\frac{3}{15} \qquad 1 = \frac{15}{15}$$
$$2\frac{1}{3} = 2\frac{5}{15} \qquad \frac{15}{15} + \frac{3}{15} = \frac{18}{15}$$
$$1\frac{13}{15}, \textit{ Difference.} \qquad \frac{18}{15} - \frac{5}{15} = \frac{13}{15}.$$

Find the difference between—

1. $\frac{3}{8}$ and $\frac{1}{4}$.
2. $\frac{7}{9}$ and $\frac{1}{3}$.
3. $\frac{1}{2}$ and $\frac{1}{3}$.
4. $\frac{1}{8}$ and $\frac{1}{4}$.
5. $\frac{1}{4}$ and $\frac{1}{5}$.
6. $\frac{1}{5}$ and $\frac{1}{6}$.
7. $\frac{1}{6}$ and $\frac{1}{7}$.
8. $\frac{1}{7}$ and $\frac{1}{8}$.
9. $\frac{1}{8}$ and $\frac{1}{9}$.
10. $\frac{1}{9}$ and $\frac{1}{10}$.

Subtract—

11. $\frac{3}{4}$ from $\frac{4}{5}$.
12. $\frac{2}{5}$ from $\frac{5}{6}$.
13. $\frac{7}{8}$ from $\frac{8}{9}$.
14. $\frac{3}{4}$ from $1\frac{5}{16}$.
15. $\frac{3}{8}$ from $\frac{5}{7}$.
16. $\frac{1}{6}$ from $\frac{3}{9}$.
17. $\frac{3}{4}$ from $\frac{7}{8}$.
18. $\frac{5}{8}$ from $1\frac{1}{12}$.

How much is—

19. $5\frac{1}{3} - 2\frac{1}{4}$.
20. $8\frac{3}{4} - 6\frac{2}{3}$.
21. $9\frac{5}{6} - 7\frac{3}{4}$.
22. $12\frac{7}{8} - 2\frac{1}{3}$.
23. $14\frac{1}{4} - 6\frac{1}{3}$.
24. $11\frac{3}{8} - 6\frac{3}{4}$.
25. $12\frac{1}{6} - 7\frac{1}{5}$.
26. $15\frac{1}{7} - 4\frac{5}{6}$.
27. $18\frac{3}{4} - 12\frac{5}{6}$.
28. $16\frac{1}{9} - 7\frac{2}{3}$.
29. $22\frac{5}{8} - 16\frac{4}{5}$.
30. $16\frac{2}{3} - 5\frac{7}{8}$.
31. $17\frac{1}{5} - 6\frac{2}{3}$.
32. $24\frac{1}{12} - 18\frac{1}{8}$.
33. $10\frac{7}{8} - 6\frac{11}{12}$.
34. $16\frac{4}{7} - 6\frac{4}{5}$.
35. $19\frac{5}{12} - 7\frac{5}{7}$.
36. $12\frac{1}{16} - 7\frac{1}{5}$.

37. A dealer bought a bushel of potatoes for $\$\frac{7}{8}$, and sold them for $\$\frac{9}{10}$. How much did he gain?

38. If a grocer draws $\frac{4}{5}$ of a gallon of oil from a can containing $\frac{9}{10}$ of a gallon, how much is left?

39. From a bottle containing $\frac{2}{3}$ of a quart of wine, there was taken $\frac{3}{8}$ of a quart. How much remained in it?

40. A boy gathered $\frac{3}{4}$ of a bushel of chestnuts, and sold $\frac{5}{8}$ of a bushel. What part of a bushel remained?

41. A merchant owned $\frac{3}{8}$ of a vessel, and sold $\frac{3}{5}$ of it. What part did he then own?

42. I bought a ton of coal for $\$7\frac{3}{10}$, and a cord of wood for $\$5\frac{7}{8}$. What was the difference in price?

43. From a cask of wine containing $60\frac{1}{16}$ gallons, $20\frac{3}{4}$ gallons were drawn. How much remained?

44. If a farmer should sell $44\frac{3}{16}$ acres from a farm containing 160 acres, how many acres would remain?

SECTION XIII.

MULTIPLICATION OF FRACTIONS.

CASE I.

To Multiply a Fraction by an Integer.

Written Exercises.

SUGGESTION.—In the multiplication of fractions, first change whole and mixed numbers to improper fractions, then multiply the numerators together for a new numerator, and multiply the denominators together for a new denominator. Change the final result, if an improper fraction, to a whole or mixed number; thus, $\frac{3}{4} \times \frac{5}{1} = \frac{15}{4} = 3\frac{3}{4}$.

Multiply—

1.	$\frac{2}{3}$ by 3.	7.	$\frac{3}{8}$ by 4.	13.	$\frac{2}{7}$ by 5.
2.	$\frac{5}{6}$ by 4.	8.	$\frac{8}{9}$ by 8.	14.	$\frac{4}{9}$ by 7.
3.	$\frac{7}{8}$ by 6.	9.	$\frac{11}{12}$ by 10.	15.	$\frac{3}{7}$ by 8.
4.	$\frac{5}{9}$ by 3.	10.	$\frac{5}{8}$ by 7.	16.	$\frac{7}{9}$ by 6.
5.	$\frac{4}{5}$ by 7.	11.	$\frac{3}{4}$ by 9.	17.	$\frac{7}{10}$ by 12.
6.	$\frac{9}{10}$ by 5.	12.	$\frac{3}{5}$ by 12.	18.	$\frac{4}{11}$ by 6.

19. Multiply $\frac{5}{15}$ by 4, by 6, by 8, by 10, by 12, and by 20.

20. Find the product of $\frac{2}{11} \times 18$; $\frac{9}{16} \times 7$; $\frac{7}{18} \times 12$.

21. How much is 7 times $\frac{3}{11}$? 8 times $\frac{4}{15}$? 9 times $\frac{7}{12}$?

22. Find the product of—

23.	$2\frac{1}{3} \times 4$.	27.	$8\frac{1}{4} \times 5$.	31.	$14\frac{4}{5} \times 9$.
24.	$3\frac{2}{5} \times 6$.	28.	$6\frac{2}{3} \times 8$.	32.	$6\frac{3}{5} \times 12$.
25.	$5\frac{3}{4} \times 7$.	29.	$7\frac{1}{5} \times 9$.	33.	$12\frac{5}{6} \times 8$.
26.	$8\frac{1}{8} \times 3$.	30.	$11\frac{2}{6} \times 12$.	34.	$9\frac{4}{5} \times 16$.

35. At $\$\frac{7}{10}$ a bushel, what do 8 bushels of corn cost?

36. At the rate of $\frac{5}{8}$ of a mile an hour, how far could you travel in 5 hrs.? In 6 hrs.? In 7 hrs.?

37. If a dealer pays $\$2\frac{3}{4}$ a barrel for flour, what will 8 barrels cost? 12 barrels? 20 barrels?

CASE II.

To Multiply an Integer by a Fraction.

Written Exercises.

Multiply—

1. 15 by $\frac{4}{5}$.	7. 36 by $\frac{4}{9}$.	13. 62 by $7\frac{5}{6}$.
2. 20 by $\frac{5}{6}$.	8. 40 by $\frac{5}{8}$.	14. 75 by $8\frac{4}{5}$.
3. 32 by $\frac{7}{8}$.	9. 36 by $\frac{9}{10}$.	15. 52 by $6\frac{3}{4}$.
4. 16 by $\frac{3}{4}$.	10. 44 by $\frac{10}{11}$.	16. 48 by $7\frac{2}{3}$.
5. 35 by $\frac{3}{5}$.	11. 52 by $\frac{13}{14}$.	17. 37 by $5\frac{1}{6}$.
6. 42 by $\frac{2}{7}$.	12. 45 by $\frac{7}{15}$.	18. 46 by $9\frac{3}{5}$.

19. Multiply 21 by $\frac{2}{3}$, by $\frac{3}{4}$, by $\frac{5}{6}$, by $\frac{6}{7}$, by $\frac{8}{9}$, by $\frac{9}{10}$.

20. Find the product of $20 \times \frac{3}{10}$; $30 \times 8\frac{3}{4}$; $45 \times 10\frac{2}{5}$.

21. How much is $\frac{3}{4}$ of 87? $108 \times \frac{3}{10}$? $9\frac{1}{3}$ times 42?

22. At \$8 a cord, what will $\frac{11}{16}$ of a cord of wood cost?

23. If a man can do a piece of work in 20 days, in what time can he do $\frac{2}{3}$ of it? $\frac{3}{4}$ of it? $\frac{7}{8}$? $\frac{8}{9}$? $\frac{9}{10}$? $\frac{11}{12}$? $\frac{14}{15}$?

24. At \$18 a ton for hay, what must be paid for $\frac{2}{3}$ of a ton? For $2\frac{5}{6}$ tons? For $3\frac{1}{2}$ tons? $10\frac{1}{3}$ tons? $12\frac{4}{5}$ tons?

25. In a piece of muslin are 34 yards. How many yards in $2\frac{1}{3}$ pieces? $5\frac{3}{4}$ pieces? $9\frac{2}{5}$ pieces?

26. If 6 men do a piece of work in $5\frac{2}{3}$ days, how many men will it take to do it in 1 day?

27. At the rate of \$7 for a ton of coal, what is the price of $\frac{8}{10}$ of a ton? Of $\frac{7}{10}$ of a ton? Of $\frac{9}{10}$ of a ton?

28. I bought 8 thousand feet of lumber, and used $\frac{4}{5}$ of it in building a barn. How many feet did I use?

29. The cleared land of a farm of 72 acres is $\frac{3}{5}$ of the whole farm. How many acres are woodland?

CASE III.

To Multiply a Fraction by a Fraction.

Written Exercises.

Find the value of—

1. $\frac{2}{3} \times \frac{3}{4}$.
2. $\frac{2}{3} \times \frac{4}{5}$.
3. $\frac{4}{5} \times \frac{5}{6}$.
4. $\frac{3}{4} \times \frac{5}{6}$.
5. $\frac{5}{6} \times \frac{7}{8}$.
6. $\frac{4}{5} \times \frac{6}{7}$.
7. $\frac{6}{7} \times \frac{8}{9}$.
8. $\frac{5}{6} \times \frac{9}{10}$.
9. $\frac{9}{10} \times \frac{10}{12}$.
10. $\frac{7}{12} \times \frac{3}{8}$.
11. $\frac{5}{12} \times \frac{6}{7}$.
12. $\frac{7}{15} \times \frac{3}{4}$.
13. $\frac{5}{14} \times \frac{7}{8}$.
14. $\frac{7}{18} \times \frac{6}{7}$.
15. $\frac{4}{15} \times \frac{5}{8}$.
16. $\frac{5}{16} \times \frac{8}{9}$.
17. $\frac{7}{18} \times \frac{6}{7}$.
18. $\frac{12}{19} \times \frac{5}{6}$.

Find the product of—

19. $6\frac{1}{2} \times \frac{2}{3}$.
20. $2\frac{1}{4} \times \frac{6}{7}$.
21. $2\frac{4}{5} \times \frac{3}{7}$.
22. $4\frac{1}{2} \times \frac{8}{9}$.
23. $2\frac{2}{3} \times \frac{3}{5}$.
24. $4\frac{1}{5} \times \frac{2}{3}$.

25. At $\$\frac{7}{10}$ a yard, what is the cost of $\frac{5}{8}$ of a yard of cloth?

26. If a bushel of apples is worth $\$\frac{7}{12}$, what is the value of $\frac{3}{4}$ of a bushel? $\frac{6}{7}$ of a bushel? $\frac{3}{5}$ of a bushel?

27. What is the cost of $\frac{4}{7}$ of a pound of flour at $5\frac{1}{4}$ cents a pound? Of $\frac{4}{5}$ of a yard of calico at $12\frac{1}{2}$ cents a yard?

28. Find the cost of $\frac{3}{4}$ of a barrel of apples at $\$2\frac{2}{3}$ a barrel; at $\$3\frac{1}{4}$ a barrel; at $\$3\frac{1}{6}$; at $\$4\frac{4}{5}$; at $\$5\frac{5}{6}$; at $\$6\frac{2}{5}$?

29. At $\$\frac{3}{4}$ a pound, what will $3\frac{2}{3}$ pounds of tea cost?

30. What are $3\frac{2}{3}$ times $\$\frac{4}{5}$? $4\frac{3}{4}$ times $\frac{3}{7}$ of a day? $5\frac{3}{5}$ times $\$\frac{3}{10}$? $6\frac{5}{6}$ times $\frac{5}{12}$ of a yard? $7\frac{6}{7}$ times $\frac{7}{18}$?

31. If a farmer can mow $\frac{7}{8}$ of an acre of grass in a day, how much can he mow in $2\frac{1}{4}$ days? In $3\frac{3}{7}$ days? In $2\frac{3}{10}$ days?

32. Emma is $15\frac{3}{4}$ years old, and her sister is $4\frac{2}{3}$ times as old. What is the sister's age? Find the sum of their ages.

33. What will $2\frac{1}{4}$ yards of silk cost at $\$2\frac{2}{3}$ a yard?

SECTION XIV.

DIVISION OF FRACTIONS.

CASE I.

To Divide a Fraction by an Integer.

Written Exercises.

SUGGESTION.—In the division of fractions, first change whole and mixed numbers to improper fractions, then invert the divisor and proceed as in multiplication. Change the final result, if an improper fraction, to a whole or mixed number; thus, $\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$.

Divide—

1. $\frac{3}{5}$ by 6.
2. $\frac{4}{5}$ by 8.
3. $\frac{2}{3}$ by 10.
4. $\frac{6}{7}$ by 3.
5. $\frac{7}{8}$ by 4.
6. $\frac{5}{7}$ by 10.
7. $\frac{5}{6}$ by 15.
8. $\frac{8}{9}$ by 6.
9. $\frac{3}{8}$ by 9.
10. $\frac{7}{8}$ by 14.
11. $\frac{10}{11}$ by 15.
12. $\frac{1}{12}$ by 16.
13. $\frac{5}{12}$ by 20.
14. $\frac{7}{12}$ by 21.
15. $\frac{4}{15}$ by 8.
16. $\frac{7}{15}$ by 14.
17. $\frac{3}{16}$ by 9.
18. $\frac{5}{16}$ by 10.

What is the quotient of—

19. $\frac{12}{13}$ of a bale divided by 6?
20. $\frac{14}{15}$ of a cord divided by 7?
21. $\frac{15}{16}$ of an acre divided by 8?
22. $\frac{19}{20}$ of a mile divided by 9?
23. $12\frac{1}{2}$ tons $\div$ 4 tons?
24. $13\frac{1}{3}$ feet $\div$ 8 feet?
25. $16\frac{2}{3}$ days $\div$ 10 days?
26. $18\frac{3}{4}$ weeks $\div$ 12 weeks?

27. How much is $\frac{3}{11} \div 9$? $12\frac{3}{4} \div 6$? $28\frac{5}{6} \div 8$?
28. If 4 tons of coal cost $\$21\frac{3}{5}$, what will 1 ton cost?
29. If 9 yards of cloth cost $\$37\frac{1}{2}$, what is the cost of 1 yard?
30. In 25 rods are $137\frac{1}{2}$ yards. How many yards in 1 rod?

CASE II.

To Divide an Integer by a Fraction.

Written Exercises.

Divide—

1.	6 by $\frac{4}{5}$.	9.	12 by $\frac{3}{5}$.	17.	14 by $\frac{5}{18}$.
2.	7 by $\frac{6}{7}$.	10.	20 by $\frac{5}{8}$.	18.	15 by $\frac{11}{14}$.
3.	8 by $\frac{8}{9}$.	11.	21 by $\frac{3}{7}$.	19.	20 by $\frac{7}{15}$.
4.	9 by $\frac{6}{11}$.	12.	14 by $\frac{3}{8}$.	20.	24 by $\frac{9}{20}$.
5.	14 by $\frac{7}{8}$.	13.	16 by $\frac{5}{12}$.	21.	12 by $3\frac{2}{3}$.
6.	12 by $\frac{9}{10}$.	14.	15 by $\frac{7}{12}$.	22.	14 by $4\frac{2}{5}$.
7.	15 by $\frac{5}{6}$.	15.	25 by $\frac{15}{16}$.	23.	15 by $6\frac{3}{4}$.
8.	16 by $\frac{4}{7}$.	16.	12 by $\frac{7}{20}$.	24.	20 by $5\frac{1}{6}$.

25. Divide 25 feet by $\frac{6}{7}$, by $\frac{3}{5}$, by $1\frac{3}{5}$, by $3\frac{5}{6}$, by $4\frac{4}{5}$.

26. Find the quotient of $32 \div 4\frac{3}{4}$; $35 \div 3\frac{2}{3}$; $28 \div 6\frac{1}{5}$; $24 \div 5\frac{5}{6}$; $21 \div 7\frac{4}{5}$; $30 \div 8\frac{1}{6}$.

27. At $\$\frac{7}{8}$ each, how many books can be bought for \$14? \$21? \$35? \$42? \$49? \$63? \$77? \$84?

28. Into how many fields can a farm of 132 acres be divided that each field shall contain $6\frac{3}{5}$ acres?

29. A builder paid a mason \$48 for labor at $\$3\frac{1}{5}$ a day. How many days did he work?

CASE III.

To Divide a Fraction by a Fraction.

Written Exercises.

Divide—

1.	$\frac{4}{5}$ by $\frac{2}{3}$.	7.	$\frac{5}{12}$ by $\frac{3}{4}$.	13.	$2\frac{1}{3}$ by $3\frac{1}{4}$.
2.	$\frac{5}{6}$ by $\frac{4}{5}$.	8.	$\frac{8}{15}$ by $2\frac{2}{3}$.	14.	$4\frac{3}{4}$ by $5\frac{2}{3}$.
3.	$\frac{7}{8}$ by $\frac{6}{7}$.	9.	$\frac{4}{9}$ by $3\frac{2}{5}$.	15.	$6\frac{4}{5}$ by $3\frac{5}{6}$.
4.	$\frac{8}{9}$ by $\frac{7}{8}$.	10.	$\frac{5}{8}$ by $5\frac{1}{6}$.	16.	$2\frac{1}{6}$ by $4\frac{3}{5}$.
5.	$\frac{3}{7}$ by $\frac{1}{6}$.	11.	$\frac{3}{8}$ by $4\frac{3}{4}$.	17.	$7\frac{3}{4}$ by $3\frac{4}{5}$.
6.	$\frac{9}{10}$ by $\frac{3}{5}$.	12.	$\frac{7}{10}$ by $6\frac{4}{5}$.	18.	$12\frac{3}{5}$ by $4\frac{2}{3}$.

19. How many times is $2\frac{2}{3}$ contained into $4\frac{1}{5}$? $6\frac{1}{4}$? $8\frac{3}{4}$? $10\frac{2}{5}$? $12\frac{4}{5}$? $15\frac{5}{6}$?

20. If a pound of sugar cost $5\frac{1}{2}$ cents, how many pounds can be bought for $38\frac{1}{2}$ cents?

21. At $\$\frac{3}{8}$ a yard, how many yards of cloth can be bought for $\$\frac{7}{8}$?

22. How many bottles, each containing $\frac{7}{12}$ of a gallon, can be filled with $4\frac{2}{3}$ gallons of vinegar?

23. How many lengths of carpet, each containing $6\frac{2}{3}$ square yards, are needed to cover $73\frac{1}{3}$ square yards of floor?

24. What fraction multiplied by $\frac{11}{12}$ equals $7\frac{5}{6}$?

25. John divided $\frac{8}{9}$ of a bushel of chestnuts among a number of boys. If each boy received $\frac{1}{18}$ of a bushel, what was the number of boys?

26. A man had $\frac{5}{6}$ of an acre of ground which he divided into lots of $\frac{1}{12}$ of an acre each. How many lots did it make?

SECTION XV.

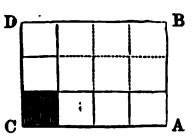
MEASUREMENT OF SURFACES.

A ***Rectangle*** is a flat surface which has four straight sides and four square corners.

A rectangle has two dimensions: ***length*** and ***breadth.***

The ***Area*** of a rectangle is the surface bounded by the sides of the rectangle.

Thus, the area of a blackboard is the surface bounded by the sides or edges of the blackboard.



Let the figure A B C D represent a surface 4 in. long and 3 in. wide. Now, there can be placed along the length of the surface as many square inches as there are inches in the length, making a row of 4 sq. in.; and there can be as many rows of square inches as there are inches in the width, making

3 rows. Hence the surface contains 3 rows of 4 sq. in. each; and 3 times 4 sq. in. or *12 sq. in.*, must be the area of the surface.

The area of a rectangle is the product of the length and breadth.

How many square feet are in a floor 16 ft. long and 10 ft. wide?

Operation.	Explanation.
16 ft. $\times$ *10 ft.* = *160 sq. ft.*	Since the area of a rectangle is the product of the length and the breadth, the product of the length, 16 ft., multiplied by the breadth, 10 ft., must be 160 sq. ft., the area required.

Written Exercises.

Find the areas of the surfaces having the following dimensions:

1. 4 in. by 3 in.; 6 in. by 4 in.
2. 2 ft. by 3 ft.; 3 ft. by 4 ft.
3. 6 in. by 8 in.; 9 in. by 6 in.
4. 3 ft. by 5 ft.; 4 ft. by 6 ft.
5. 3 ft. by 4 ft.; 2 ft. by 5 ft.
6. 2 yd. by 3 yd.; 3 yd. by 4 yd.

7. How many feet of boards 1 foot wide will it take to make the top of a table 4 ft. long and 3 ft. wide?

8. How many feet of boards 6 inches wide will it take to lay the floor of a room 16 ft. long and 12 ft. wide?

9. How many yards of carpet 1 yard wide will it take to cover the floor of a room 18 ft. long and 15 ft. wide?

10. How many feet of boards 3 in. wide will it take to make a blackboard 10 ft. long and 4 ft. high?

11. How many feet of tiles 12 in. wide will be required for the floor of a cellar 28 ft. long and 16 ft. wide?

SECTION XVI.

ARITHMETICAL ANALYSIS.

1. If 4 yards of cloth cost \$12, what will 7 yards cost?

OPERATION.	EXPLANATION.
4 yd. cost \$12.	If 4 yards of cloth cost \$12, 1 yard of cloth will cost $\frac{1}{4}$ of \$12, or \$3, and 7 yards will cost 7 times \$3, or \$21.
1 yd. cost $\frac{1}{4}$ of \$12 = \$3.	
7 yd. cost 7 times \$3 = \$21.	

2. If 7 pencils cost 28 cents, how much will 4 pencils cost?

3. How much will 9 tons of hay cost, if 4 tons cost \$36?

4. If 6 men earn \$18 in 1 day, how much will 8 men earn?

5. If a boy can walk 12 miles in 3 hours, how far can he walk in 5 hours?

6. I gave \$24 for 8 hats. What is the cost of 5 hats?

7. How much must I pay for 12 barrels of flour if 9 barrels cost \$54?

8. If 12 pounds of coffee last a family 36 days, how long will 5 pounds last them?

9. If 9 bags of grain weigh 108 pounds, what will 7 bags weigh?

10. A boy earns \$7.50 in 5 days. How long will it take him to earn \$18?

11. If a gallon of oil cost 12 cents, how much will $\frac{3}{4}$ of a gallon cost?

OPERATION.	EXPLANATION.
1 gal. cost 12 cents.	If a gallon of oil cost 12 cents, $\frac{1}{4}$ of a gallon will cost $\frac{1}{4}$ of 12 cents, or 3 cents, and $\frac{3}{4}$ of a gallon will cost 3 times 3 cents, or 9 cents.
$\frac{1}{4}$ of a gal. cost $\frac{1}{4}$ of 12 cents = 3 "	
$\frac{3}{4}$ of a gal. cost 3 times 3 cents = 9 "	

12. If a ton of coal cost \$6, what will $\frac{2}{3}$ of a ton cost?

13. There are 60 minutes in an hour. How many minutes in $\frac{2}{3}$ of an hour?

14. How much will $\frac{4}{5}$ of a yard of cloth cost at \$1.50 a yard?

15. At 36 cents a pound, how much will $\frac{5}{6}$ of a pound of coffee cost?

16. If a basket of peaches cost 72 cents, what will $\frac{3}{4}$ of a basket cost?

17. A train travels 54 miles an hour. How far does it travel in $\frac{5}{6}$ of an hour?

18. John is 24 years old. Mary is $\frac{7}{8}$ as old. How old is Mary?

19. A tank contains 84 gallons of water. $\frac{3}{4}$ leaked out. How many gallons leaked out?

20. There are 64 pints in 1 bushel. How many pints in $\frac{3}{8}$ of a bushel?

21. If $\frac{3}{4}$ of a yard of cloth cost \$6, what will 1 yard cost?

Operation.	Explanation.
$\frac{3}{4}$ *of a yd. cost* \$6.	If $\frac{3}{4}$ of a yard of cloth cost \$6, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of \$6, or \$2, and $\frac{4}{4}$ of a yard, or 1 yard, will cost 4 times \$2, or \$8.
$\frac{1}{4}$ *of a yd. cost* $\frac{1}{3}$ *of* \$6 = \$2.	
$\frac{4}{4}$ *of a yd. cost 4 times* \$2 = \$8.	

22. If I sell $\frac{2}{3}$ of a pound of coffee for 18 cents, what will I receive for 1 pound?

23. John is 9 years old. If he is $\frac{2}{3}$ as old as Mary, how old is Mary?

24. A man spends \$18 in $\frac{5}{6}$ of a week. How much does he spend in 1 week?

25. At 12 cents for $\frac{4}{5}$ of a gallon of oil, how much will 1 gallon cost?

26. How much will a bushel of corn cost if $\frac{7}{8}$ of a bushel cost 49 cents?

27. If William has 15 marbles, which is $\frac{3}{5}$ of what he had at first, how many had he at first?

28. If $\frac{4}{5}$ of a bushel of oats sells for 40 cents, how much will 1 bushel sell for?

29. If $\frac{3}{7}$ of a pound of tea costs 27 cents, how much will 1 pound cost?

30. A man walks 36 miles in $\frac{5}{6}$ of a day. How far can he walk in 1 day?

31. If 2 pounds of sugar cost \$.12, how much will $\frac{2}{3}$ of a pound cost?

Operation.	Explanation.
2 lb. cost \$.12. *1 lb. cost $\frac{1}{2}$ of \$.12 = \$.06.* *$\frac{1}{3}$ of a lb. cost $\frac{1}{3}$ of \$.06 = \$.02.* *$\frac{2}{3}$ of a lb. cost 2 times \$.02 = \$.04.*	If 2 pounds of sugar cost \$.12, 1 pound will cost $\frac{1}{2}$ of \$.12, or \$.06. If 1 pound cost \$.06, $\frac{1}{3}$ of a pound will cost $\frac{1}{3}$ of \$.06, or \$.02, and $\frac{2}{3}$ of a pound will cost twice \$.02, or \$.04.

32. If 3 yards of silk cost \$24, what will $\frac{3}{4}$ of a yard cost?

33. If 5 gallons of oil cost 75 cents, how much will $\frac{3}{5}$ of a gallon cost?

34. If a horse travels 36 miles in 3 hours, how far can it travel in $\frac{5}{6}$ of an hour?

35. A vessel sailed 28 miles in 2 hours. How far does it sail in $\frac{3}{7}$ of an hour?

36. If 3 times John's money is \$45, how much is $\frac{2}{3}$ of his money?

37. There are 48 pounds of sugar in 3 bags. How many pounds in $\frac{7}{8}$ of a bag?

38. If 5 pounds of butter are worth \$1.50, how much is $\frac{4}{5}$ of a pound worth?

39. A man earns \$7.50 in 3 days. How much does he earn in $\frac{2}{5}$ of a day?

40. How much will $\frac{3}{8}$ of a pound of candy cost if 5 pounds cost \$1?

41. If $\frac{3}{4}$ of a quart of milk cost \$.06, what will 5 quarts cost?

OPERATION.

$\frac{3}{4}$ *of a qt. cost* \$.06.

$\frac{1}{4}$ *of a qt. cost* $\frac{1}{3}$ *of* \$.06 = \$.02.

$\frac{4}{4}$ *of a qt. cost 4 times* \$.02 = \$.08.

5 qt. cost 5 times \$.08 = \$.40.

5 quarts will cost 5 times \$.08, or \$.40.

EXPLANATION.

If $\frac{3}{4}$ of a quart of milk cost \$.06, $\frac{1}{4}$ of a quart will cost $\frac{1}{3}$ of \$.06, or \$.02, and $\frac{4}{4}$ of a quart, or 1 quart, will cost 4 times \$.02, or \$.08. If 1 quart cost \$.08,

42. If $\frac{2}{3}$ of a pound of butter cost 24 cents, how much will 3 pounds cost?

43. If a man earns \$2.25 in $\frac{1}{5}$ of a day, how much will he earn in 2 days?

44. A train travels 21 miles in $\frac{3}{4}$ of an hour. How far does it travel in 3 hours?

45. If $\frac{3}{8}$ of John's money is \$9, how much is 4 times his money?

46. If $\frac{5}{8}$ of an acre of land cost \$25, how much will 4 acres cost?

47. If a family uses $\frac{1}{7}$ of a bushel of potatoes in 12 days, how long will 2 bushels last them?

48. If $\frac{3}{4}$ of a basket of apples sell for 27 cents, how much will 6 baskets sell for?

49. If $\frac{3}{5}$ of a pound of meat cost 12 cents, how much will 5 pounds cost?

50. If $\frac{2}{5}$ of a ton of hay is worth \$4, how much are 7 tons worth?

SECTION XVII.

REVIEW PROBLEMS.

1. William paid $\$\frac{5}{6}$ for a knife and $\$\frac{7}{8}$ for a pair of skates. What did he pay for both?

2. If a farmer sells $\frac{1}{4}$ of his wheat to one man, and $\frac{1}{6}$ of it to another, what part of his wheat does he sell to both?

3. If John does $\frac{1}{6}$ of a piece of work one day, and $\frac{1}{8}$ of it the next, what part does he perform in both days?

4. A man having $\frac{3}{4}$ of a ton of coal, bought $\frac{4}{5}$ of a ton more. How much coal had he then?

5. I used $\frac{4}{7}$ of a ton of coal one week, and $\frac{4}{5}$ of a ton the next. How much did I use in the two weeks?

6. A gardener has $\frac{3}{4}$ of an acre of raspberries and $\frac{4}{5}$ of an acre of strawberries. How many acres of berries has he?

7. In a village lot are $\frac{5}{8}$ of an acre, and in another are $\frac{7}{10}$ of an acre. How much land is in both lots?

8. A owns $\frac{2}{5}$ of a vessel, and B owns $\frac{3}{8}$ of it. What part of the vessel do both of them own?

9. What fraction is that from which, if $\frac{5}{6}$ be taken, the remainder will be $\frac{2}{5}$?

10. William saved $\$\frac{4}{5}$ one week, $\$\frac{7}{8}$ the next, and $\$\frac{7}{10}$ the next. How much did he save in the three weeks?

Find the sum of—

11. $\frac{7}{8}$ of a mile and $\frac{9}{10}$ of a mile.

12. $\frac{9}{10}$ of an acre and $\frac{11}{12}$ of an acre.

13. $\frac{11}{12}$ of a yard and $\frac{15}{16}$ of a yard.

14. $\frac{11}{20}$ of a ton and $\frac{11}{25}$ of a ton.

15. $\frac{13}{24}$ of a cord and $\frac{13}{32}$ of a cord.

16. $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{6}{7}$ of a bushel.

17. $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$ of a gallon.

18. A farmer sold $2\frac{3}{4}$ tons of hay to one man, and $3\frac{4}{5}$ tons to another. How much did he sell to both?

19. If the minuend is $5\frac{1}{8}$ feet, and the subtrahend is $3\frac{2}{3}$ feet, what is the remainder?

20. If the difference is $2\frac{3}{4}$ miles, and the minuend is $6\frac{5}{6}$ miles, what is the subtrahend?

21. From a field containing $5\frac{3}{8}$ acres I sold $\frac{7}{10}$ of an acre. How much land was left in the field?

22. If the cost of making a pair of boots is $\$5\frac{2}{5}$, and the selling price is $\$7\frac{1}{4}$, what is the profit?

23. A laborer who earns $\$8\frac{3}{10}$ a week expends $\$6\frac{3}{8}$. How much a week does he save?

24. From a piece of beef weighing 10 pounds a butcher sold $6\frac{3}{10}$ pounds. How many pounds remained?

25. A man owned $\frac{9}{10}$ of a flour mill, and sold $\frac{1}{3}$ of the mill to one man and $\frac{2}{5}$ of it to another. What part had he left?

26. The sum of three numbers is $8\frac{3}{5}$ tons. If two of them are $2\frac{2}{3}$ tons and $\frac{3}{4}$ of a ton, what is the third?

27. Henry had \$10, spent $\$2\frac{3}{10}$, and earned enough to make $\$11\frac{1}{2}$. What did he earn?

28. A boy had $\$9\frac{1}{2}$, then earned \$5 lacking $\$1\frac{5}{8}$. How much had he then? How much more than at first?

29. From two remnants of calico containing $3\frac{2}{3}$ yards and $4\frac{3}{4}$ yards, there were cut $8\frac{1}{12}$ yards. How much less than 1 yard remained? How much more than $\frac{2}{9}$ of a yard?

30. The sum of three numbers is $12\frac{3}{10}$. If one of the numbers is $5\frac{2}{3}$, and another is $4\frac{5}{9}$, what is the third?

31. John earned $\$5\frac{7}{8}$ one week and $\$6\frac{3}{10}$ the next. What did he then lack of having enough to pay \$15 for a coat?

32. Into three milk cans were poured $15\frac{3}{4}$ quarts of milk. If one contained $5\frac{3}{4}$ quarts, and another $4\frac{3}{8}$ quarts, what did the other contain?

33. William is $10\frac{3}{4}$ years old, and James is $12\frac{5}{12}$ years old. How many years less is the sum of their ages than their father's age, which is $35\frac{1}{2}$ years?

34. Find the cost of $7\frac{3}{4}$ pounds of rice at 7 cents a pound.

35. How many miles will a yacht sail in $4\frac{7}{8}$ hours at the rate of 10 miles an hour? 12 miles an hour?

36. What is the cost of $3\frac{5}{12}$ dozen lead pencils at 20 cents a dozen? Of $5\frac{7}{12}$ dozen? $7\frac{1}{12}$ dozen?

37. How far will a railroad train run in $5\frac{3}{4}$ hours at the rate of 24 miles an hour? In $6\frac{3}{8}$ hours? $7\frac{5}{12}$ hours?

38. A and B bought a reaper for $75, A paying $\frac{3}{10}$ of the cost, B $2\frac{1}{3}$ times as much. What did each pay?

39. A merchant had $8\frac{3}{4}$ yards of broadcloth, and sold $\frac{5}{7}$ of it at $\$2\frac{2}{3}$ a yard. What did he receive for it?

40. How much will $\frac{7}{8}$ of $5\frac{1}{3}$ tons of coal cost at $\$6\frac{3}{4}$ a ton? What will $\frac{5}{9}$ of $6\frac{3}{4}$ tons cost?

41. What number added to 4 times $\frac{2}{5}$ of 28 miles will make the number equal to $45\frac{1}{2}$ miles?

42. A laborer works 2 days for $\$1\frac{3}{4}$ and $\$1\frac{7}{8}$. At 2 dollars a day, how much more would he have received?

43. A painter used $\frac{2}{3}$ of $\frac{6}{7}$ of a keg of white lead worth $\$2\frac{5}{8}$. What part of the keg did he use, and what was it worth?

44. I paid $\$6\frac{1}{4}$ for a ton of coal, and $1\frac{1}{3}$ times as much for a barrel of flour. How much did both cost?

45. George had $\$1\frac{3}{5}$, and paid $\frac{3}{4}$ of it for a knife. What did he gain or lose by selling it for $\$1\frac{3}{8}$?

46. If I buy a cow for $\$37\frac{1}{2}$, and sell her for $\frac{9}{10}$ of the cost, how much will I lose?

47. Suppose a coat costs $\frac{6}{7}$ of $\$9\frac{1}{3}$, and a vest costs $\frac{4}{7}$ as much. How much do both cost?

48. Mary is $3\frac{3}{4}$ years old, and Helen is 5 times as old as Mary. In how many years will Helen be 21 years old?

49. If a man can put up $3\frac{3}{5}$ rods of stone wall in a day, how much can he put up in $\frac{5}{6}$ of a day?

50. If a horse trots $8\frac{3}{4}$ miles an hour, how far does he trot in $\frac{4}{5}$ of an hour? In $\frac{6}{7}$ of an hour?

51. If a pound of sugar is worth $\frac{4}{5}$ of $17\frac{1}{2}$ cents, how much will $\frac{3}{4}$ of $1\frac{1}{7}$ pounds cost? $\frac{3}{5}$ of $2\frac{2}{3}$ pounds?

52. What is the quotient of $3\frac{2}{3}$ days divided by $\frac{3}{4}$ of a day?

53. How long will it take to earn $\$4\frac{3}{4}$ at $\$\frac{7}{8}$ a day?

54. At $\$\frac{9}{10}$ a yard, how many yards of cassimere must a merchant sell to receive $\$5\frac{4}{5}$? To receive $\$6\frac{7}{8}$?

55. If $7\frac{1}{2}$ years is $\frac{5}{12}$ of Harry's age, how old is he?

56. A teamster hauled $8\frac{2}{5}$ cords of wood in $\frac{7}{8}$ of a day. How many cords a day could he haul at that rate?

57. If a canal boat averages $2\frac{1}{2}$ miles an hour, how long will it be in running $8\frac{1}{3}$ miles?

58. If a man can cut $1\frac{1}{8}$ cords of wood in a day, in how many days can he cut $4\frac{1}{4}$ cords? $6\frac{1}{2}$ cords? 10 cords?

59. How many weeks will $8\frac{1}{4}$ bushels of oats last a horse that eats $2\frac{1}{2}$ bushels a week? How long will $12\frac{1}{16}$ bushels last?

60. At $\$3\frac{1}{4}$ a pair, how many pairs of shoes will $\$19\frac{1}{2}$ buy? How many pairs can be bought for $\$22\frac{3}{4}$? For $\$32\frac{1}{2}$?

61. A farmer cut $12\frac{1}{2}$ tons of hay from a field that yielded $1\frac{3}{4}$ tons to the acre. How many acres were in the field?

62. If $6\frac{3}{4}$ bushels of wheat cost $\$10\frac{4}{5}$, what is the price of 1 bushel? If $7\frac{1}{2}$ bushels cost $\$11\frac{1}{4}$?

63. If the quotient is 7, the remainder is $1\frac{4}{5}$, and the divisor is $6\frac{1}{2}$, what is the dividend? If the divisor is $7\frac{2}{3}$?

64. If the dividend is $40\frac{3}{5}$, the quotient 7, and the remainder $5\frac{1}{2}$, what is the divisor? If the quotient is 8?

65. A newsboy bought 10 papers at $3\frac{1}{2}$ cents each, and sold them for 48 cents. What did he gain on each?