Modern Carpentry and Joinery

Vol. II

ADVANCED SERIES

BEING A COMPILATION

OF

THE VERY BEST THINGS AND MOST MODERN AND PRACTICAL METHODS KNOWN IN THE ARTS OF CARPENTRY AND JOINERY

PREPARED AND EDITED BY

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FRED T. HODGSON

PREFACE

In a previous volume—Modern Carpentry—I made a fairly successful attempt to put together a series of plain and simple examples of working problems and their solutions suited to the capacity of all ordinary mechanics, and owing to the fact that many thousands of that volume have found their way into the hands of carpenters and joiners in the United States, Mexico and Canada, and to the other fact that the author, as well as the publishers, have received hundreds of letters asking for something more on the same lines, of a higher grade, it has been determined to make another or more volumes, on the same subject, hence "Modern Carpentry and Joinery" advanced course.

In the present work I have drawn largely from acknowledged authorities and from workmen of recognized ability to which have been added the results of my own experience and observation and my knowledge of the kinks and secrets of the woodworking trades. In a work of this kind the reader must expect to find something he has met in other places, and perhaps in other adaptations, but I think that upon careful analysis he will find the presentation of the cases somewhat improved, and rendered in a more simple and understandable manner. The selections, too, I think will be found more suitable and more appropriate to the present-day practice than most of the matter found in recent technical literature on the subject. At any rate, it has been my endeavor in the formation of the present volume to place in a handy form, instructive examples of the better class of work of the carpenters and joiners' "Art."

It is not intended to repeat what has already been published in the first volume of Modern Carpentry, unless such repetition will be necessary to explain or formulate some similar matter.

Young men are apt to think that because they have a fair knowledge of their trades, they know all that is required and if they peruse the first volume of this work, and have mastered its contents, they have reached the limit. This, of course, is a great mistake, as will readily be discovered by a glance over the matter of the present volume, and I can say right here, and now, that even the present volume does not by any means cover the whole subject, for, at least a half dozen other volumes could be written without touching on the other two already in the market, and the subject would not nearly be exhausted.

It is not necessary for me to quote the authorities from whom I have drawn, unless the matter is of such importance as to demand special recognition. I may say, however, that in very few books written during the last 25 years on the subject of carpentry, there has been but very little presented that had not been published before in some form or other, but the descriptions generally, in most cases—not in all—have been improved more or less: and the present book does not differ a great deal from most others that have preceded it, only that it is more up-to-date, and more suited to 20th century requirements.

It must also be understood, that while this book goes out to the public under my name as author, I do not claim authorship, for really, it is more of a compilation than an original work, but, I do claim that the selections and compilations made are better and more suited to the wants of the present-day workman than can be found in any other similar work published in this or any other country.

FRED T. HODGSON,
Editor and Compiler.

Collingwood, July 1, 1906.

PART I.

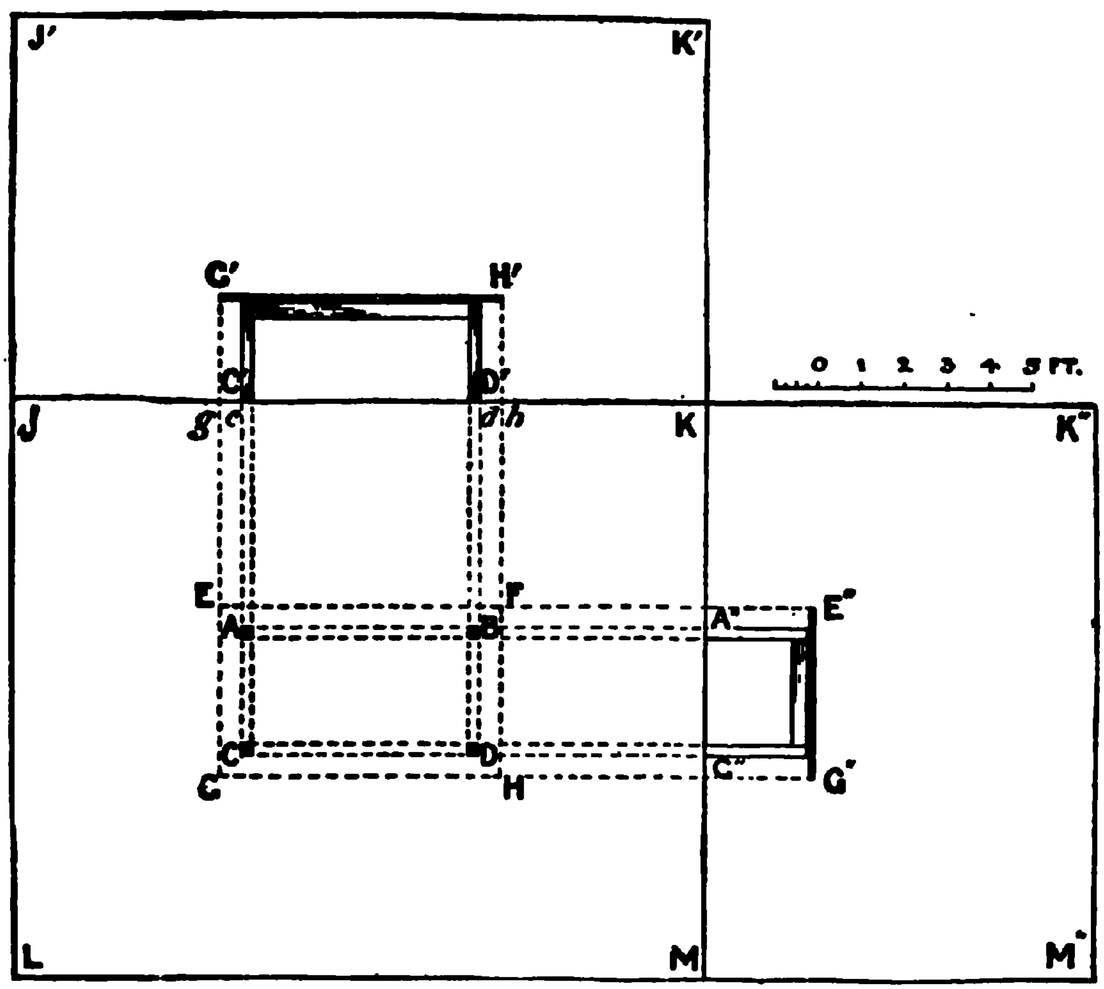
SOLID GEOMETRY.

INTRODUCTORY.

In the first volume of Modern Carpentry I gave a short treatise on plain or Carpenters' Geometry, which I trust the student has mastered, and thus prepared himself for a higher grade in the same science, namely, **solid Geometry**: and to this end, the following treatise has been selected, as being the most simple and the most through available.

Solid geometry is that branch of geometry which treats of solids—i. e., objects of three dimensions (length, breadth and thickness). By means of solid geometry these objects can be represented on a plane surface, such as a sheet of paper, in such a manner that the dimensions of the object can be accurately measured from the drawing by means of a rule or scale. The "geometrical" drawings supplied by the architect or engineer for the builder's use are, with few exceptions, problems in solid geometry, and therefore a certain amount of knowledge of the subject is indispensable, not only to the draughtsman who prepares the drawings, but also to the builder or workman who has to interpret them.

As the geometrical representations of objects consist entirely of lines and points, it follows that if projections of lines and points can be accurately drawn, the representation of objects will present no further difficulty. A study of lines and points, however, is somewhat confusing, unless the theory of projection has first been grasped, and for this reason the subject will be introduced by a simple concrete example, such, as a



Vertical Projections (or Elevations) and Horizontal Projection (or Plan) of a Table in the Middle of an Oblong Room

Fig. 1.

table standing in the middle of a room. The four legs rest on the floor at A, B, C, and D (Fig. 1), and the perpendiculars let fall from the corners of the table-top to meet the floor at E, F, G, and H. The oblong E F G H represents the horizontal projection or "plan" of the table-top; the small squares at A, B, C, and D represent the plan of the four legs, and the lines correcting them represent the plan of the frame work under the top. The large oblong J K L M is a plan of the room.

The plan or horizontal projection of an object is therefore a representation of its horizontal dimensions —in other words, it is the appearance which an object presents when every point in it is viewed from a position vertically above that point.

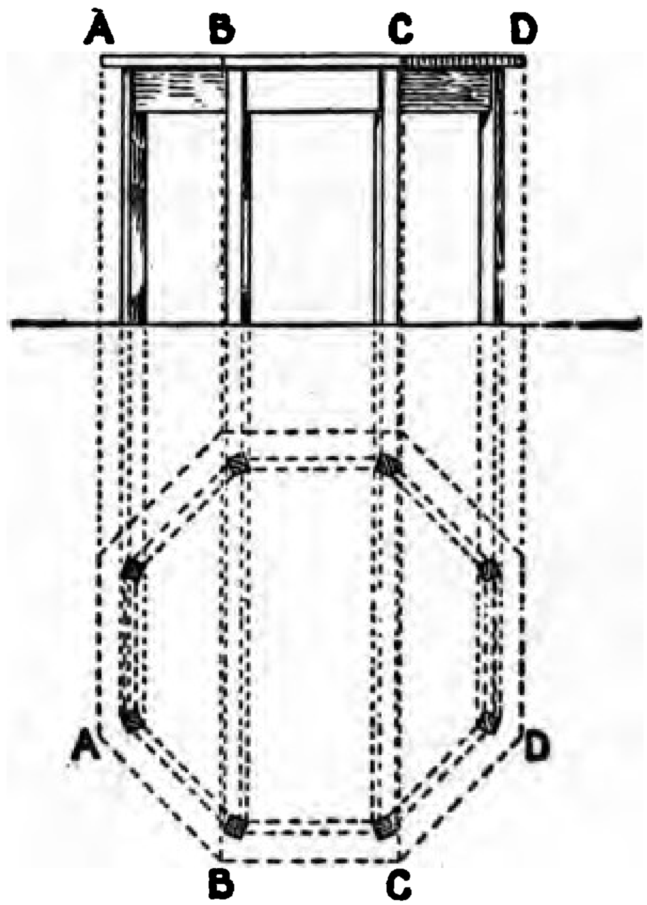
In a similar manner an elevation or vertical projection of an object is a representation of its vertical dimensions, and also, it should be added, of some of its horizontal dimensions,—i. e., it is the appearance which an object presents when every point in it is viewed from a position exactly level with that point, all the lines of sight being parallel both horizontally and vertically. Thus, the front elevation of the table (or the vertical projection of the side G H) will be as shown at G′ H′ C′ D′, and the vertical projection of the side J K of the room will be J K J K. The vertical projection of the end of E G of the table will be as shown E″ G″ A″ C″ and of the end K M of the room will be K″ M″ K M; the drawing must be turned until the line K M is horizontal, for these and projections to be properly seen.

By applying the scale to the plan, we find that the length of the table-top is six and a half feet, the breadth 4 feet, and the distance of the table from each wall 4¾ feet. From the **front elevation** we can learn the height of the table, and also give its length and distance from the end walls of the room. From the **end elevation** we can ascertain the breadth of the table and its distance from the sides of the room, as well as its height.

To make the drawing clearer, let us imagine that the walls of the room are of wood and hinged at the level of the floor. On the wall J K draw the front elevation of the table and then turn the wall back on its hinges until it is horizontal,—i. e., in the same plane as the floor. Proceed in a similar manner with the end K M, and we get the three projections of the room and table on one plane, as shown in the diagram. To avoid confusion the end elevation will not be further considered at the present.

It will be seen that the line J K represents the angle formed by the wall and floor,—in other words, it represents the intersection of the vertical and horizontal "planes of projection," it is known as "the line of intersection," or "the ground line." If a line is let fall from G′ perpendicular to J K, the two lines will meet at G, and they will be in the same straight line. Similarly, the perpendiculars H′ h and h H are in the same straight line. Lines of this kind perpendicular to the planes of projections are known as "projectors," and are either horizontal or vertical G′ g and H′ h are vertical projectors; G g, C c, D d, and H h are horizontal projectors.

Vertical projectors are not always parallel to one of the sides of the object represented, or, if parallel to one side, are not parallel to other sides which must be represented; thus, a vertical projector or "elevation" of an octagonal object, if parallel to one of the sides of the octagon, must be oblique to the two adjacent sides. In Fig. 2 an octagonal table is shown. The plan must first be drawn, and from the principal points of the plan projectors must be drawn perpendicular to the vertical plane of the projection, until they cut the ground line, and from this perpendiculars must be erected to the height of the several parts of the table. The elevation



Plan and Elevation of an Octagonal Table.
Fig. 2.

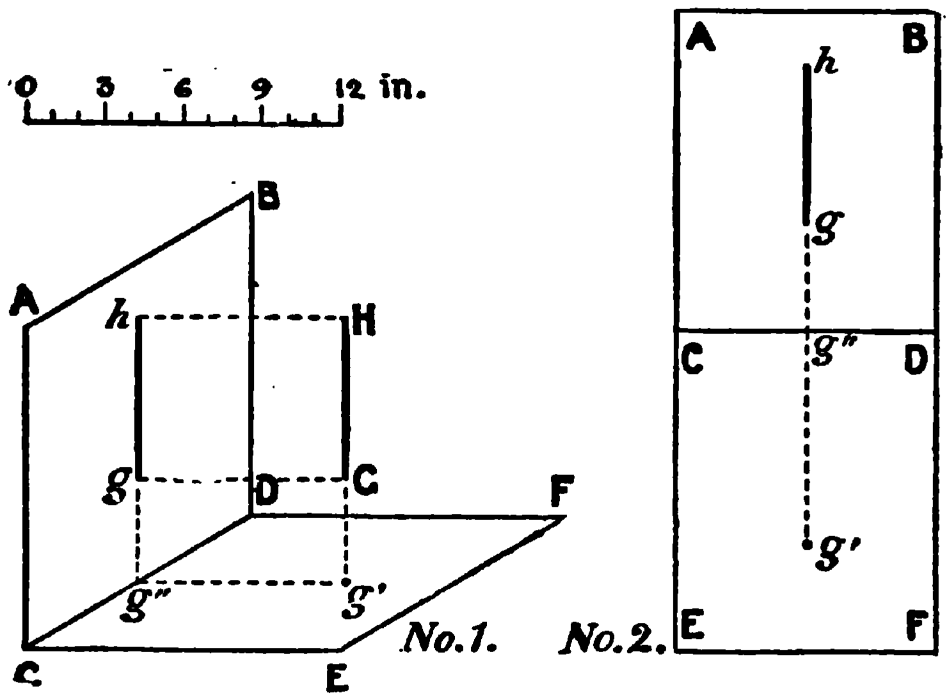
can then be completed without difficulty. The side B C of the table is parallel to the vertical plane of projection, but the adjacent sides A B and C D are oblique.*

*A distinction must be made between "perpendicular" and "vertical." The former means, in geometry, a line or plane at right angles to another line or plane, whether these are horizontal, vertical or inclined; whereas a vertical line or plane is always at right angles to a horizontal line or plane. The spirit-level gives the horizontal line or plane, the plumb-rule gives the vertical.

2. POINTS, LINES, AND PLANES.

1. To determine the position and length of a given straight line, parallel to one of the planes of the projection.

Let G H (Fig. 3) be the given straight line. To determine its position (i. e., in regard to horizontal and vertical planes), it is only necessary to determine the position of any two of the extreme points.

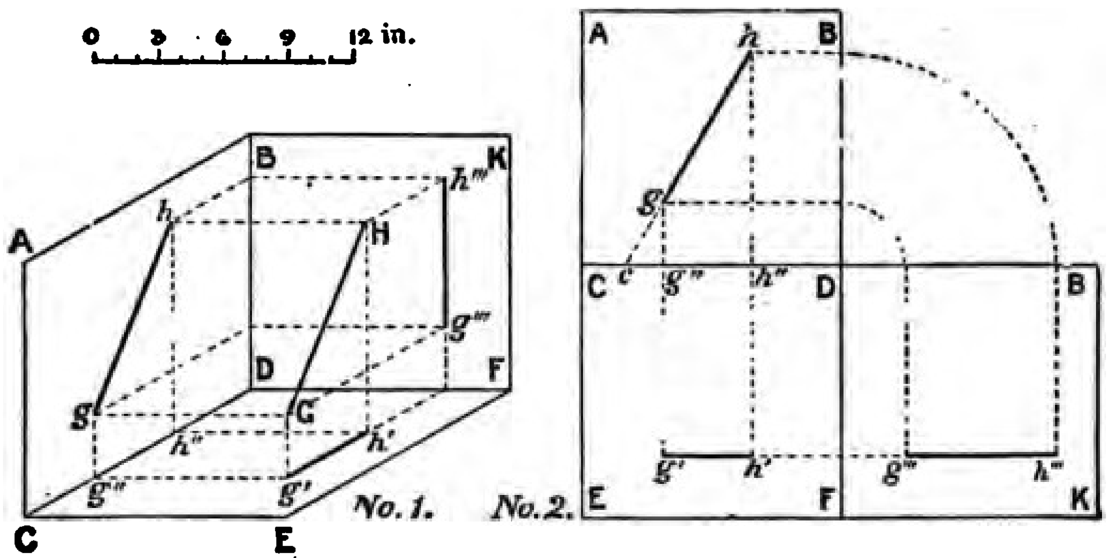


·No. 1. Perspective View. No. 2. Geometrical Projections Horizontal and Vertical

Fig. 3.

Let A B C D be a vertical plane parallel to the given line, and C D E F a horizontal plane. The vertical projection or elevation of the line is represented by the line h g, and the horizontal projection or plan by the point g′, the various projection being shown by dotted lines. The given line is proved to be vertical, because its horizontal projection is a point; its length as measured by the scale, is 6 inches; its height (g g″) above the line of intersection C D is 4 inches; and its horizontal distance (g′ g″) from the same line is 8 inches.

If the illustrations are turned so that C D E F become a vertical plane, and A B C D the horizontal plane then G H will be horizontal line, because one of its vertical projections is a point. Other vertical projections of the line can be made,—as, for example, a side elevation,—in which the projection will appear as a line and not a point, but a line must be horizontal if **any** vertical projection of it is a point.



-No. 1. Perspective View. No. 2. Vertical and Horizontal Projections

Fig. 4.

Let the given line G H (Fig. 4) be parallel to the vertical plane, but inclined to the horizontal plane. Then g h will be a vertical projection, and g′ h′ its horizontal projection or plan. By producing h g till it cuts C D at c, it will be found that the given line is inclined at an angle of 60° to the horizontal plane; its length, as measured by the scale along the vertical projection g h, is 8 inches; the height of G above the horizontal plane (measured at g g″) is 3 inches, and the height of H (measured at h h″) is 9¾ inches.

II. To determine the length of a given straight line, which is oblique to both planes of projection.

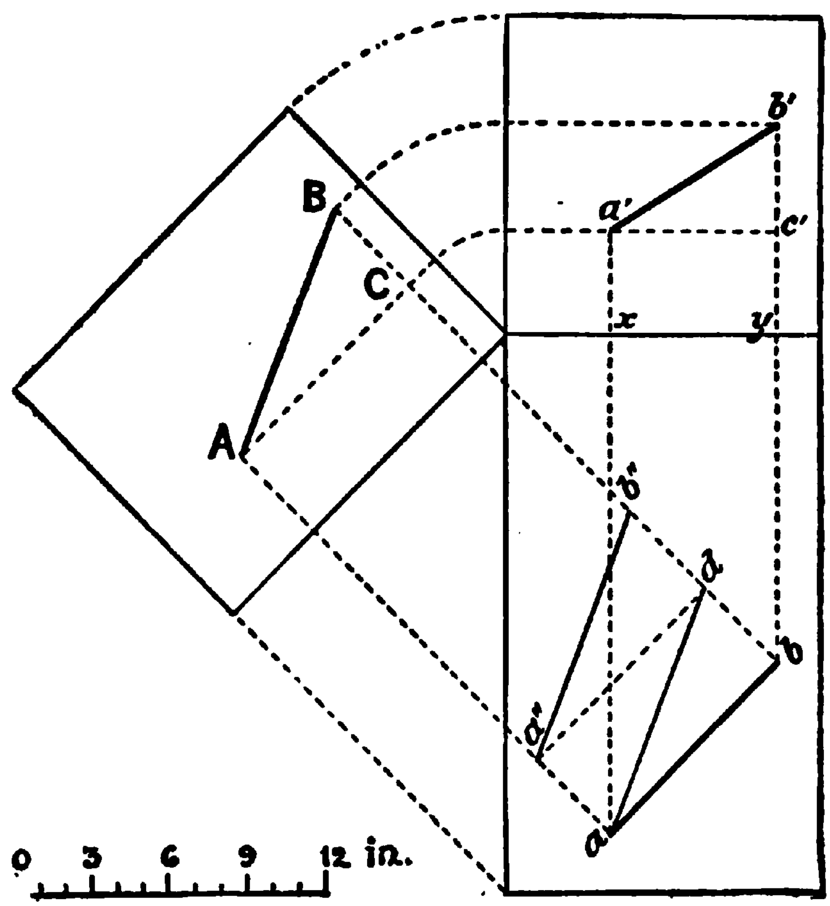
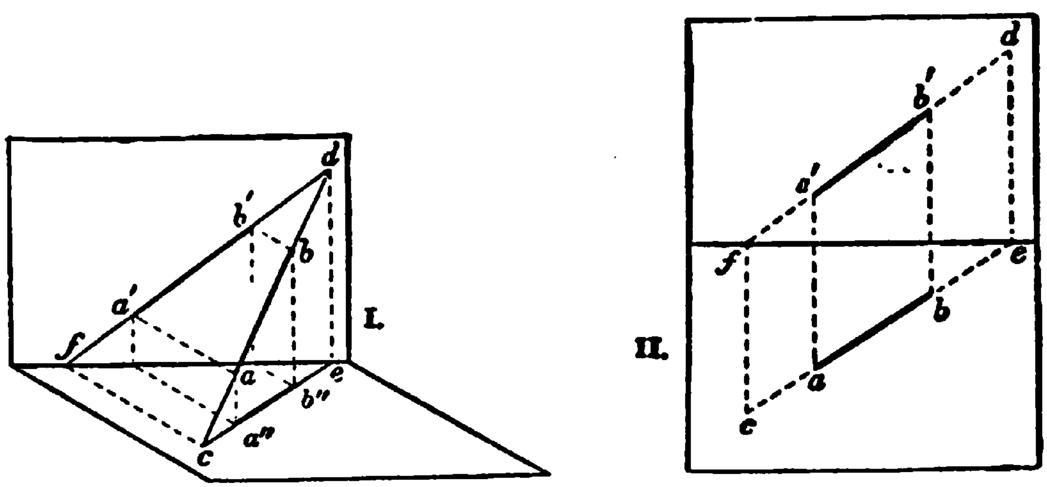


Fig. 5.

Let a b (Fig. 5) be the horizontal projection and a′ b′ the given vertical projection of the given line. Draw the projection A B on a plane parallel to a b in the manner shown. From this projection the length of the given line will be found (by applying the scale) to be 10 inches. The height C B is of course equal to the height c′ b′, and the horizontal measurement A C is equal to the horizontal measurement A C is equal to the horizontal projection a b, and the angle A C B is a right angle. It follows therefore that, if from b the line b d is drawn perpendicular to a b and equal to the height c′ b′, the line joining a d will be the length of the given line. To avoid drawing the horizontal line a′ c′ the height of a′ and b′ above x y are usually set up from a and b as at a″ and b″, the line a″ b″ is the length of the given line.*

III. The projections of a right line being given, to find the points wherein the prolongation of that line would meet the planes of projection.



-I. Perspective View. II. Vertical and Horizontal Projections

Fig. 6.

Let a b and a′ b′ (Fig. 6, II) be the given projection of the line A B. In the perspective representation of the problem it is seen that A B, if prolonged, cuts the horizontal plane in c, and the vertical plane in d, and the projections of the prolongation becomes c e and f d. Hence, if a b, a′ b′ (Fig. 6, II) be the projections

*The application of this problem to hips is obvious. Suppose that *a b* is the plan of a hip-rafter, and *a′ b′* an elevation, the length of the rafter will be equal to *a d* or A B.

of A B, the solution of the problem is obtained by producing these lines to meet the common intersection of the planes in f and e, and on these points raising the perpendiculars f c and e d, meeting a b produced in c and a′ b′ produced in d; c and d are the points sought.*

IV. If two lines intersect each other in space, to find from their given projections the angles which they make with each other.

Let a b, c d, and a′ b′ c′ d′ (Fig. 7) be the projections of the lines. Draw the projectors e′ f′ f′ e′, perpendicular to the line of intersection a′ c′, and produce it indefinitely towards E″; from e draw indefinitely, e E′ perpendicular to the line e′ f, and make e E′ equal to f″ e, and draw f E′. From f as a centre describe the arc E″ g E″, meeting e f produced in E″, and join a E″, c E″. The angle a E″ c is the angle sought. This problem is little more than a development of problem II. If we consider e f, e′ f′, as horizontal and vertical projections of an imaginary line lying in the same plane as a e and c e, we find the length of this line by problem II to be f E′; in other words f E′ is the true altitude of the triangle a e c, a′ e′ c′. Construct a triangle on the base a e with an altitude f E′ equal to f E′, and the problem is solved. The practical application of this problem will be understood if we imagine a e c to be the plan and a′ e′ c′ the elevation of a hipped roof; f E′ gives us the

*The points *d* and *c* are known respectively as the vertical and horizontal "traces" of the line A B, the trace of a straight line on a plane being the point in which the straight line, produced if necessary, meets or intersects the plane. A horizontal line cannot therefore have a horizontal trace, as it cannot possibly, even if produced, meet or intersect the horizontal plane; for a similar reason, a vertical line cannot have a vertical trace.

length and slope of the longest common rafter or spar, and a E″ C is a true representation of the whole hip, i. e., on a plane parallel to the slope of the top. It will be observed that the two projections (e and e′) of the

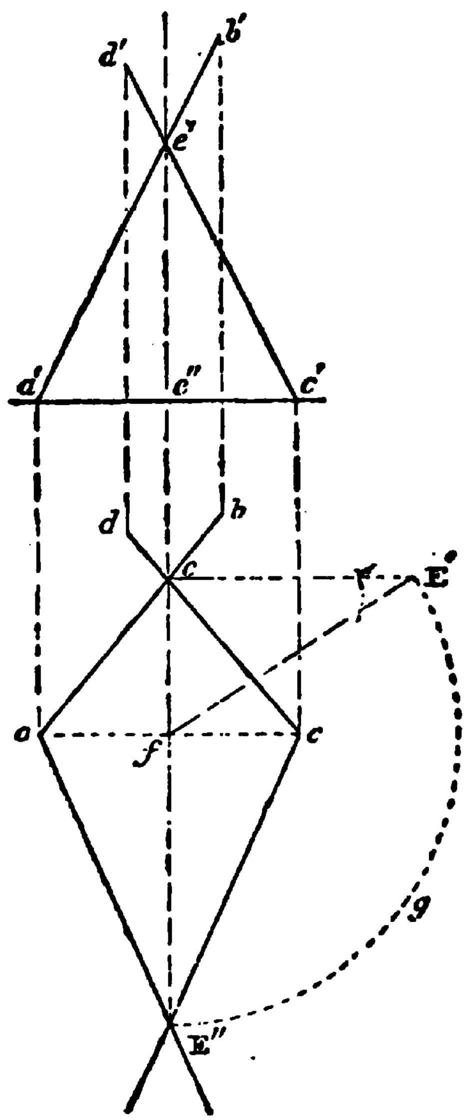
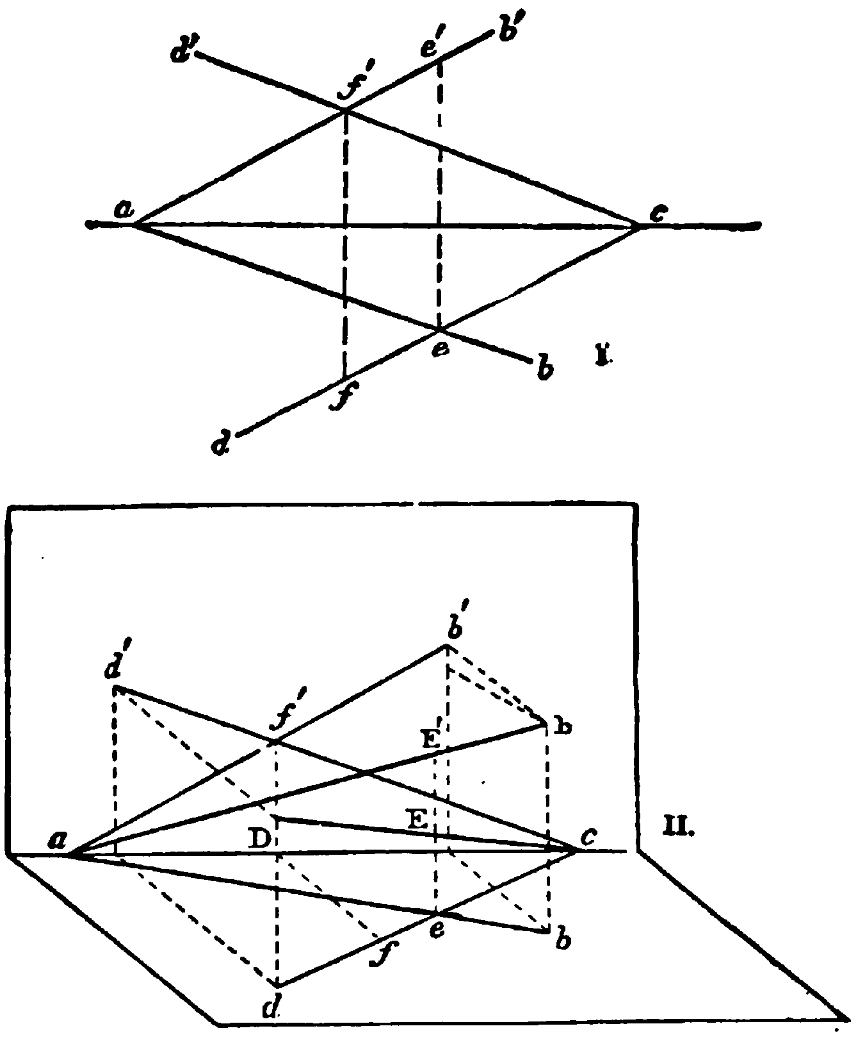


Fig. 7.

point of intersection of the two lines are in a right line perpendicular to the line of intersection of the plains of projection. Hence this corollary.—**The projections of the point of intersections of two lines which cut**

each other in space are in the same right line perpendicular to the common intersection of the planes of projection. This is further illustrated by the next problem.



·I. Vertical and Horizontal Projections. II. Perspective View

Fig. 8.

V. To determine from the projection of two lines which intersect each other in the projections, whether the lines cut each other in space or not.

Let a b, c d, a b′, c d′ (Fig. 8, I) be the projections of the lines. It might be supposed, as their projections intersect each other that the lines themselves intersect each other in space, but on applying the corollary of the preceding problem, it is found that the intersections are not in the same perpendicular to the line of intersection a c of the planes of projection. This is represented in perspective in Fig. 8, II. We there see that the original lines a B c D do not cut each other, although their projections a b, c d, a b′, c d′, do so. From the point of intersection e raise a perpendicular to the horizontal plane, and it will cut the original line c D in E, and this point therefore belongs to the line c D, but c belongs equally to a B. As the perpendicular raised on e passes through E on the line c D, and through E on the line a B, these points E E′ cannot be the intersection of the two lines, since they do not touch; and it is also the same in regard to f f. Hence, when two right lines do not cut each other in space, the intersections of their projections are not in the same right line perpendicular to the common intersection of the planes of projection.

VI. To find the angle made by a plane with the horizontal plane of projection.

Let a b and a c, Fig. 9, be the horizontal and vertical traces of the given plane, i. e., the lines on which the given plane would, if produced, cut the horizontal and vertical planes of projection. Take any convenient point d in a b, and from it draw d e perpendicular to a b, and cutting the line of intersection x y in e, from e draw e d perpendicular to x y and cutting a c in d′. The angle made by the given plane with the horizontal plane of projection is such that, with a base d e, it has a vertical height e d′. Draw such an angle on the vertical plane of projection by setting off from e the distance e d″ equal to e d, and joining d′ d″. The angle d′ d″ e is the angle required.*

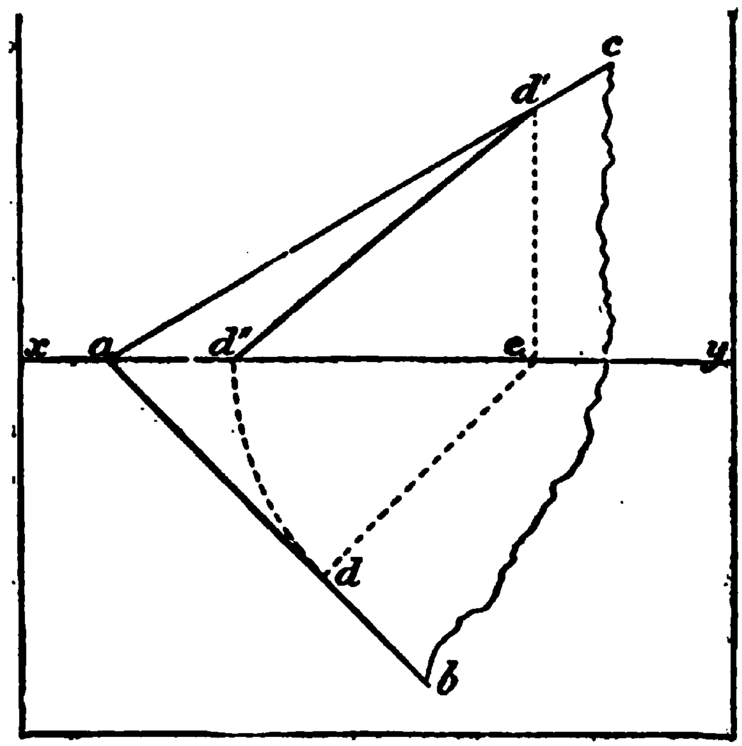


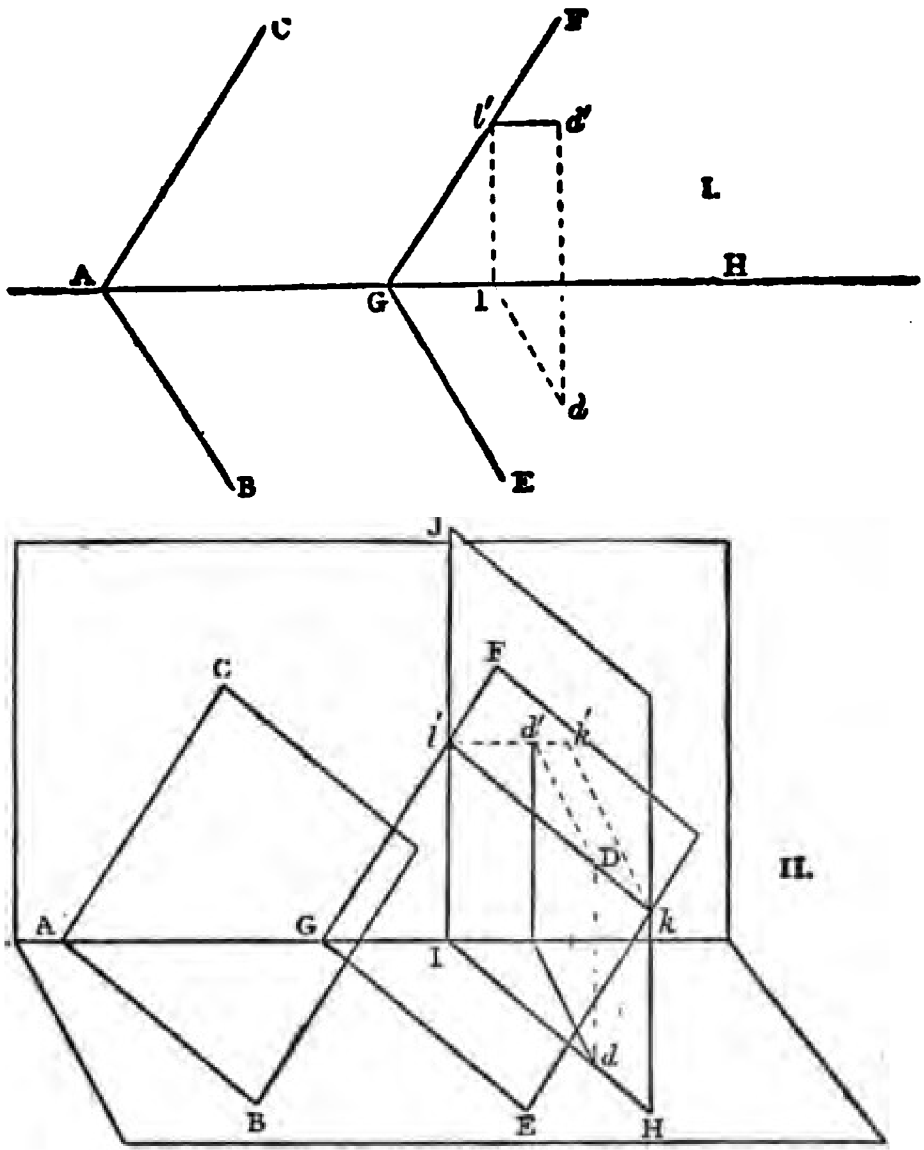
Fig. 9.

VII. The traces of a plane and the projections of a point being given, to draw through the point a plane parallel to the given plane.

In the prospective representation (Figs. 10 and 11) suppose the problem solved, and let B C be the given

*The angle made by a plane with the vertical plane of projection can be found in a similar manner. If we imagine the part above *x y* in fig. nine to be the horizontal projection and the part below *x y* to be the vertical projection—in other words, if fig nine is turned upside down—*a b* becomes the vertical trace and *a c* the horizontal trace, and the angle *d′ d″ e* is the angle made by the given plane with the vertical plane of projection.

plane, and A C, A B its vertical and horizontal traces, and E F a plane parallel to the given plane, and G F, G E its traces. Through any point D, taken at pleasure



I. Vertical and Horizontal Projections. II. Perspective View

Fig. 10.

on the plane E F, draw the vertical plane H J, the horizontal trace of which, I H, is parallel to G E. The plane H J cuts the plane E F in the line k l′, and its vertical trace G F in l′. The horizontal projection of k′ l′ is H I, and its vertical projection k′ l′; and as the point D is in k l′, its horizontal and vertical projections will be d and d′. Therefore, if through d be traced a line d l, parallel to A B, that line will be the horizontal projection of a vertical plane passing through the original point D; and if an l be drawn the indefinite perpendicular L″ l′, and through d′, the vertical projection of the given point, be drawn the horizontal line d′ l′, cutting the perpendicular in l′, then the line F G drawn through l parallel to A C, will be the vertical trace of the plane required; and the line G E drawn parallel

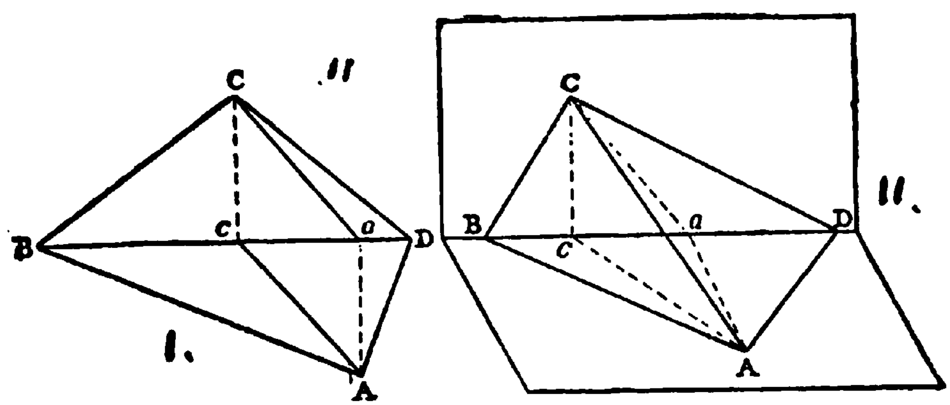


Fig. 11.

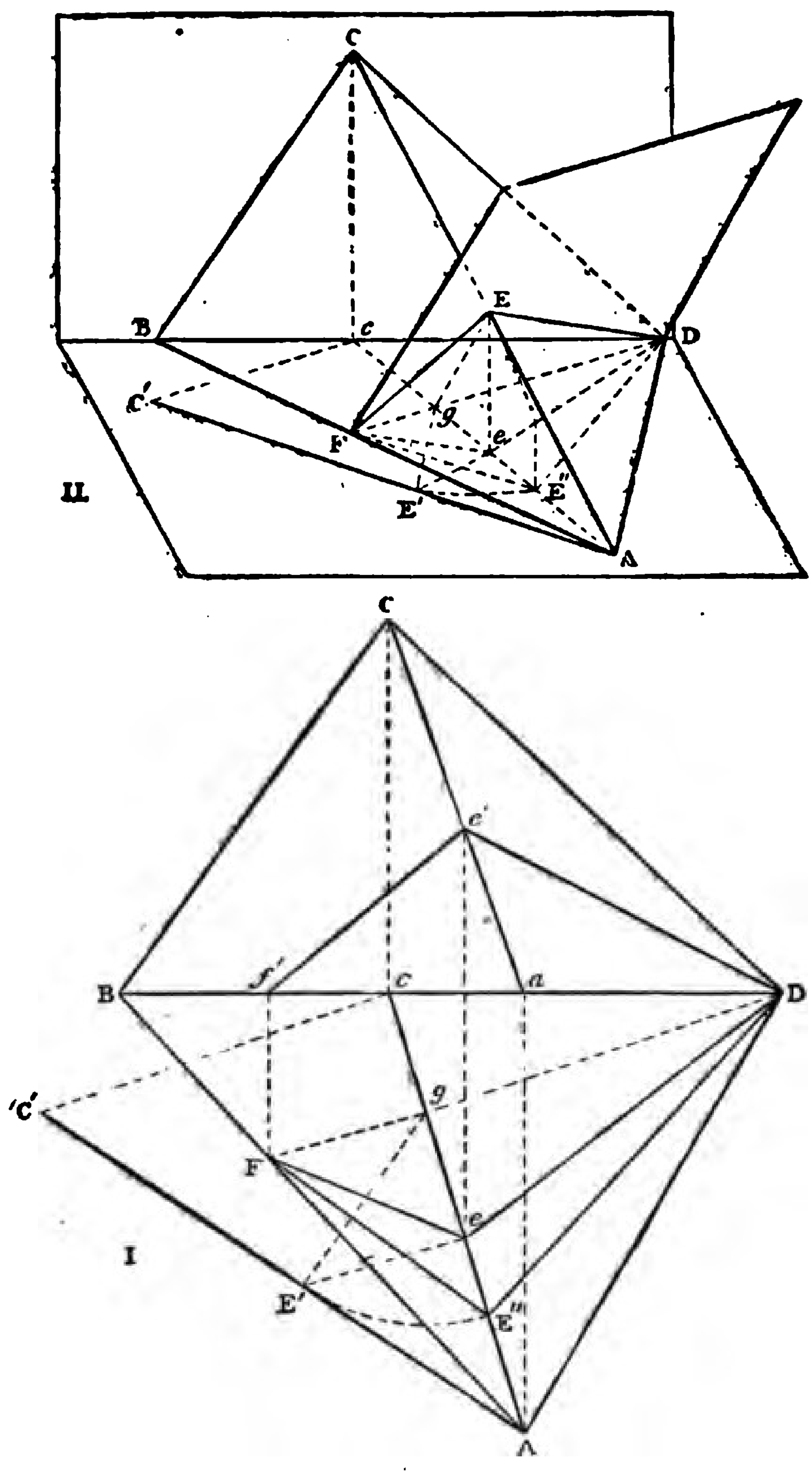
to A B, its horizontal trace. Hence, **all planes parallel to each other have their projections parallel, and reciprocally.** In solving the problem, let A B, A C (Figs. 10, I) be the traces of the given plane, and d d the projections of the given point. Through d draw d l parallel to A B, and from I draw I l′ perpendicular to A H. Join d d′ and through d′ draw d′ l′ parallel to the line of intersection A H. Then F l′ G drawn parallel to A C, and G E parallel to A B, are the traces of the required plane.

VII. The traces A B, B C, and A D, D C, of two planes which cut each other being given, to find the projections of their intersections.

The planes intersect each other in the straight line A C (Fig. II, 11), of which the points A and C are the traces, since in these points this line intersects the planes of projection. To find these projections, it is only necessary to let fall on the line of intersection in Fig. II, 1, the perpendiculars A a, C c, from the points A and C, and join A c, C a. A c will be the horizontal projection, and C a the vertical projection of C A, which is the line of intersection or arris of the planes.

XI. The traces of two intersecting planes being given, to find the angle which the planes make between them.

The angle formed by two planes is measured by that of two lines drawn from the same point in their intersection (one along each of the planes), perpendicular to the line formed by the intersection. This will be better understood by drawing a straight line across the crease in a double sheet of note-paper at right angles to the crease; if the two leaves of the paper are then partly closed so as to form an angle, we have an angle formed by two planes, and this angle is the same as that formed by two lines which have been drawn perpendicular to the line of intersection of the two planes. These lines in effect determine a third plane perpendicular to the arris. If, therefore, the two planes are cut by a third plane at right angles to their intersection the solution of the problem is obtained.

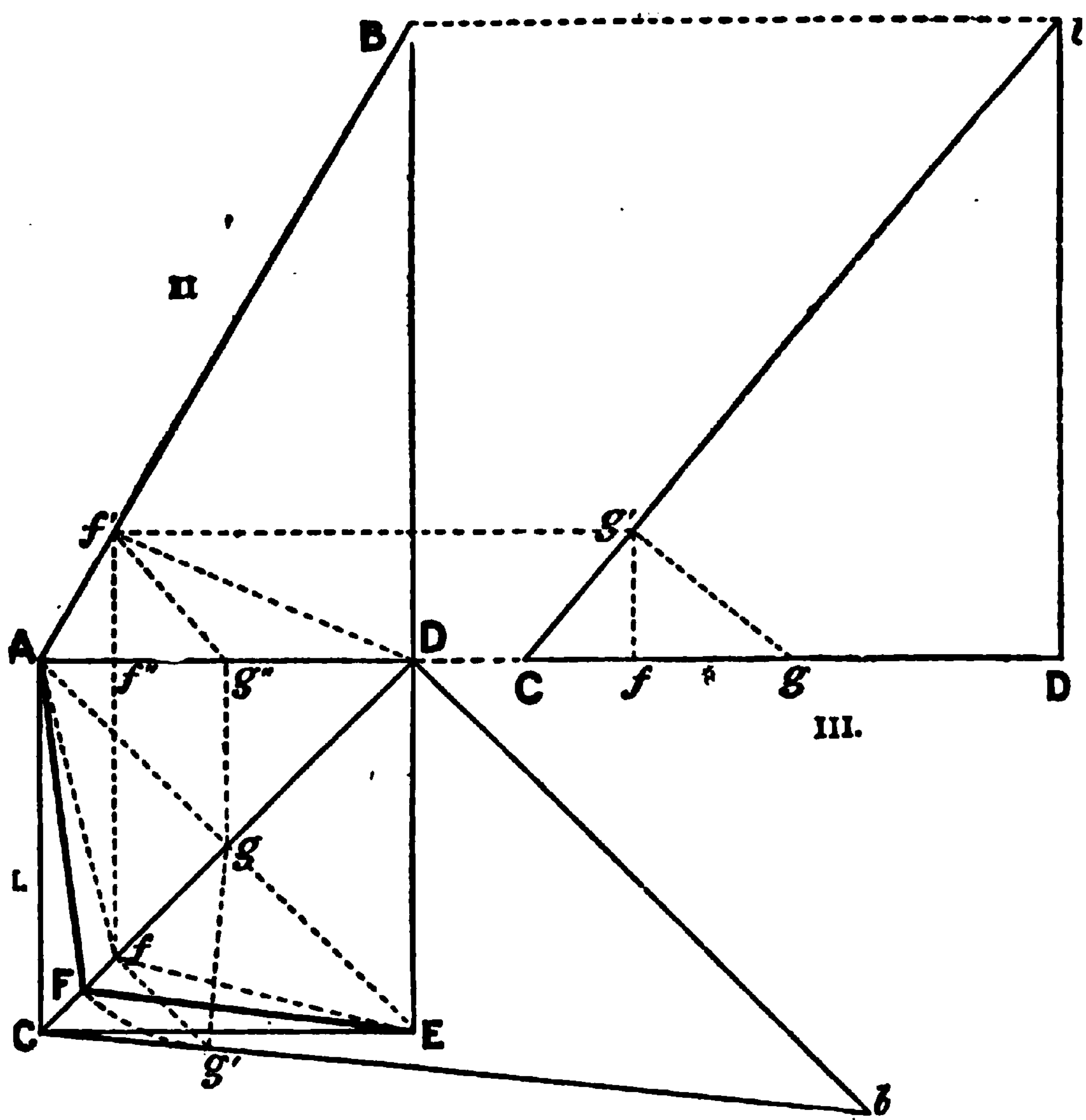


I. Vertical and Horizontal Projections. II. Perspective View

Fig. 12.

On the arris A C (Fig. 12, 11) take at pleasure any point E, and suppose a plane passing through that point cutting the two given planes perpendicular to the arris. There results from the section a triangle D E F, inclined to the horizontal plane, and the angle of which, D E F, is the measure of the inclination of the two planes. The horizontal projection of that triangle is the triangle D e F, the base of which, F D, is perpendicular to A c (the horizontal projection of the arris A C), and cuts it in the point g, and the line E g is perpendicular to D F. The line g E is necessary perpendicular to the arris A C, as it is in the plane D E F, and its horizontal projection is g e. Now, suppose the triangle D E F turned on D F as an axis, and laid horizontally, its summit will then be at E″, and D E″ F is the angle sought. The perpendicular g E is also in the vertical triangle A C c, of which the arris is the hypothenuse and the sides A c, C C are the projections. This description introduces the solution of the problem.

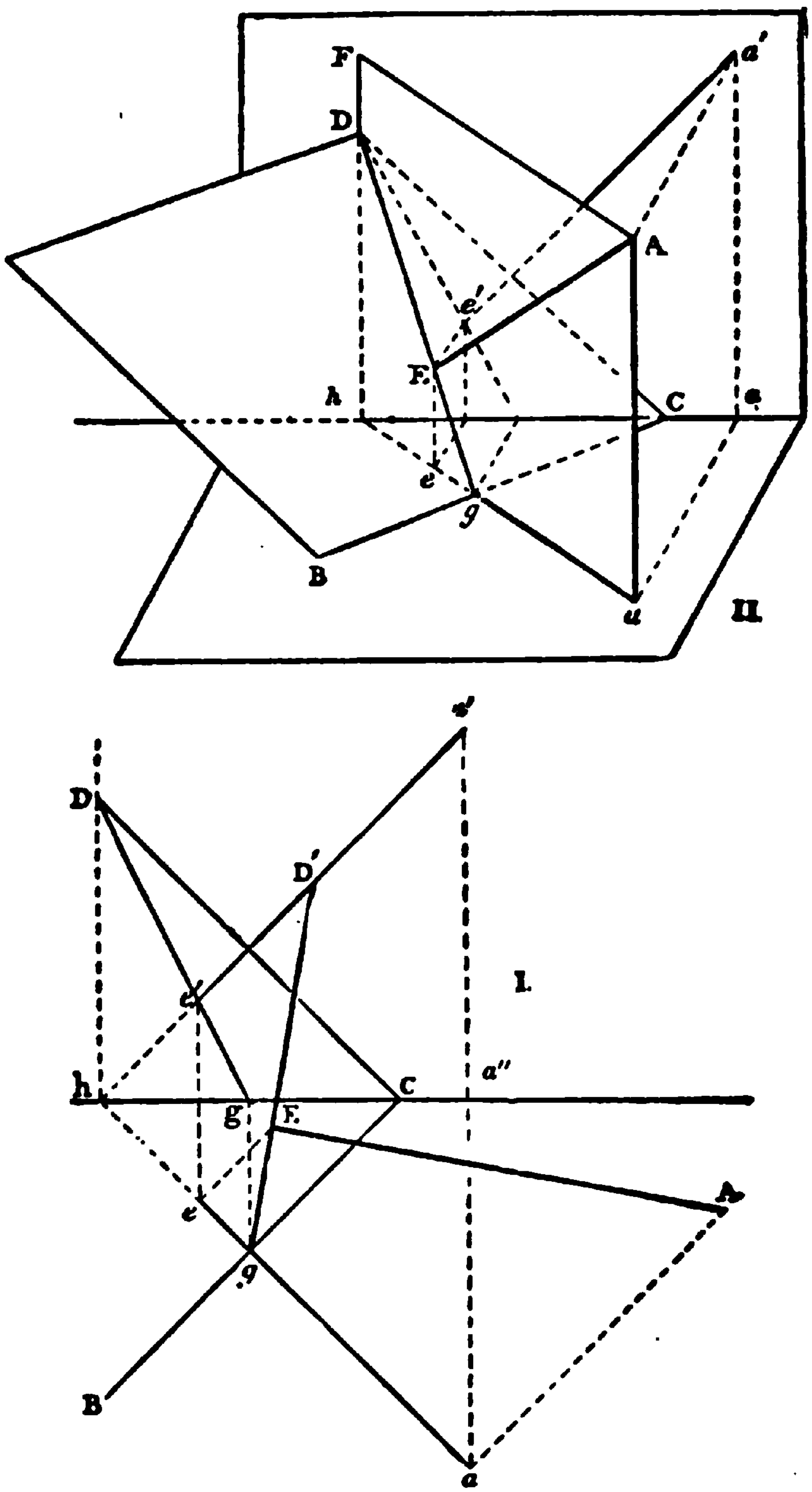
Through any point g (Fig. 12, 1) on the line A c, the horizontal projection of the arris or line of intersection of the two planes, draw F D perpendicular to A c; on A c (which for the moment must be considered as a "line of intersection" or "ground line" for a second vertical projection) describe the vertical projection of the arris by drawing the perpendicular c C, and then joining A C′. A C gives the true length and inclination of the arris. From g draw g E′ perpendicular to A C′, and meeting it in E′; from E′ let fall a perpendicular E e′ or A c, meeting A c in e. F e D is the horizontal projection of the triangle F E D (see No. 11) and from this the vertical projection f′ e′ D can be drawn as shown. We have now obtained the vertical and horizontal projections of two intersecting lines, namely, F e, e D, and f′ e′, e′ D, and by problem (4) the triangle which they make with each other can be found. It will be seen that g E′ is the true altitude of the triangle f′ e′ D, as c E is equal to e′ e. Set off therefore from g



I. Horizontal Projection. II. Vertical Projection on Plane A D.
III. Vertical Projection on Plane C D.

Fig. 13.

towards A a distance g E equal to g E, and join F E, E″ D; F E″ D is the angle made by the two intersecting planes.



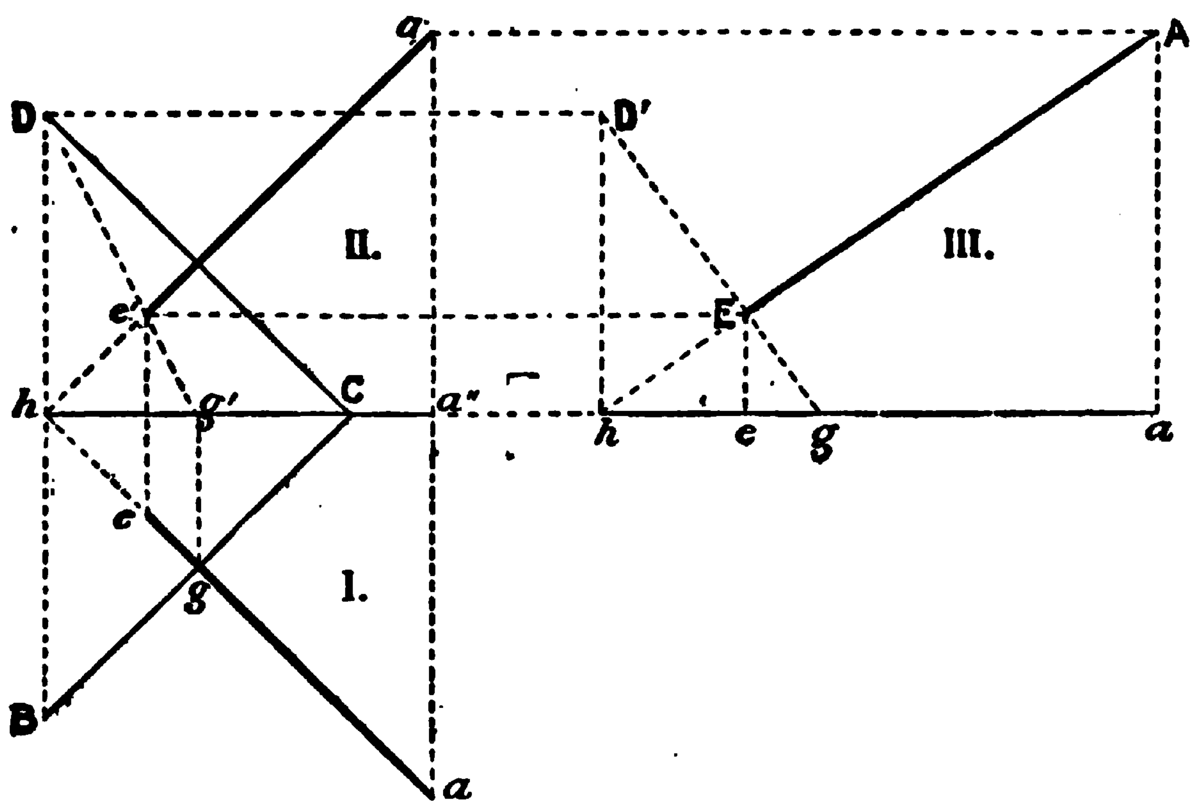
I. Vertical and Horizontal Projections. II. Perspective View

Fig. 14.

X. Through a given point to draw a perpendicular to a given plane.

Let a and a′ (Fig. 14, II.) be the projections of the given point, and B C, C D the horizontal and vertical traces of the given plane. Suppose the problem solved, and that A E is the perpendicular drawn through the A to the plane B D, and that its intersection with the plane is the point E. Suppose also a **vertical** plane a F to pass through A E′, this plane would cut B D in the line g D, and its horizontal trace a h would be perpendicular to the trace B C. In the same way a′ e′, the vertical projection of A E, would be perpendicular to C D, the vertical trace of the plane B D. Thus we find that if a line a h is drawn from a, perpendicular to B C, it will be the horizontal trace of the plane in which lies the required perpendicular A E, and h F will be the vertical trace of the same plane. From a′, draw upon C D an indefinite perpendicular, and that line will contain the vertical projection of A E, as a h contains its horizontal projection. To find the point of intersection of the line A E with the given plane, construct the vertical projection g D of the line of intersection of the two planes, and the point of intersection of that line with the right line drawn through a′, will be the point sought. If from that point a perpendicular is let fall on a h, the point e will be the horizontal projection of the point of intersection E.

In Fig. 15, 1, let B C, C D be the traces of the given plane, and a, a′ the projections of the given point. From the point a, draw a h perpendicular to B C; a h will be the horizontal projection of a plane passing vertically through a, and cutting the given plane. On a h as "ground line" draw a vertical projection as follows: From a, draw a A perpendicular to a g, and make it equal to a″ a′; from h draw h D′ perpendicular to h a, and make h D′ equal to h D; draw g D, which will be the section of the given plane by a vertical D g, and the angle h g D′ will be the



I. Horizontal Projection. II. Vertical Projection on Plane parallel to *h a″*. III. Vertical Projection on Plane parallel to *h a*

Fig. 15.

measure of the inclination of the given plane with the horizontal plane; there is now to be drawn, perpendicular to this line, a line A E through A, which will be the vertical projection on a h of the line required. From the point of intersection E let fall upon a h a perpendicular, which will give e as the horizontal projection of E. Therefore a e is the horizontal projection of the required perpendicular, and a′ e′ its vertical projection on the original "ground line" h a″. It follows from this problem that—**Where a right line in space is perpendicular to a plane, the projections of that line are respectively perpendicular to the traces of the plane.**

XI. Through a given point to draw a plane perpendicular to a given right line.

Let a, a′ (Fig. 16, 1) be the projections of the given point A, and b c, b′ c′ the projections of the given line B C.

The foregoing problem has shown that the traces of the plane sought must be perpendicular to the projections of the line, and the solution of the problem consists in making to pass through A, a vertical plane A f (Fig. 16, 11), the horizontal projection of which will be perpendicular to b c.

Through a (Fig. 16, 1) draw the projection a f perpendicular to b c. From f raise upon K L the indefinite perpendicular f f, which will be the vertical trace of the plane a f f perpendicular to the horizontal plane, and passing through the original point A (in No. 11). Then draw through a in the vertical projection a horizontal line, cutting f f in f, which point should be in the trace of the plane sought; and as that plane must be perpendicular to the vertical projection of the given right line draw through f a perpendicular to b c, and produce it to cut K L in G. This point G is in the horizontal trace of the plane sought. All that remains therefore, is from G to draw

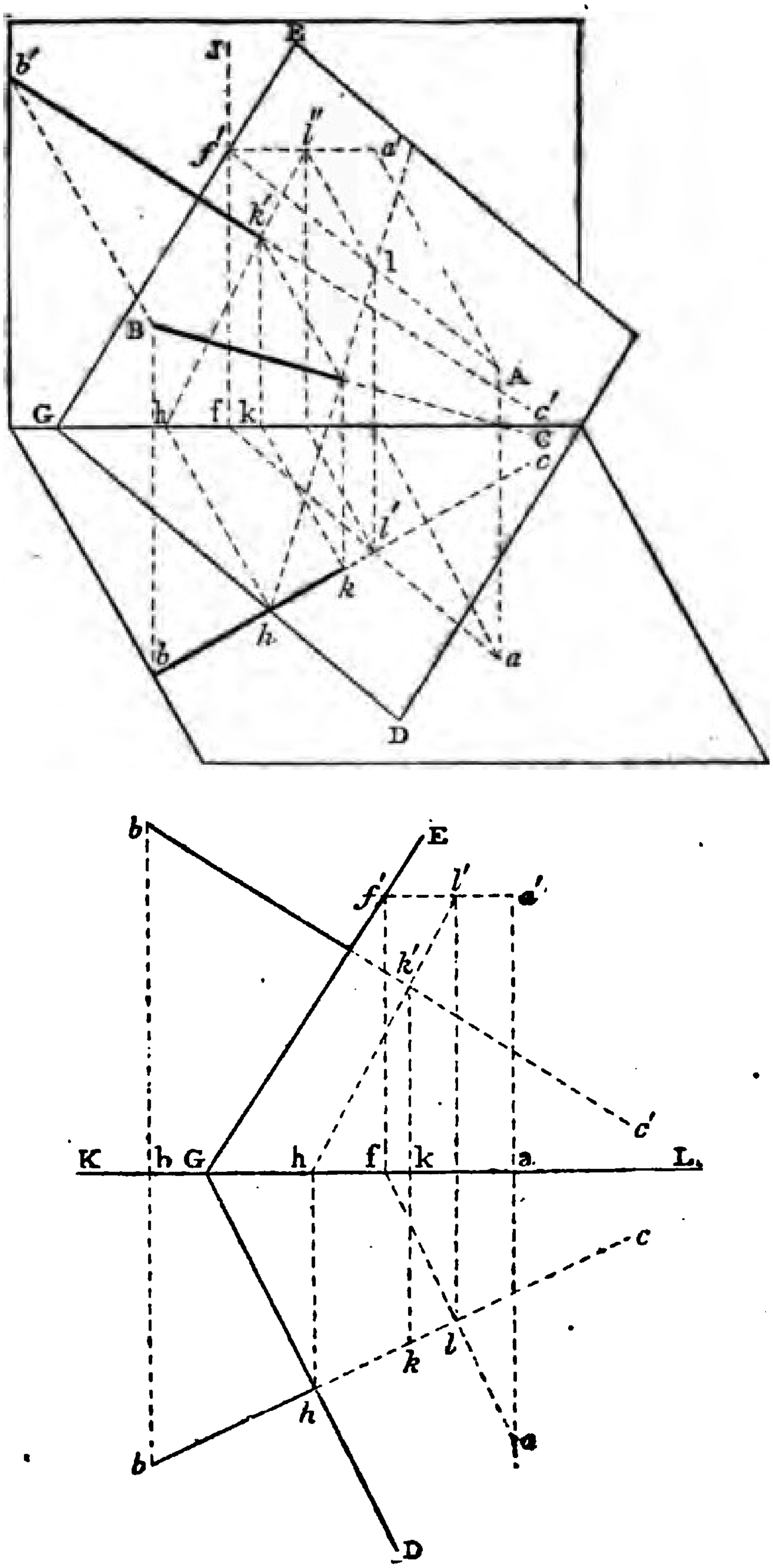


Fig. 16.

G D perpendicular to b c. If the projections of the straight line are required, proceed as in the previous problem, and as shown by the dotted lines; the plane will cut the given line at k in the horizontal projection, and at k in the vertical projection.*

XII. A right line being given in projection, and also the traces of a given plane, to find the angle which the line makes with the plane.

Let A B (Fig. 17, 11) be the original right line intersecting the plane C E in the point B. If a vertical plane a B pass through the right line, it will cut the plane C E in the line f B, and the horizontal plane in the line a b. As the plane a B is in this case parallel to the vertical plane of projection,its projection on that plane will be a quadrilateral figure a b of the same dimensions, and f B contained in the rectangle will have for its vertical projection a right line D b, which will be equal and similar to f B. Hence the two angles a b D, A B f, being equal, will equally be the measure of the angle of inclination of the right

*The diagram will be less confusing if the projection on *a h* is drawn separately, as in Fig. 16, where I is the horizontal projection or plan, II the vertical projection or elevation on a plane parallel to *ha''*, and III the vertical projection or elevation on a plane parallel to *h a*. To draw No. III draw first the ground-line *h a* equal to *h a* on No. I, and mark on it the point *g*; from *h* draw the vertical *h* D' equal to *h* D, and from *a* draw the vertical *a* A equal to *a'' a'*; join D' *g* and A *h*, and from the point of intersection E let fall a perpendicular on *h a*, cutting it in *e*. A E is the actual length and inclination of the required line, and *a e* its horizontal length. Transfer the length *a e* to No. I, and from *e* draw *e e'* perpendicular to *h a''*, and cutting *a' h* in *e'*. If the drawing has been correctly made, a line from E parallel to the ground-line *h a''* will also intersect *a' h* in *e'*.

line A B to the plane C E. Thus the angle a b D (Fig. 17, 1), is the angle sought.

This case presents no difficulty; but when the line is in a plane which is not parallel to the plane of

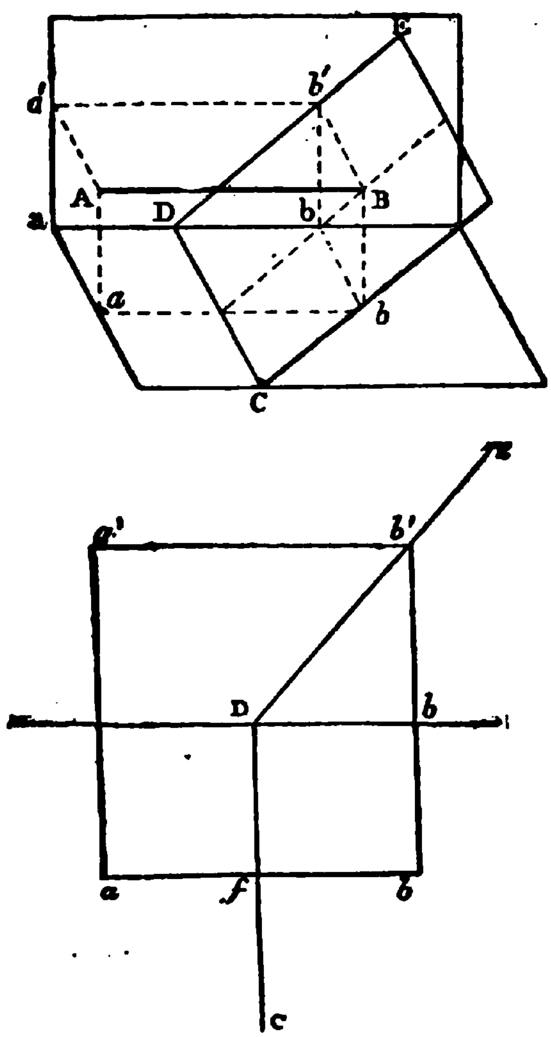


Fig. 17.

projection, the problem is more difficult; as, however, the second case is not of much practical value, it will not be considered.

3. STRAIGHT-SIDED SOLIDS.

XIII. Given the horizontal projection of a regular tetrahedron, to find its vertical projection.

Let A B C d (Fig. 18) be the given projection of the tetrahedron, which has one of its faces coincident with the ground-line. From d draw an indefinite line

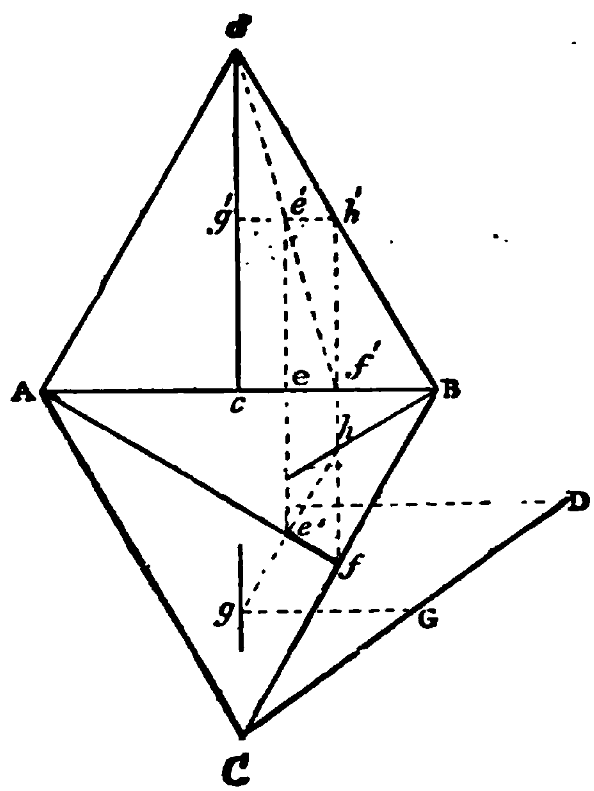


Fig. 18.

d D perpendicular to d C, and make C D equal to C B, C A, or A B; d D will be the height sought, which is carried to the vertical projection from c to d. Join A d and B d, and the vertical projection is complete. This problem might be solved in other ways.

XIV. A point being given in one of the projections of a tetrahedron, to find the point on the other projection.

Let e be the point given in the horizontal projection (Fig. 18). It may first be considered as situated in the plane C B d, which is inclined to the horizontal plane, and of which the vertical projection is the triangle c B d. According to the general method, the vertical projection of the given point is to be found somewhere in a perpendicular raised on its horizontal projection e. If through d and the point e be drawn a line produced to the base of the triangle in f, the point e will be on that line, and its vertical projection will be on the vertical projection of that line f e d, at the intersection of it with the perpendicular raised on e. If through e be drawn a straight line g h, parallel to C B, this will be a horizontal line, whose extremity h will be on B d. The vertical projection of d B is d B; therefore, by raising on h a perpendicular to A B, there will be obtained h, the extremity of a horizontal line represented by h g in the horizontal plane. If through h is drawn a horizontal line h g this line will cut the vertical line raised on e in e, the point sought. If the point had been given in g on the arris c d, the projection could not be found in the first manner; but it could be found in the second manner, by drawing through g, a line parallel to C B, and prolonging the horizontal line drawn through h, to the arris c d, which it would cut in g, the point sought. The point can also be found by laying down the right-angled triangle C d D (which is the development of the triangle formed by the horizontal projection of the arris C d, the height of the solid, and the length of the arris as a hypotenuse), and by drawing through g the line g G perpendicular to C d, to intersect the hypotenuse in G, and carrying the height g G from c to g in the vertical projection. One or other of these means can be employed according to circumstances. If the point had been given in the vertical instead of the horizontal projection, the same operations inverted would require to be used.

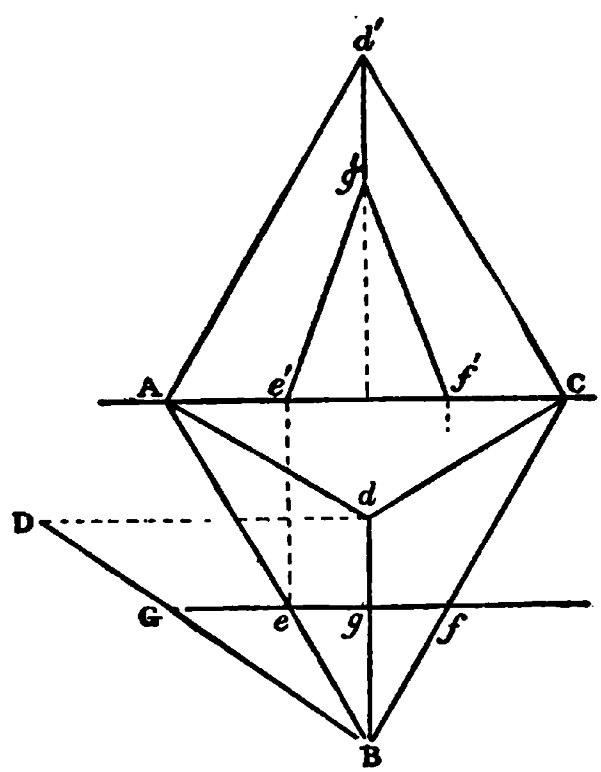


Fig. 19.

XV. Given a tetrahedron, and the trace of a plane (perpendicular to one of the planes of projection) cutting it, by which it is truncated, to find the projection of the section.

First when the intersecting plane is perpendicular to the horizontal plane (Fig. 19), the plane cuts the base in two points e f, of which the vertical projections are e and f; and the arris B d is cut in g, the vertical projection of which can readily be found in any of the ways detailed in the last problem. Having found g join e g f g and the triangle e g f is the projection of the intersection sought.

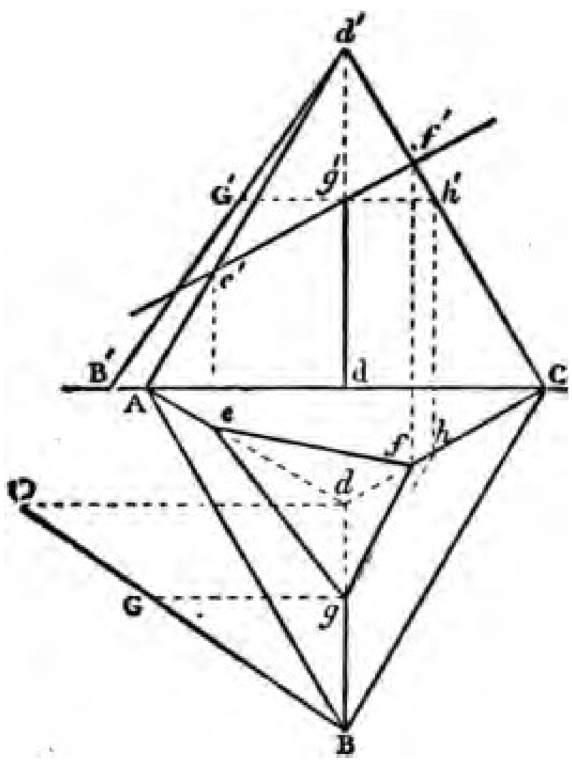


Fig. 20.

When the intersecting plane is perpendicular to the vertical plane, as e f in Fig. 20, the horizontal projections of the three points e g f have to be found. The point g in this case may be obtained in several ways. First by drawing G g h through g, then through h drawing a perpendicular to the base, produced to the arris at h, in the horizontal projection, and then drawing h g parallel to C B, cutting the arris B d in g, which is the point required. Second, on d B, the horizontal projection of the arris, construct a triangle d D B, d B being the altitude of the tetrahedron, and B D the arris, and transfer this triangle to the vertical projection at d d B. From g draw the horizontal line cutting B d in G; g G is the horizontal distance of the required point in the arris, from the vertical axis of the tetrahedron as d is horizontal projection of the vertical axis, and d B the horizontal projection of the arris, it follows that the length gG, transferred to d g, will give the required point g. The points e and f are found by drawing lines from e and f perpendicular to the ground-line, and producing them till they meet the horizontal projections of the arrises in e and f. The triangle e f g is the horizontal projection of the section made by the plane e f.

XVI. The projections of a tetrahedron being given, to find its projections when inclined to the horizontal plane in any degree.

Let A B C d (Fig. 21) be the horizontal projection of a tetrahedron, with one of its sides coincident with the horizontal plane, and e d B its vertical projection; it is required to find its projections when turned round the arris A B as an axis. The base of the pyramid being a horizontal plane, its vertical projection is the right line c B. If this line is raised to c by turning on B, the horizontal projection will be A c2 B. When the point c, by the raising of B c, describes the arc c c, the point d will have moved to d, and the perpendicular let fall from that point on the horizontal plane will give d3, the horizontal projection of the extremity of the arris C d; for as the summit d moves in the same plane as C, parallel to the vertical plane of projection the projection of the summit will evidently be in the prolongation of the arris C d, which is the horizontal projection or trace of that plane. The process therefore, is very simple and is as follows: Construct at the point B the angle required, c B c, and make the triangle c d B equal to c d B;

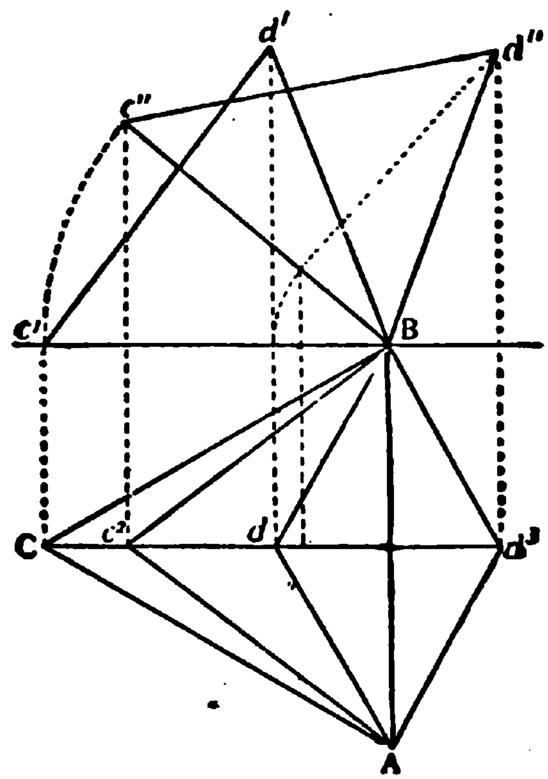


Fig. 21.

from d let fall a perpendicular cutting the prolongation of the arris C d in d3, and from c a perpendicular cutting the same line in c2; join B c2, A c2, B d3, A d3.

The following is a more general solution of the problem: Let A B C d (Fig. 22), be the horizontal projection of a pyramid resting with one of its sides on the horizontal plane, and let it be required to raise, by its angle C, the pyramid by turning around the arris A B, until its base makes with the horizontal plane any required angle, as 50°. Conceive the right line C e turning round e, and still continuing to be perpendicular to A B, until it is raised to the required angle, as at e C. If a perpendicular be now let fall from C, it will give the point C as the horizontal projection of the angle C in its new position. Conceive

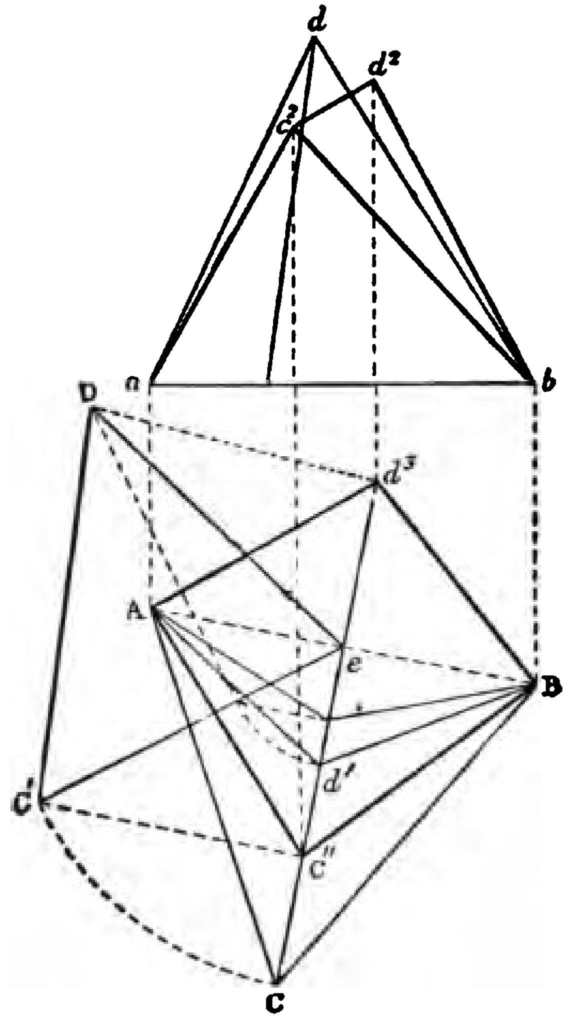


Fig. 22.

a vertical plane to pass through the line C e. This plane will necessarily contain the required angle. Suppose, now we lay this plane down in the horizontal projection thus: Draw from e the line e C, making with e C an angle of 50°, and from e with the radius e c describe an arc cutting it in C. From C let fall on C e, a perpendicular on the point C, which will then be the horizontal projection of C in its raised position. On C e draw the profile of the tetrahedron C D e inclined to the horizontal plane. From D let fall a perpendicular on C e produced, and it will give d as the horizontal projection of the summit of the pyramid in its inclined position. Join A d, B d, A c, B c to complete the figure. The vertical projection of the tetrahedron in its original position is shown by a d b, and in its raised position by a, c2, d2, b, the points c2, and d2 being found by making the perpendiculars c3 c2 and d3 d2 equal to C C and d D respectively.

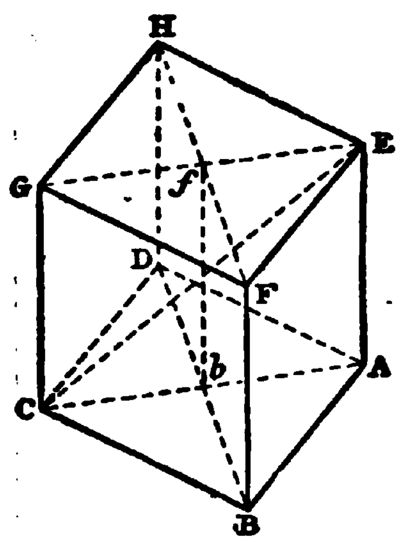


Fig. 23.

XVII. To construct a vertical and horizontal projections of a cube, the axis 1 of which are perpendicular to the horizontal plane.

If an arris of the cube is given, it is easy to find its axis, as this is the hypotenuse of a right-angled triangle, the shortest side of which is the length of an arris, and the longest the diagonal of a side. Conceive the cube cut by a vertical plane passing through its diagonals E G, A C (Fig. 23), the section will be the rectangle A E G C. Divide this into two equal right-angled triangles, by the diagonal E C. If in the upper and lower faces of the cube, we draw the diagonals F H, B D, they will cut the former diagonals in the points f and b. Now, as the lines b B, b D, f F, f H, are perpendicular to the rectangular plane A E G C, f b may be considered as the vertical projection of B F and D H, and from this consideration we may solve the problem.

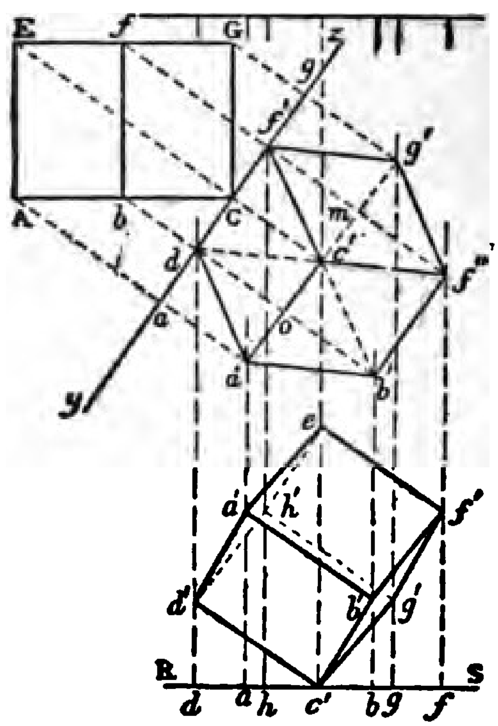


Fig. 24.

Let A E (Fig. 24) be the arris of any cube. The letters here refer to the same parts as those of the preceding diagram (Fig. 23). Through A draw an indefinite line, A C, perpendicular to A E. Set off on this line, from A to C, the diagonal of the square of A E, and join E C, which is then the axis of the cube. Draw the lines E G, G G, parallel respectively to A C and A E, and the resulting rectangle, A E G C, is the section of a cube on the line of the diagonal of one of its faces. Divide the rectangle into two equal parts by the line b f, which is the vertical projection of the lines B F, D H (Fig. 23), and we obtain, in the figure thus completed, the vertical projection of the cube, as a c b d (Fig. 25).

Through C (Fig. 24), the extremity of the diagonal E C draw y z perpendicular to it, and let this line represent the common section or ground-line of the two planes of projection. Then let us find the horizontal projection of a cube of which A E G C is the vertical projection. In the vertical projection the axis E C is perpendicular to y z, and consequently, to the horizontal plane of projection, and we have the height above this plane of each of the points which terminate the angles. Let fall from each of these points perpendiculars to the horizontal plane, the projections of the points will be found on these perpendiculars. The horizontal projection of the axis E C will be a point on its prolongation, as c. This point might have been named e with equal correctness, as it is the horizontal projection of both the extremeties of the axis, C and E. Through c draw a line parallel to y z, and find on it the projections of the points A and G, by continuing the perpendiculars A a, G g, to a and g. We have now to find the projections of the points b f (representing D B F H, Fig. 23,), which will be somewhere on the perpendiculars b b, f f, let fall from them. We have seen in Fig. 23 that B F, D H are distant from b f by an extent equal half the diagonal of the square face of the cube. Set off, therefore, on the perpendiculars b b and f f, from o and m, the distance. A b in d, b, and f, f and join d a, a b, b f, f g, g f, to complete the hexagon which is the horizontal projection of the cube. Join f e, f e, and a c, to give the arrises of the upper half of the cube. The dotted lines d c, b c, g c, show the arrises of the lower side. Knowing the heights of the points in these vertical projections, it is easy to construct a vertical projection on any line whatever, as that on R S below.

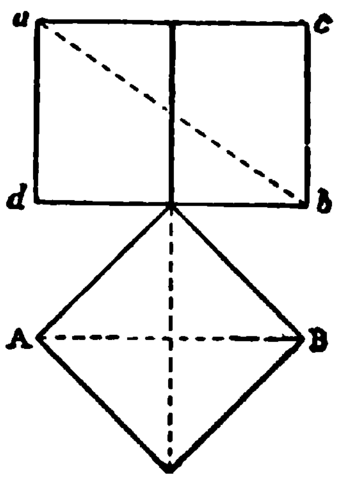


Fig. 25.

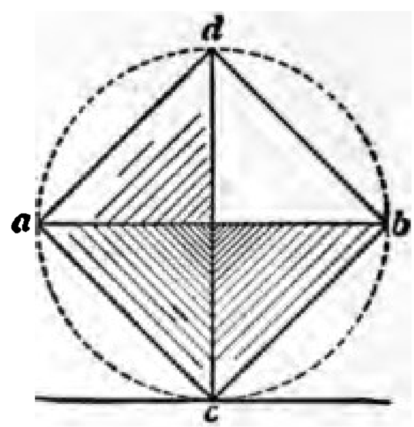


Fig. 26.

XVIII. To construct the projections of a regular octahedron, when one of its axis is perpendicular to either plane of projection.

Describe a circle (Fig. 26), and divide it into four equal parts by the diameters, and draw the lines a d, d b, b c, c a; a figure is produced which serves for either the vertical or the horizontal projection of the octahedron, when one of its axis is perpendicular to either plane.

XIX. One of the faces of an octahedron being given, coincident with the horizontal plane of projection, to construct the projections of the solid.

Let the triangle A B C (Fig. 27) be the given face. If A be considered to be the summit of one of the two pyramids which compose the solid, B C will be one of the sides of the square base, k C B i. The base makes with the horizontal plane an angle, which is easily found. Let fall from A a a perpendicular on B c, cutting it in d, with the length B c as a radius, and from

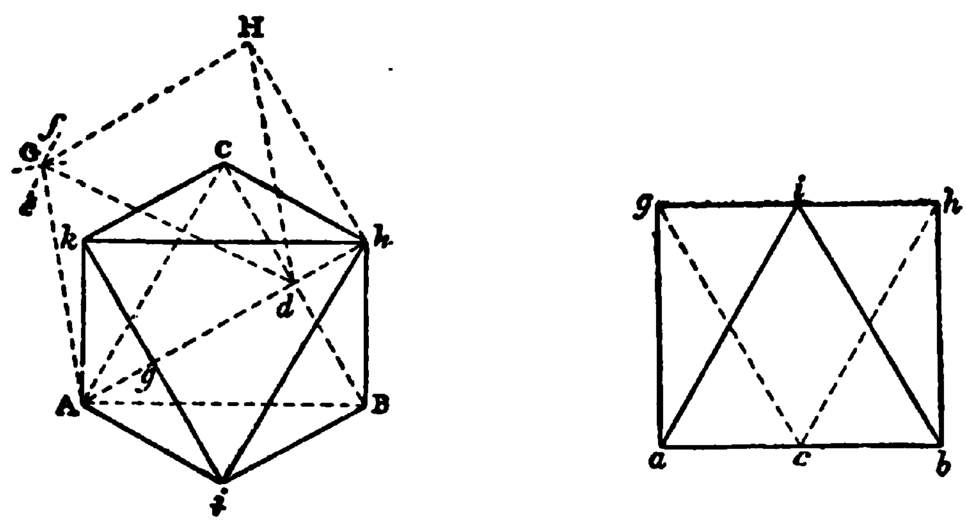


Fig. 27.

d as a centre, describe the indefinite arc e f. The perpendicular A d will be the height of each of the faces, and, consequently, of that which, turning on A, should meet the side of the base which has already turned on d. Make this height turn on A, describing from that point as a centre, with the radius A d, an indefinite arc, cutting the first arc in G, the point of meeting of one of the faces with the square base; draw the line G A, G d: the first is the profile or inclination of one of the faces on the given face A B C according to the angle d A G;

the second, d G, is the inclination of the square base, which separates the two pyramids in the angle A d G. The face adjacent to the side B C is found in the same manner. Through G, draw the horizontal line G H equal to the perpendicular A d. This line will be the profile of the superior face. Draw d H, which is the profile of the face adjacent to B C. From H let fall a perpendicular on A d produced, which gives the point h for the horizontal projection of H, or the summit of the superior triangle parallel to the first, draw h i parallel to C A, h k parallel to A B, A k and B h perpendicular to A B, and join k C, C h, B i, and A i and the horizontal projection is complete. From the heights we have thus obtained we can now draw the vertical projection shown in No. II, in which the parts have the same letters of reference.

The finding of the horizontal projection may be abridged by constructing a hexagon and inscribing in it the two triangles A C B, h i k.

XX. Given in the horizontal plane the projection of one of the faces of a dodecahedron, to construct its projections.

The dodecahedron is a twelve-sided solid, all the sides being regular and equal pentagons. It is necessary, in order to construct the projection, to discover the inclination of the faces among themselves. Let the pentagon A B C D E (Fig. 28) be the side on which the body is supposed to be seated on the plane. Conceive two other faces, E F G H D and D I K L C, also in the horizontal plane, and then raised by being turned on their bases, E D, D C. By their movement they will describe in space arcs of circles, which will terminate by the meeting of the sides D H, D I.

To find the inclination of these two faces.—From the points I and H let fall perpendiculars on their bases produced. If each of these pentagons were raised vertically on its base, the horizontal projection of H and I would be respectively in z z; but as both are raised

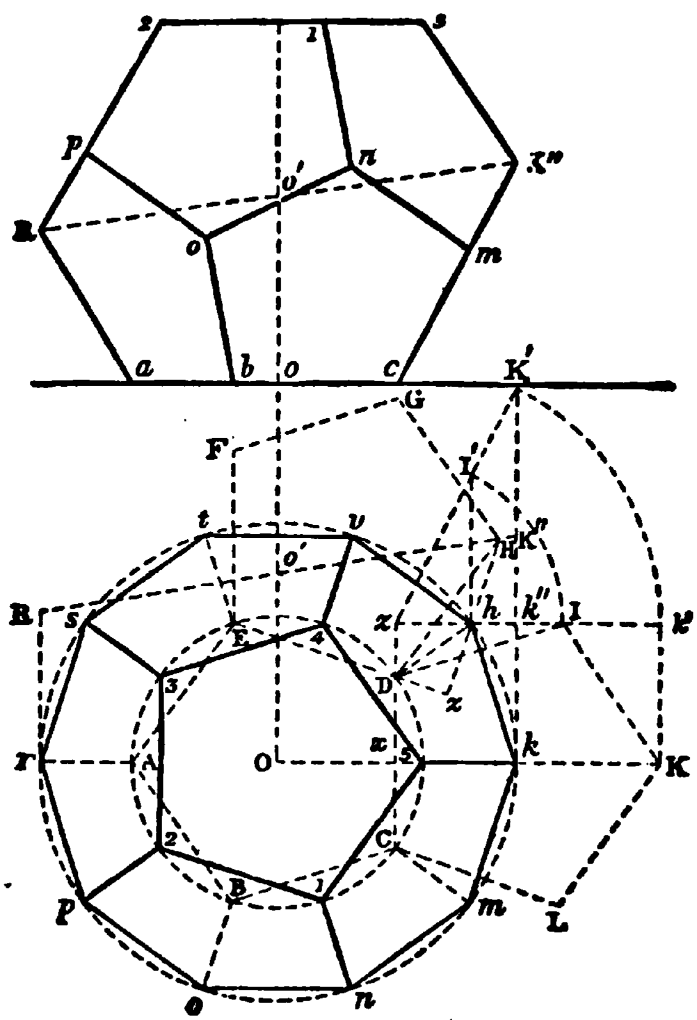


Fig. 28.

together, the angles H and I would meet in space above h where the perpendiculars intersect therefore, h will be the horizontal projection of the point of meeting of the angles. To find the horizontal projection of K, prolong indefinitely z I, and set off from z on z I the length x K in k; from z as centre with radius z I, describe the arc I I cutting the perpendicular h I in I; join z I; then from z as centre, with the radius z k, describe an arc cutting z I produced in the point K, from which let fall on z k a perpendicular K k, and produce it to k in x K. If, now, the right-angled triangle z k K, were raised on its base, k would be the projection of K. Conceive now the pentagon C D I K L turned round on C D, until it makes an angle equal to k z K with the horizontal plane, the summit K will then be raised above k by the height k K, and will have for its horizontal projection the point k. In completing the figure practically;—from the centre o, describe two concentric circles passing through points h D. Draw the lines h D, h k, and carry the last round the circumference in m n o p r s t u: through each of these lines draw radially the lines m C, o B, r A, t E, and these lines will be the arrises analogous to h D. This being done, the inferior half of the solid is projected. By reason of the regularity of the figure, it is easy to see that the six other faces will be similar in those already drawn, only that although the superior pentagon will have its angles on the same circumference as the inferior pentagon the angles of the one will be in the middle of the faces of the other. Therefore, to describe the superior half; —through the angles n p s v k, draw the radial lines n 1, p 2, s 3, v 4, k 5, and join them by the straight lines 1 2, 2 3, 3 4, 4 5, and 5 1.

To obtain the length of the axis of the solid, observe that the point k is elevated above the horizontal plane by the height k K: carry that height to k K: the point r, analogous to h, is raised the same height as that point, that is to say h i, which is to be carried from r to R; and the line R K is the length sought. As this axis should pass through the centre of the body, if a vertical projection of the axis in O, and therefore O o is the half of the height of the solid vertically. By doubting this height, and drawing a horizontal line to cut the vertical lines of the angles of the superior face is obtained, as in the upper portion of (Fig. 28), in which the same letters refer to the same parts.

XXI. One of the faces of a dodecahedron being given, to construct the projections of the solid, so that its axis may be perpendicular to the horizontal plane.

Let A B E D C (Fig 29, 1) be the given face. The solid angles of the dodecahedron are each formed by the meeting of three pentagonal planes. If there be conceived a plane B C passing through the extremities of the arrises of the solid angle A, the result of the section would be a triangular pyramid, the sides of whose base would be equal to one of the diagonals of the face, such as B C. An equilateral triangle b c f (Fig. 29, 11) will represent the base of that pyramid inverted, that is, with its summit resting on the horizontal projection, it is required to find the height of that pyramid, or which is the same thing, that one of the three points of its base b c f, for as they are all equally elevated, the height of one of them gives the others. There is necessarily a proportion between the triangle A b c (No. 11) and A B C (No. 1), since the first is the horizontal projection of the second. A g is the horizontal proection of A G; but A G is a part of A H, and the projection of that line is required for one of the faces of the solid; therefore as A G: A g:: A H: x. In other words, the length, the length of x may be obtained by drawing a fourth proportion of the three lines it will be found to be equal to A h; or it may be obtained graphically thus:—Raise on A g at g an indefinite perpendicular, take the length A G (No. 1) and carry it

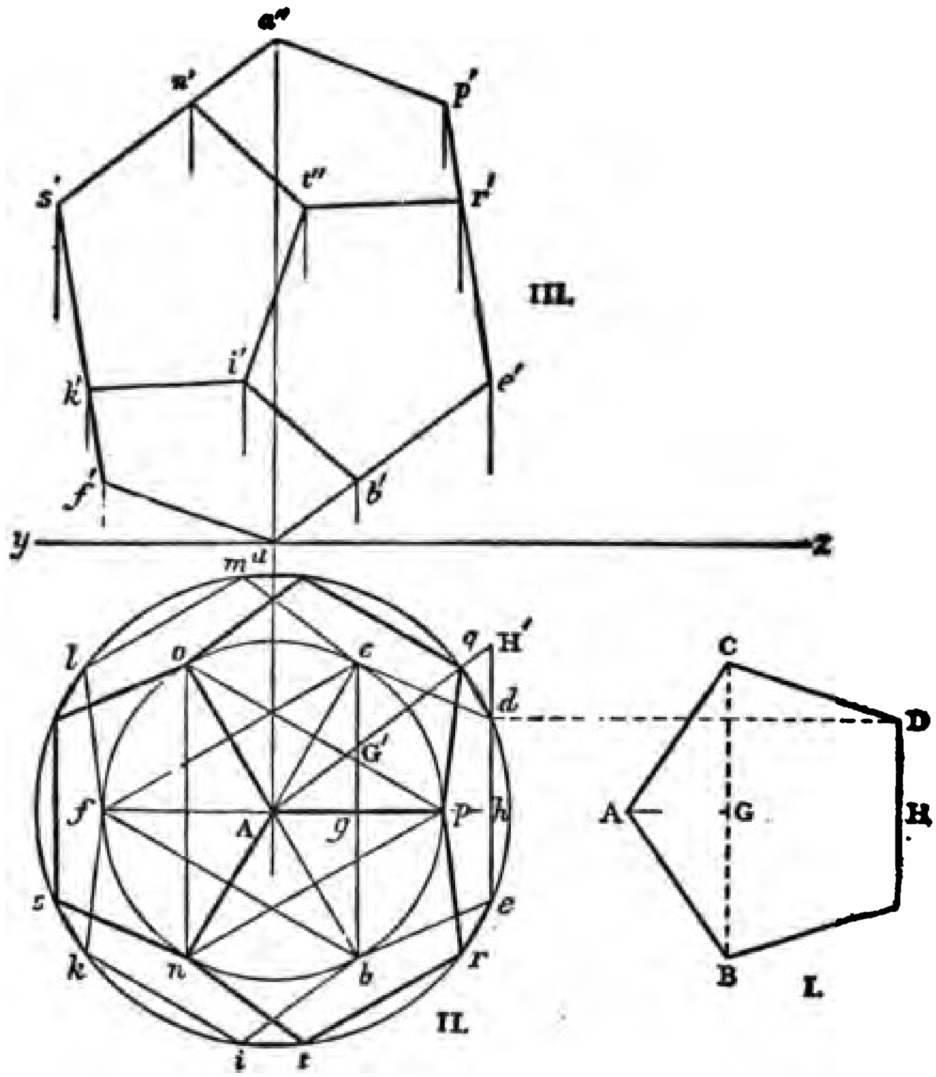


Fig. 29.

from A to G (No. 11); g is a point in the assumed pyramidal base b c f. Since A G is a portion of A H, A G will be so also. Produce A G, therefore, to H, making A H equal to A H (No. 1) and from H let fall a perpendicular on A g produced, which gives h the point sought. Produce H h, and carry on it the length H D or H E from h to d and h to e; draw the lines c d, b e, and the horizontal projection of one of the faces is obtained inclined to the horizontal plane, in the angle H A h. As the other two inferior faces are similar to the one found, the three faces should be found on the circumference of a circle traced from A as centre, and with A d or A e as a radius. Join f A, prolong A n, A o, perpendicular to the sides of the triangle f c b, and make them equal to A h, and through their extremeties draw perpendiculars, cutting the circumference in the points i k, l m. Draw the lines i b, k f, l f, m c, and the horizontal projections of the three inferior faces obtained. The superior pyramid is similar and equal to the inferior, and solely opposed by its angles. Describe a circle passing through the three points of the first triangle, and draw within it a second equilateral triangle n o p, of which the summits correspond to the middle of the faces of the former one. Each of these points will be the summit of a pentagon, as the points b c f. These pentagons have all their sides common, and it is only necessary therefore to determine one of these superior pentagons to have all the others. Six of the faces of the dodecahedron have now been projected; the remaining six are obtained by joining the angular points already found, as q d, e r, t i, k s, &c.

To obtain the vertical projection (No. 111) begin with the three inferior faces. The point A in the horizontal projection being the summit of the inferior solid angle, will have its vertical in a; the points b c f, when raised to the height g G, will be in b c f, or simply b f. The points b g c being in a plane perpendicular to the vertical plane, will necessarily have the same vertical projection, b. The line a f will be the projection of the arris A f, and a b will be that of the arrises A b, A c, and of the line A g, or rather that of the triangle A b c, which is in a plane perpendicular to the vertical plane. But this triangle is only a protion of the given pentagonal face (No. 1), of which A H is the perpendicular let fall from A on the side E D. Produce a b to e, making a e equal to A H; e is the vertical projection of the arris e d. This arris is common to the inferior pentagon, and to the superior pentagon e d q p r, which is also perpendicular to the vertical plane, and, consequently, its vertical projection will be e p, equal to a e.

This projection can now be obtained by raising a vertical line through p, the summit of the superior pentagon, and from e as a centre, and with the radius A H or A H, describing an arc cutting this line in p, the point sought. But p n o belong to the base of the superior pyramid; therefore, if a perpendicular is drawn from n through y z to n, and the height p is transferred to n by drawing through p′ a line parallel to y z, n will be the projection of the points n and o. Through n draw s n a parallel to a e cutting perpendiculars drawn through s and A in the horizontal projection. Through s draw s f parallel to p e, and join a f, a p; set off on the perpendicular from r the height of s above y z at r, and draw r t parallel to y z, cutting the perpendicular from t, and join n t. Draw perpendiculars from k and i though y z to k and i, make k and i the same height as e, and draw k i, and join i b, i t. The vertical projection is now complete.

XXII. In a given sphere to inscribe a tetrahedron, a hexahedron or cube, an octahedron and a dodecahedron.

Let A B (Fig. 30) be the diameter of the given sphere. Divide it into three equal parts, D B being one of these parts. Draw D E perpendicular to A B, and

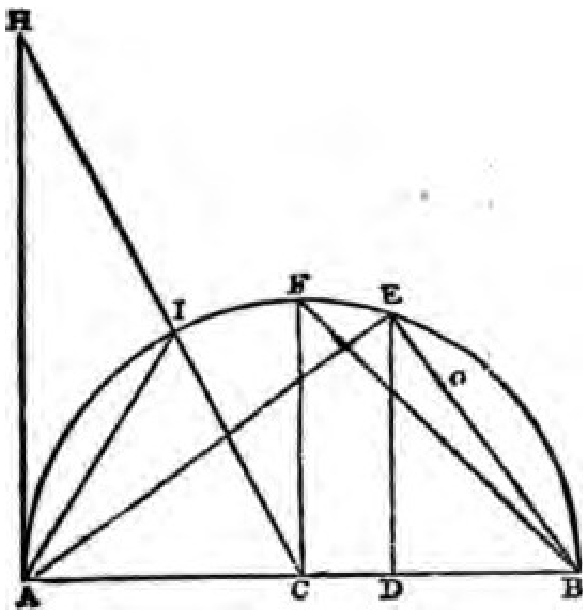


Fig. 30.

draw the chords A E, E B. A E is the arris of the tetrahedron, and E B the arris of the hexahedron or cube. From the centre C draw the perpendicular radius C F, and the chord F B is the arris of the octahedron. Divide B E in extreme and mean proportion in G, and B G is the arris of the dodecahedron. The arrises being known, the solids can be drawn by the help of the problems already solved. Draw the tangent A H equal to A B; join H C and A I; A I is the arris of an icosahedron which can be inscribed in the sphere, an icosahedron being a solid with twenty equal sides, all of which are equilateral triangles. See Fig. 30½.

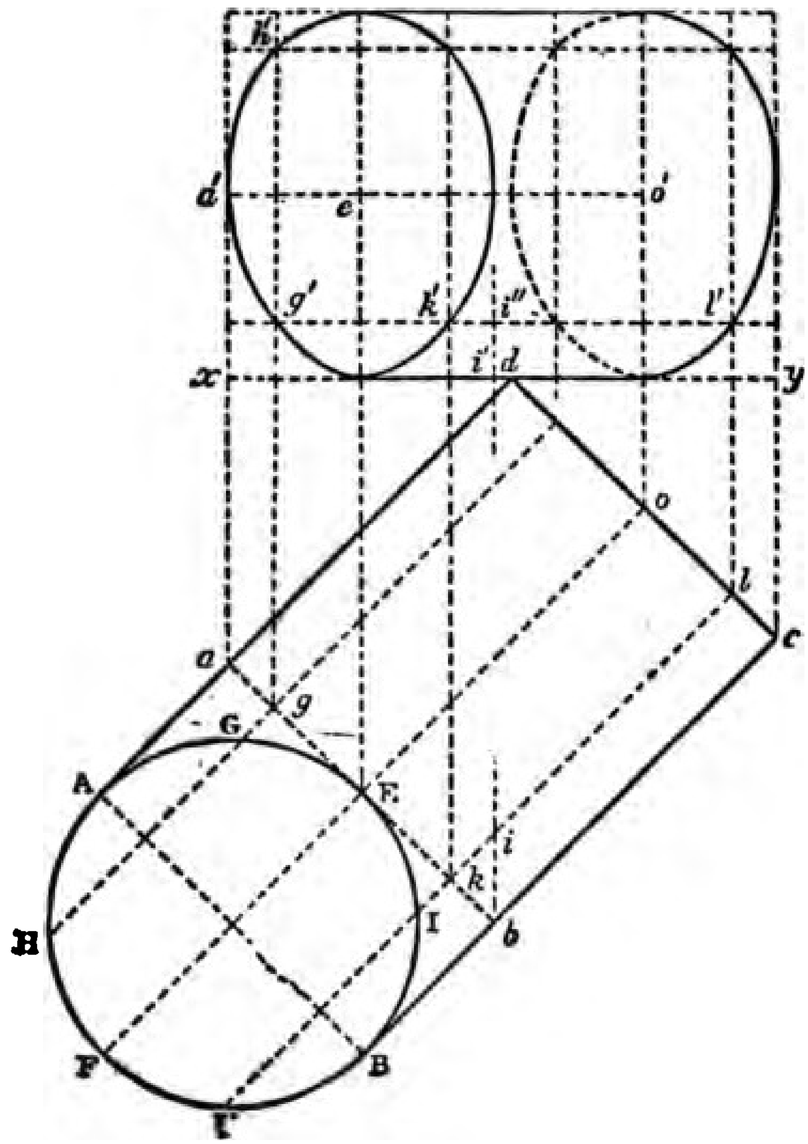


Fig. 30½

4. THE CYLINDER, CONE, AND SPHERE.

XXIII. The horizontal projection of the cylinder, the axis of which is perpendicular to the horizontal plane, being given to find the vertical projection.

Let the circle A B C D (Fig. 31) be the base of the cylinder, and also its horizontal projection. From the points A and C raise perpendiculars to the ground-line

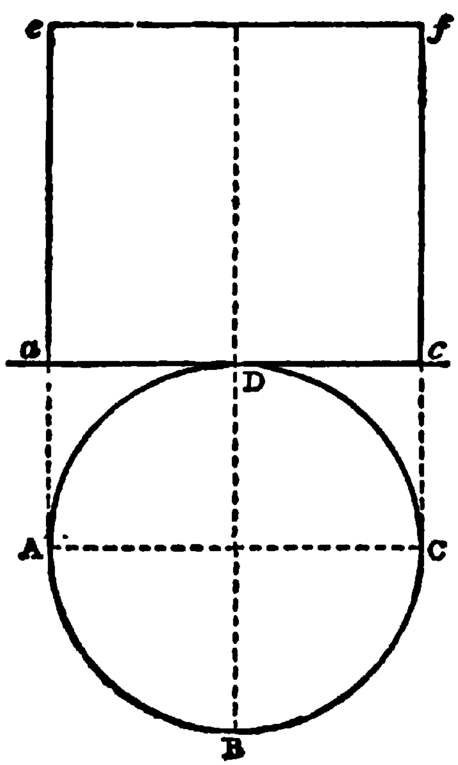


Fig. 31.

a c, and produce them to the height of the cylinder—say, for example, a e, c f. Draw e f parallel to a c, and the rectangle a e f c is the vertical projection required.

XXIV. Given the traces of an oblique plane, to determine the inclination of the plane to both the H. P. and the V. P.

Let v t and h t (Fig. 32) be the traces of the given plane. Draw the projections of a semi-cone having its axis a′ b′ in the vertical plane, the apex a′ in the given v t and its base (a semi-circle) c e d in the h p and lying tangentially to the given h t. Then the base angle

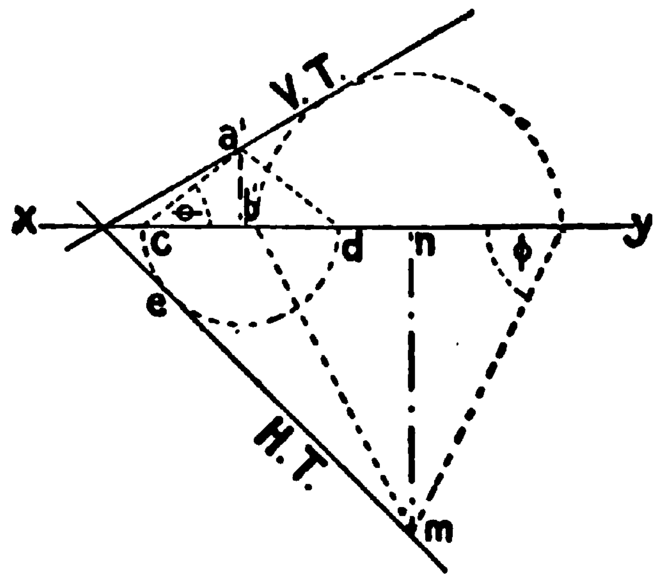


Fig. 32.

(0) of the cone gives the inclination of the plane to the h p. To determine the inclination of the plane to the v. p., draw the projections of a second semi-cone, having the axis m n in the h p, and the apex m in the given h t, while the base is in the v p and tangential to the v t. The base angle (0) of this cone gives the inclination to the v p.

XXV. The base of a cylinder being given, and also the angles which the base makes with the planes of projection, to construct the projections of the cylinder.

Let the circle A G B H (Fig. 33) be the given base, and let each of the given angles be 45°. Draw the diamater A B making an angle of 45° with the ground-line or vertical plane, and draw the line A B, making with A B the given angle; and from A as a centre, with A B and A C as radii, describe arcs cutting A B in B and C. Then draw A D, B E perpendicular to A B,

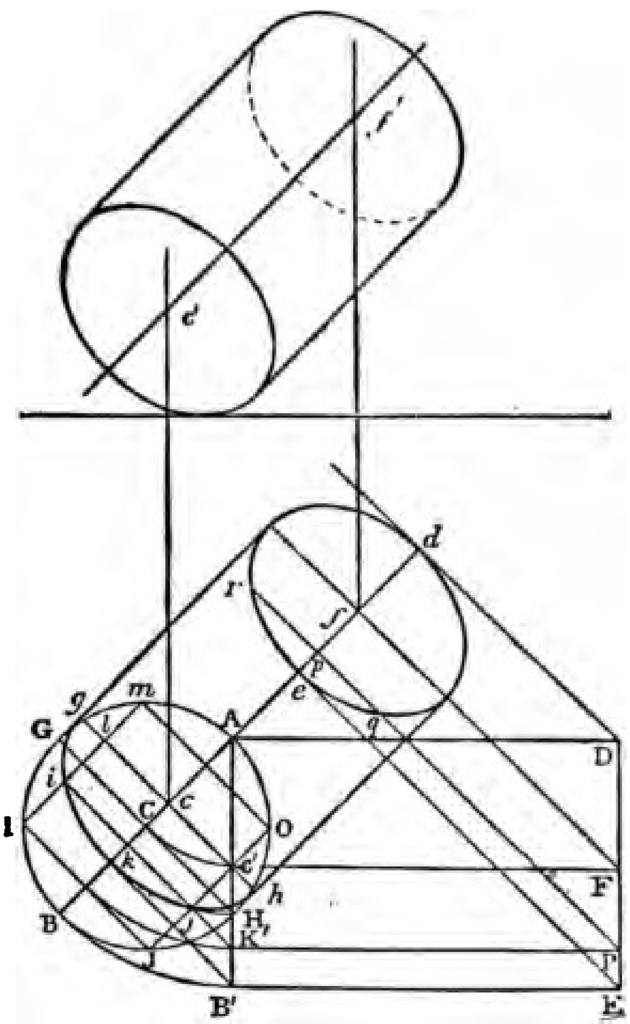


Fig. 33.

and equal to the length of the cylinder; the rectangle A E is the vertical projection of the cylinder parallel to the vertical plane and inclined to the horizontal plane A B in an angle of 45°. Now prolong indefinitely the diameter B A, and this line will represent the projection on the horizontal plane of the line in which the generating circle moves to produce the cylinder. If from B and C perpendiculars be let fall on A B, k will be the horizontal projection of B, A k that of the diameter A B, and c that of the centre C. Through c draw h g perpendicular to A B, and make c h, c g equal to C H, G G; and the two diameters of the ellipse, which is the projection of the base of the cylinder, will be obtained, namely, A k and h g.

In like manner, draw D F E the lines D d, F f, E e, perpendicular to the diameter A B produced, and their intersections with the diameter and the sides of the cylinder will give the means of drawing the ellipse which forms the projection of the farther end of the cylinder. The ellipses may also be found by taking any number of points in the generating circle as I J, and obtaining their projections i j. The method of doing this, and also of drawing the vertical projection c f, will be understood without further explanation.

XXVI. A point in one of the projections of a cone being given, to find it in the other projection.

Let a (Fig. 34) be the given point. This point belongs equally to the circle which is a section of the cone by a plane passing through the point parallel to the base, and to a straight line forming one of the sides of a triangle which is the section of the cone by a plane perpendicular to its base and passing through its vertex and through the given point, and of which f a g is the horizontal, and f a g the vertical projection. To find the vertical projection of a, through a draw a a perpendicular to b c, and its intersection with f g is the point required; and reciprocally, a in the horizontal projection may be found from a in the vertical projection, in the same manner.

Otherwise, through a, in the horizontal projection, describe the circle a d c, and draw e e or c c, cutting

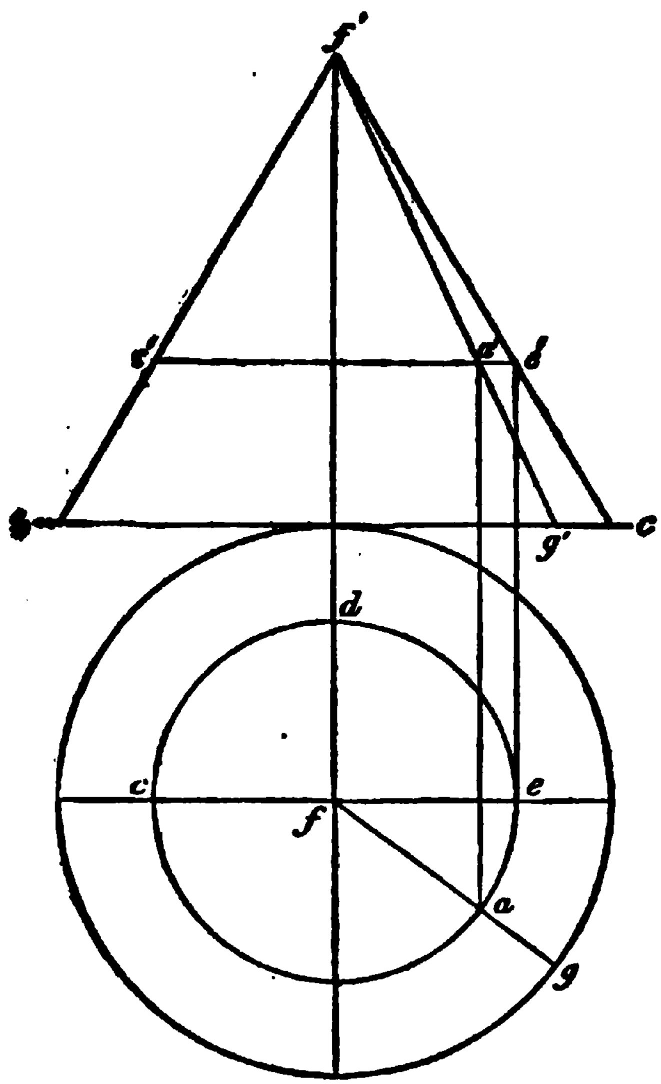


Fig. 34.

the sides of the cone in e and c; draw c e parallel to the base, and draw a a, cutting it in a, the point required.

XXVII. On a given cylinder to describe a helix.

Let a b c d, &c. (Fig. 35), be the horizontal projection of the given cylinder. Take on this curve a series of equal distances, a b, b c c d, &c., and through each

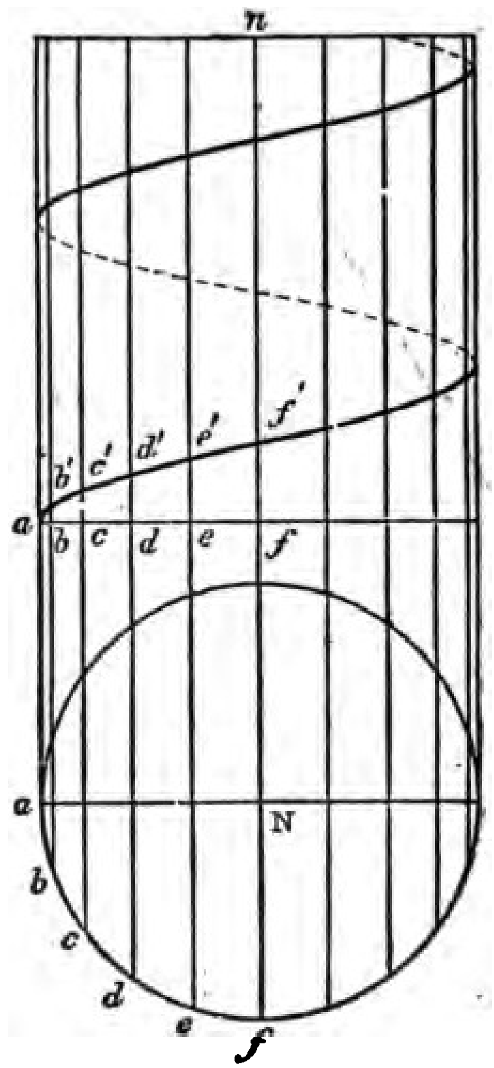


Fig. 35.

of the points a, b, c, &c. draw a vertical line, and produce it along the vertical projection of the cylinder. Then conceive a curve cutting all these verticals in the points a b c d, in such a manner that the height of the point above the ground-line may be in constant relation to the arcs a b, b c, c d; for example, that a may be the zero of height, that bb may be 1, c c 2, d d3, &c.; then this curve is named a helix. To construct this curve, carry on the vertical projection on each vertical line such a height as has been determined, as 1 on b, 2 on c, 3 on d; and through these points will pass the curve sought. It is easy to see that the curve so traced is independent of the cylinder on which it has been supposed to be traced; and that if it be isolated, its horizontal projection will be a circle. The helix is named

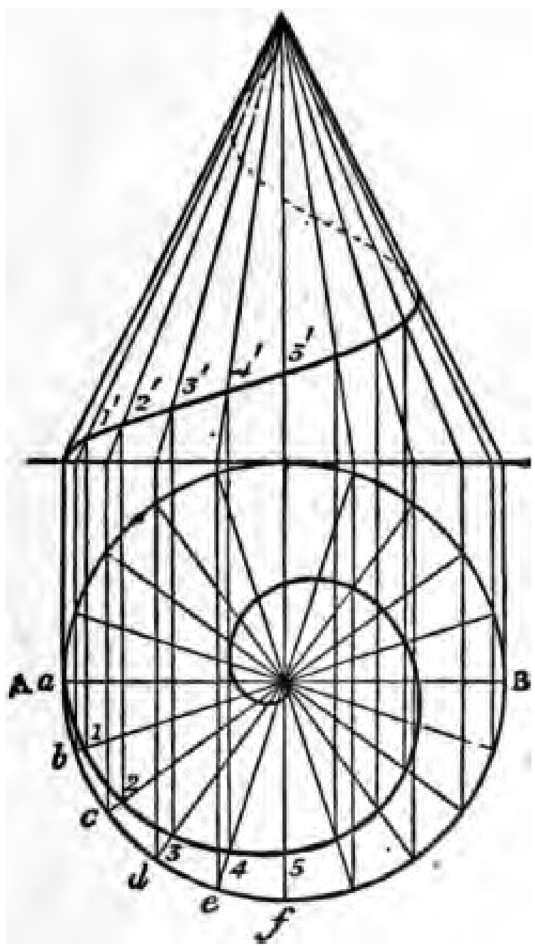


Fig. 36.

after the curve which is its horizontal projection. Thus the helix in the example is a helix with a circular base. The vertical line f n is the axis of the helix, and the height b b, comprised between two consecutive intersections of the curve with a vertical, is the pitch of the helix.

XXVIII. On a given cone to describe a helix.

Let the projections of the given cone be as shown in (Fig. 36.) Divide the base of the cone in the horizontal projection into any number of equal parts, as a b, b c, c d, &c., and draw lines from the vertex to the points thus obtained. Set off along these lines a series of distances increasing in constant ratio, as 1 at b, 2 at c, 3 at d, &c. The curve then drawn through these points, when supposed to be in the same plane, is called a spiral. If these points, in addition to approaching the centre in a constant ratio, are supposed also to rise above each other by a constant increase of height, a helical curve will be obtained on the vertical projection of the cone.*

XXIX. A point in one of the projections of the sphere being given, to find it in the other projection.

Let a be the given point in the horizontal projection of the sphere h b c i (Fig. 38). Any point on the surface of a sphere belongs to a circle of that sphere. Therefore, if a is the point, and a vertical plane b c is made to pass through that point to A B, the section of

*The octahedron is formed by the union of eight equilateral triangles; or, more correctly, by the union of two pyramids with square bases, opposed base to base, and of which all the solid angles touch a sphere in which they may be inscribed.

It is essential that the diagram should be clearly *seen* as a solid, and not as a mere set of lines in one plane. Imagine *h* as the apex of one pyramid on the base *k* C B *i*, and A as the apex of the other pyramid on the opposite side of the same base. The octahedron is shown to be lying on its side A B C.

the sphere by this plane will be a circle, whose diameter will be b c, and the radius consequently, d b or d c; and the point a will necessarily be in the circumference of this circle. Since the centre d of this circle is situated on the horizontal axis of the sphere, and as

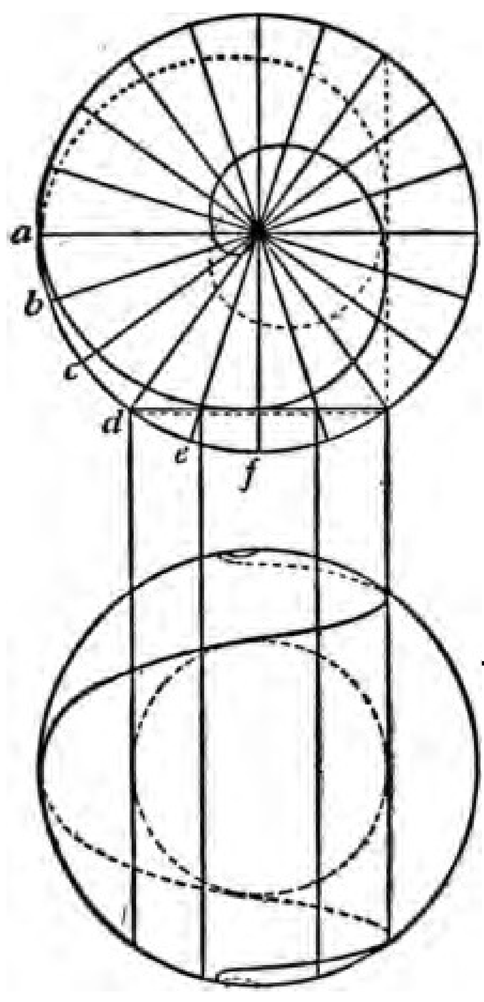


Fig. 37.

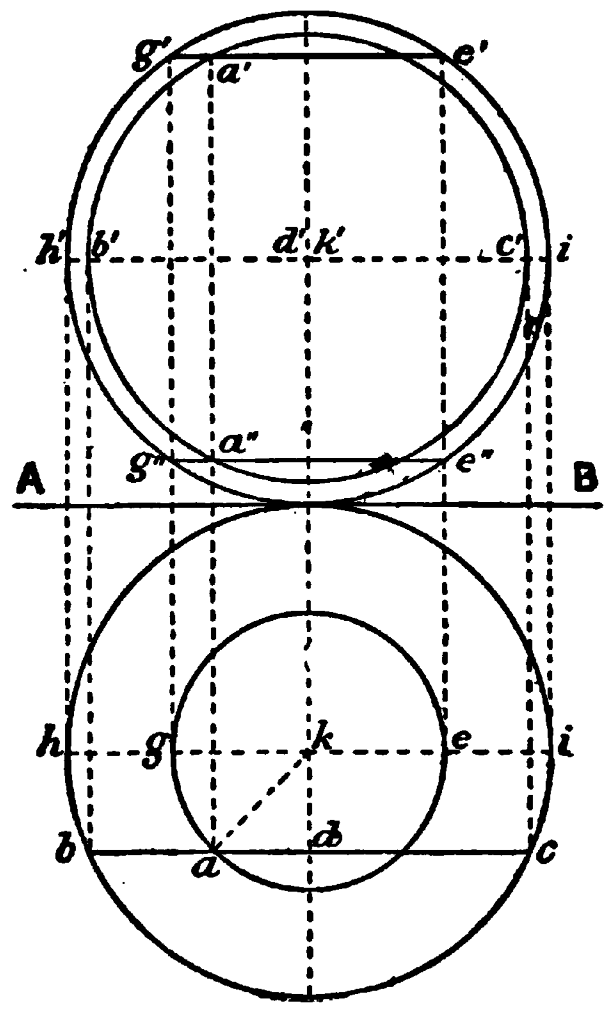


Fig. 38.

this axis is perpendicular to the vertical plane, its vertical projection will be the point d. It is evident that the vertical projection of the given point a will be found in the circumference of the circle described from d with the radius d b or d c and at that point of it where it is intersected by the line drawn through a, perpendicular to A B. Its vertical projection will therefore be either a or a, according as the point a is on the superior or inferior semi-surface of the sphere.

The projection of the point may also be found thus: Conceive the sphere cut by a plane parallel to the horizontal plane of projection passing through the given point a. The resulting section will be the horizontal circle described from k, with the radius k a; and the vertical projection of this section will be the straight line g e, or g e; and the intersections of these lines with the perpendicular drawn through a, will be the projection of a, as before.

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5. SECTIONS OF SOLIDS.

To draw sections of any solid requires little more than the application of the method described in the foregoing problems. Innumerable examples might be given, but a few selected ones will suffice.

XXX. The projections of a regular tetrahedron being given, to draw the section made by a plane perpendicular to the vertical plane and inclined to the horizontal plane.

Let A B C D and a b c d (Fig. 39) be the given projections, and E F G the given plane perpendicular to the vertical plane and inclined to the horizontal plane at an angle of 30°. The horizontal projection e f g of the section is easily found as shown. To find the correct section draw through e, f, and g, lines parallel to the ground-line A C, and a c one of them as E e set off the distances E F, E G at E f and E g, and through f and g draw perpendiculars cutting the other lines in F and

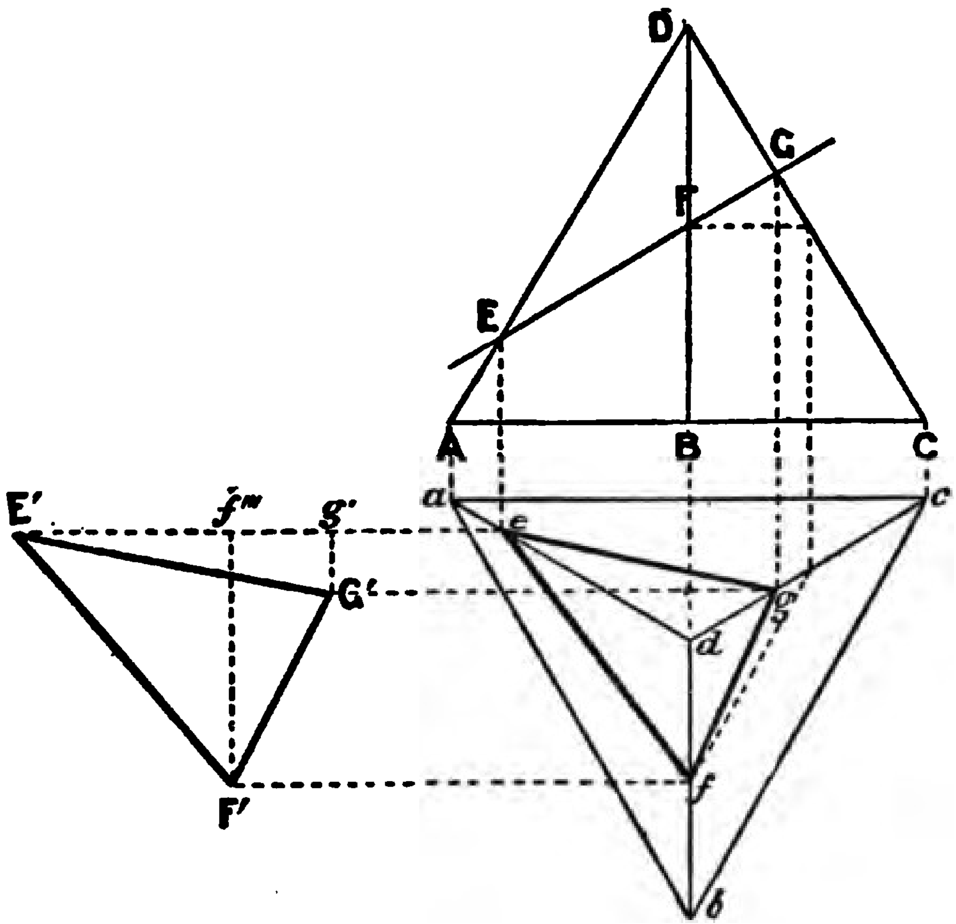


Fig. 39.

G′. Join E G, G F, and F E. E F G is the correct section made by the plane.

XXXI. The projections of a hexagonal pyramid being given, to draw the section made by a plane perpendicular to the vertical plane and inclined to the horizontal plane.

Let A B C D and a b c d e f g (Fig. 40) be the given projections, and H I J K the given plane. The horizontal projection h i j k l m of the section is easily found as shown. Through m, l, h, j, and i draw lines parallel to the ground-line A D, and on one of them as h m, set off the distances H I, H J, H K, at h M, h l, h k. From

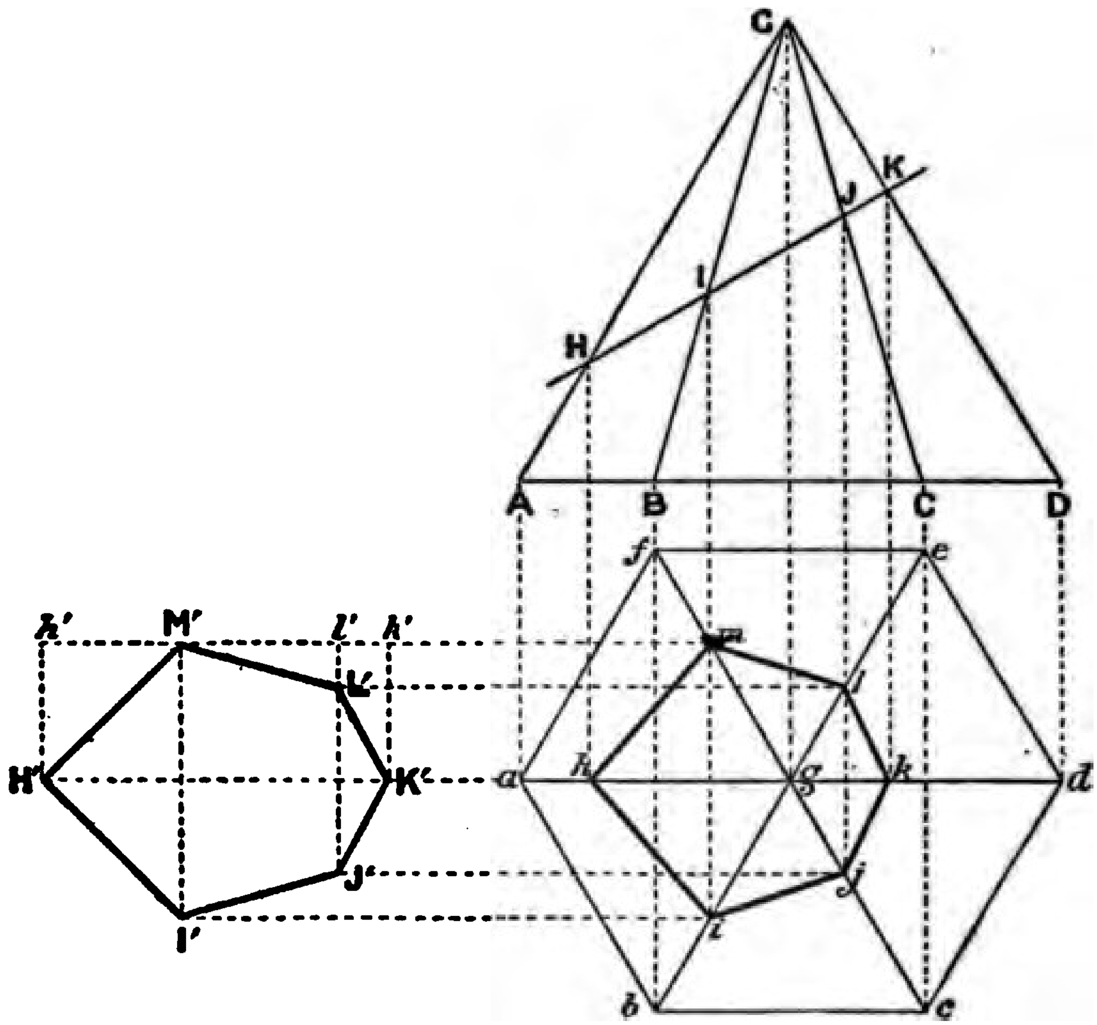


Fig. 40.

h, M, l, and k draw perpendiculars meeting the other lines in H, I, L, J, and K, and join the points of intersection. H I J K L M is the true section made by the plane.

XXXII. The projections of an octagonal pyramid being given, to draw the section made by a vertical plane.

Let A B C D F and a b c d e f (Fig. 41) be the given projections, and g h i j k of the section made by the plane is easily found by drawing g g, h h, i i, &c. per-

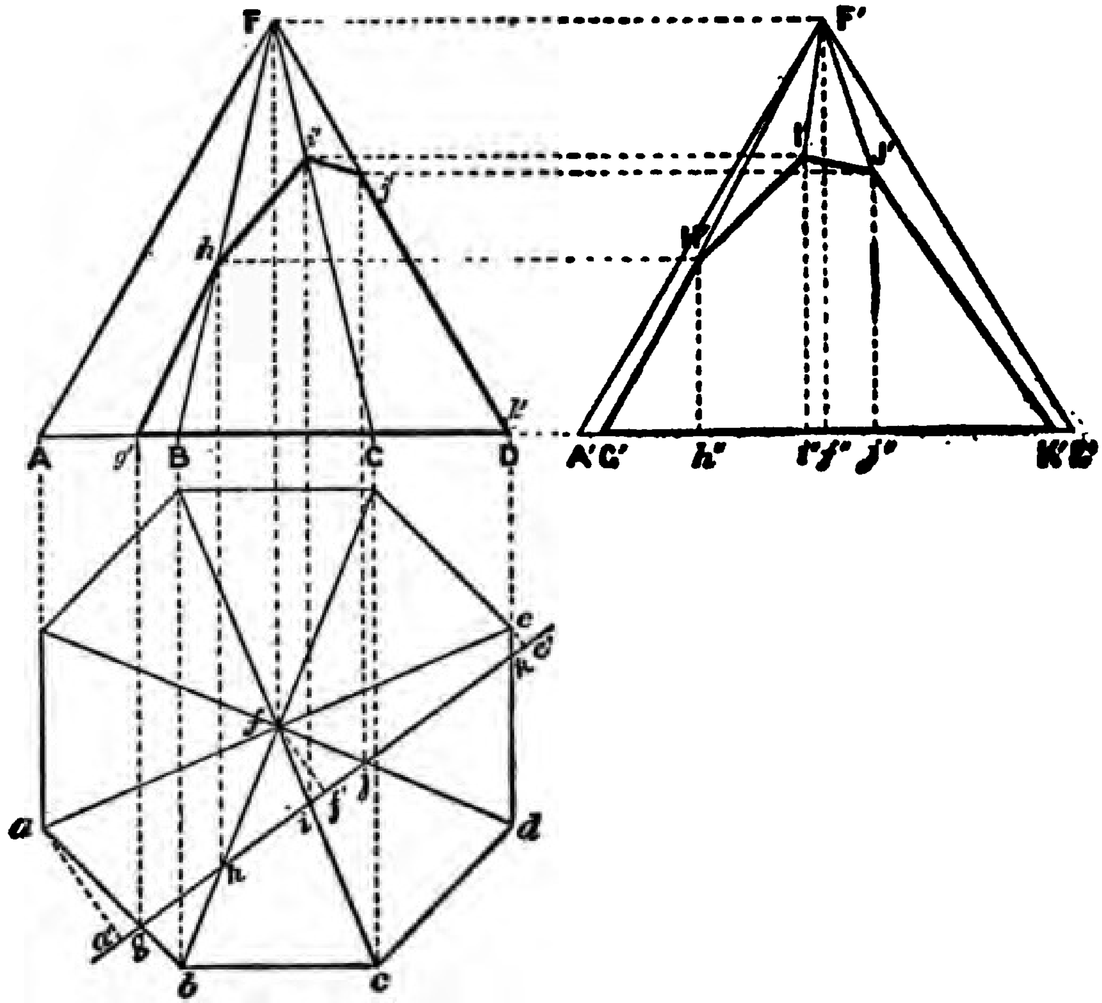


Fig. 41.

pendicular to the ground-line A D. On A D produced set off the distances g h, h i, i j, and j k at G h, h i, &c., and from the points thus found draw perpendiculars to G K meeting lines drawn from h, i, and j parallel to A D, in H, I, and J. Join G H, H I, I J, and J K. G H I J K is the true section made by the plane.

A **cylinder** may be cut by a plane in three different ways—1st, the plane may be parallel to the axis; 2nd, it may be parallel to the base; 3rd, it may be oblique to the axis or the base.

In the first case, the section is a parallelogram, whose length will be equal to the length of the cylinder, and whose width will be equal to the chord of the circle of the base in the line of section. Whence it follows, that the largest section of this kind will be that made by a plane passing through the axis; and the smallest will when the section plane is a tangent—the section in that case will be a straight line.

When the section plane is parallel to the base, the section will be a circle equal to the base. When the section plane is oblique to the axis or the base, the section will be an ellipse.

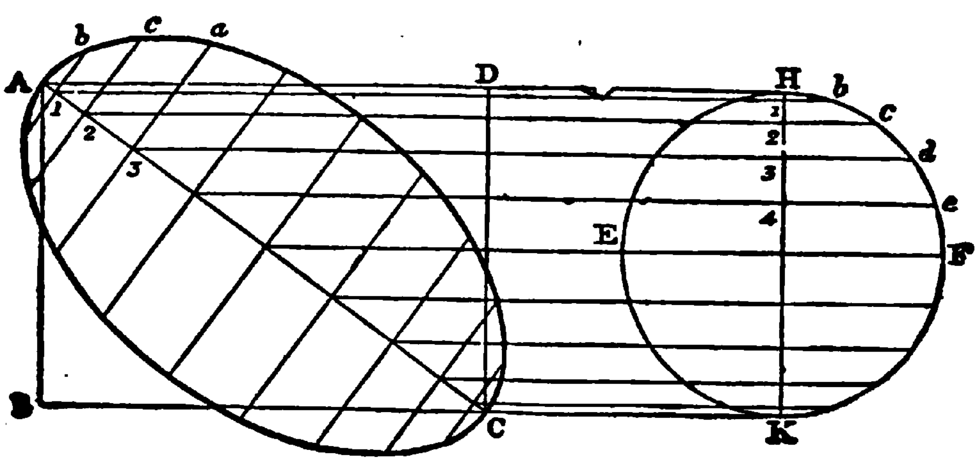


Fig. 42.

XXXIII. To draw the section of a cylinder by a plane oblique to the axis.

Let A B C D (Fig. 42) be the projection of a cylinder, of which the circle E H F K represents the base divided into twenty equal parts at b c d e, &c., and let it be required to draw the section made by the plane A C.

The circular base must be drawn in such a position that the axis of the cylinder when produced meets the centre of the circle. Through the centre of the circle draw the diameter H K perpendicular to the axis produced. Then through the divisions of the base, b c d, &c. draw lines parallel to the axis, and meeting the section plane in 1, 2, 3, &c., and through these points draw perpendiculars to A C making them equal to the corresponding perpendiculars from H K, i e, 1 b, 2 c, 3 d, &c. A curve drawn through the points thus found will be an ellipse, the true section of A B C D on the plane A C.*

The Cone. A cone may be cut by a plane in five different ways, producing what are called the conic sections: 1st. If it is cut by a plane passing through the axis, the section is a triangle, having the axis of a cone as its height, the diameter of the base for its base, and the sides for its sides. If the plane passes through the vertex, without passing through the axis, as c e (Fig. 43), the section will still be a triangle, having for its base the chord c e, for its altitude the line c e, and for its sides the sides of the cone, of which the lines c e, o e are the horizontal and the line c e the vertical projections. 2nd. If the cone is cut parallel to the base, as in g h, the section will be circle, of which g h will be the diameter. 3rd. When the section plane is oblique to the axis, and passes through the opposite sides of the cone, as m p h the section will be an ellipse, m n h. 4th. When the plane is parallel to one of the sides of the cone, as r h, the resulting section is a parabola r s h t u. 5th. When the section plane is such as to pass through the sides of another cone formed by producing

*The point is assumed to be in the inferior half of the cylinder.

the sides of the first beyond the vertex, as the plane q h, the resulting curve in each cone is a hyperbola.

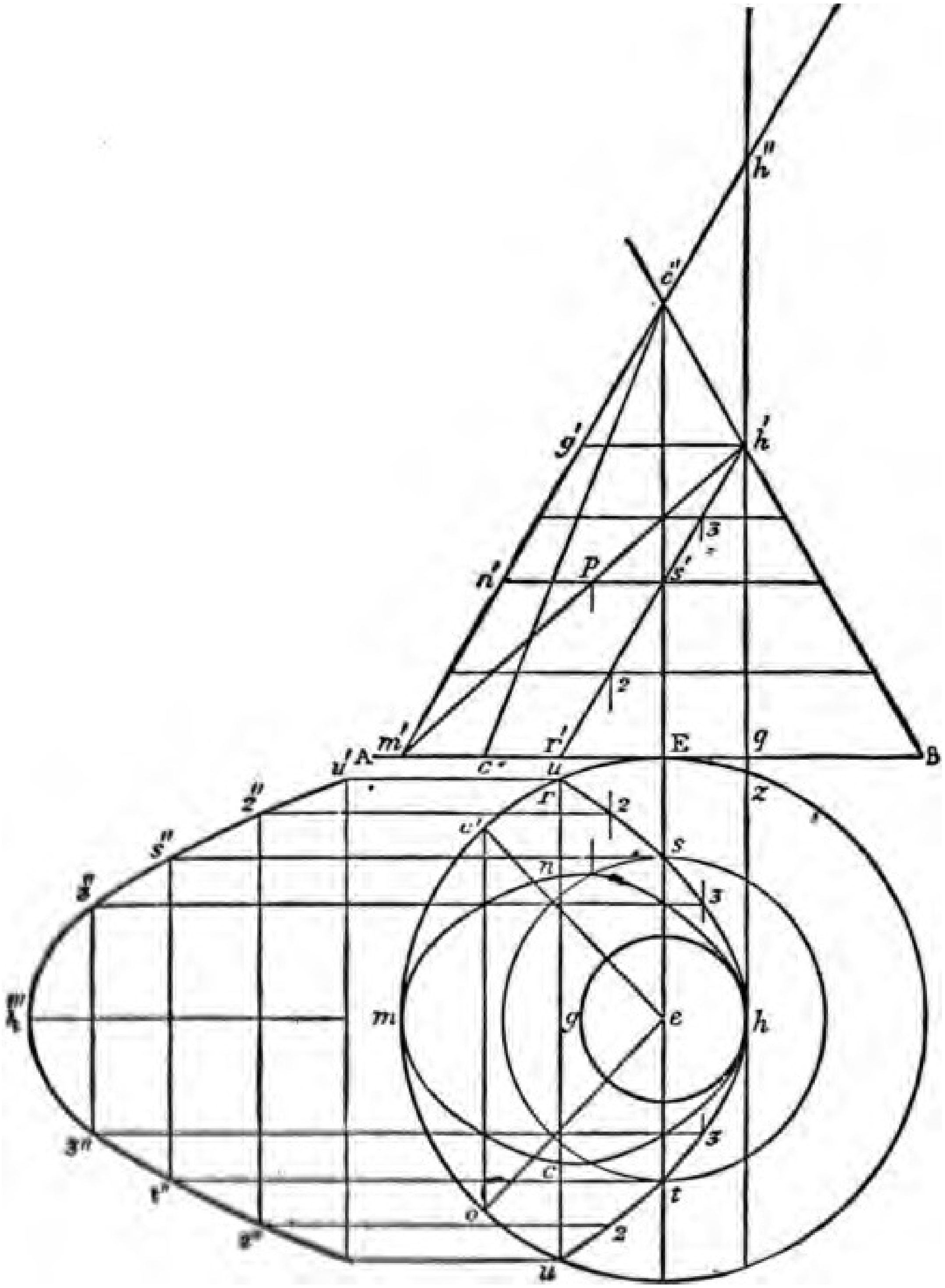


Fig. 43.

Several methods of drawing the curves of the conic sections have already been given in Plane Geometry, Vol. 1. Here their projections, as resulting from the sections of the solid by planes, are to be considered. If the mode of finding the projections of a point on the surface of a given cone be understood, the projections of the curves of the conic sections will offer no difficulty. Let the problem be: First, to find the projections of the section made by the plane m h. Take at pleasure upon the plane the several points, as p, &c. Let fall from these points perpendiculars to the horizontal plane, and on these will be found the horizontal projection of the points; thus, in regard to the point p ——. Draw through p a line parallel to A B: this line will be the vertical projection of the horizontal plane cutting the cone, and its horizontal projection will be a circle, with s n for its radius. With this radius, therefore, from the centre e, describe a circle cutting, twice, the perpendicular let fall from p; the points of intersection will be two points in the horizontal projection of the circumference of the ellipse. In the same manner, any other points may be obtained in its circumference. The operation may often be abridged by taking the point p in the middle of the line m h; for then m h will be the horizontal projection of the major axis, and the two points found on the perpendicular let fall from the central point p will give the minor axis.

To obtain the projections of the parabola, more points are required, such as r, 2, s, 3, h, but the mode of procedure is the same as for the ellipse. The vertical projection of the parabola u s h t u is shown at **u s h t u.**

The projections of the section plane which produces the hyperbola are in this case straight lines, q h, z h.

XXXIV. To draw the section of a cone made by a plane cutting both its sides, i. e., an ellipse.

Let A D B (Fig. 44) be the vertical projection of the cone A C B the horizontal projection of half its

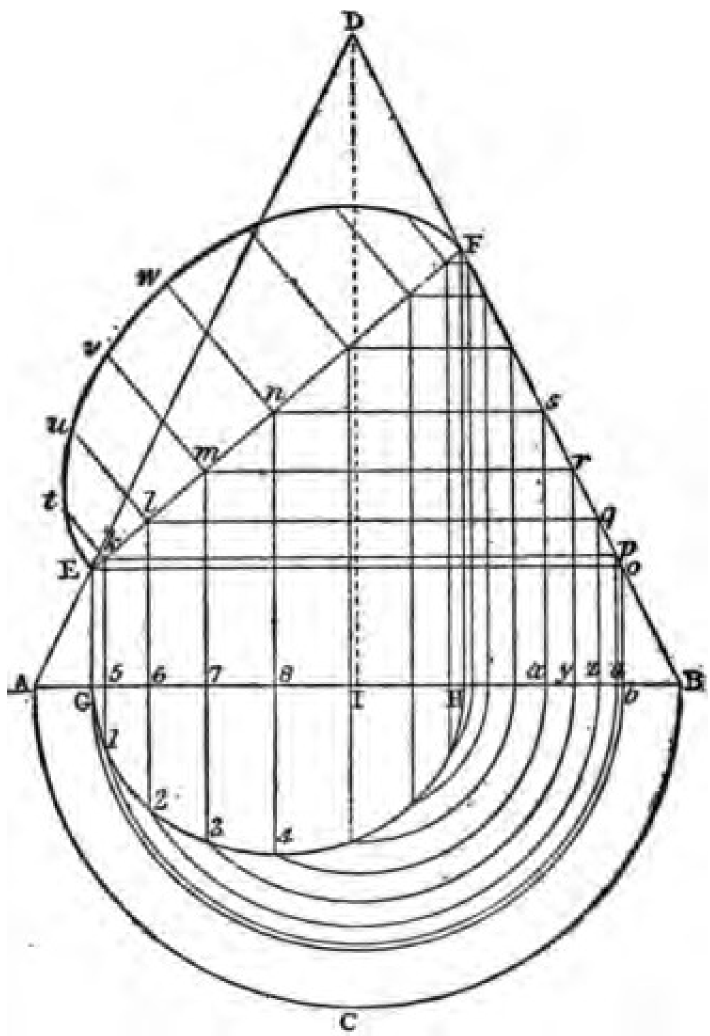


Fig. 44.

base, and E F the line of section. From the points E and F let fall on A B the perpendiculars E G, F H. Take any points in E F, as k, l, m, n, &c., and from them draw lines parallel to A B, as k p, l q, m r, &c.,

and also lines perpendicular to A B, as k l, 12 m 3, &c. Also from p, q, r, &c., let fall perpendiculars on A B, namely, p a, q z, r y, &c. From the centre of the base of the cone, l, with radius la, lz, ly, &c., describe arcs cutting the perpendiculars let fall from k, l, m, &c., in 1, 2, 3, &c. A curve traced through these points will be the horizontal projection of the section made by the plane E F. To find the true section,—Through k, l, m, &c., draw kt, lu, m v, n w, perpendicular through E F, and make them respectively equal to the corresponding ordinates, 5 1, 6 2, 7 3, &c., of the horizontal projection G 4 H, and points will be obtained through which the half E w F of the required ellipse can be traced. It is obvious that, practically it is necessary only to find the minor axis of the ellipse, the major axis E F being given.

XXXV. To draw the section of a cone made by a plane parallel to one of its sides, i. e. a parabola.

Let A D B (Fig. 45) be the vertical projection of a right cone, and A C B half the plan of its base; and let E F be the line of section. In E F take any number of points, E, a, b, c, e, F, and through them draw lines E H, a 61, b 7 2, &c., perpendicular to A B, and also lines parallel to A B, meeting the side of the cone in f, g, h, k, 1: from these let fall perpendiculars on A B, meeting it in m n, o, p, q. From the centre of the base l, with the radii lm, ln, lo, &c., describe arcs cutting the perpendiculars let fall from the section line in the points 1, 2, 3, 4, 5; and through the points of intersection trace the line H 12345G, which is the horizontal projection of the section. To find the true section, from E, a, b, c, d,e, raise perpendiculars to E F, and make them respectively equal to the ordinates in the horizontal projection, as E r equal to E H, a s equal to 6 1, &c., and the points r s t u v w in the curve will be

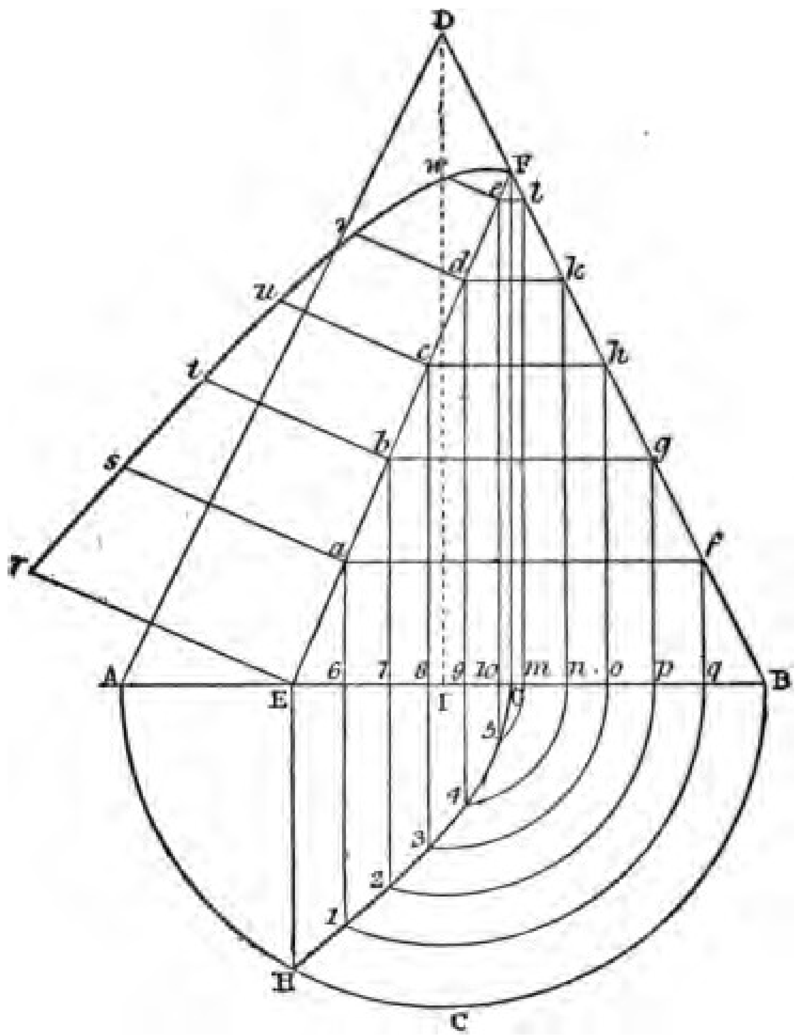


Fig. 45.

obtained. The other half of the parabola can be drawn by producing the ordinates w e, v d, &c., and setting the same distances to the right of E F.

XXXVI. To draw the section of a cone made by a plane parallel to the axis i. e. an hyperbola.

Let d c d (Fig. 46) be the vertical projection of the cone, d q r d one half of the horizontal projection of the base, and q r the section plane. Divide the line r q into any number of equal parts in 1, 2, 3, h, &c., and through them draw lines perpendicular to d d. From c as centre, with the radii c1, c2, &c., describe the arcs

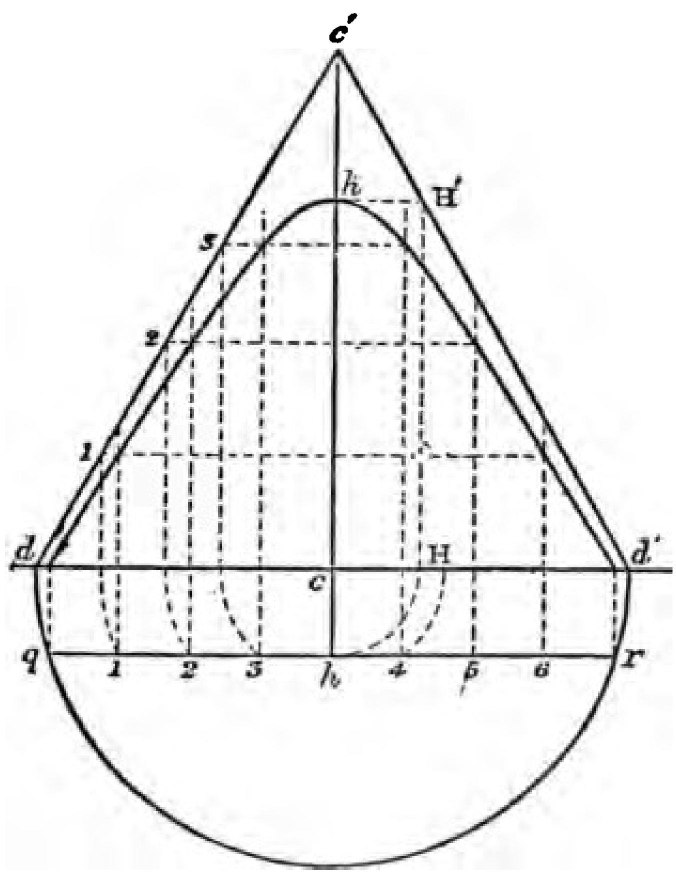


Fig. 46.

cutting d d; and from the points of intersection draw perpendiculars cutting the sides of the cone in 1, 2, 3, and these heights transferred to the corresponding perpendiculars drawn directly from the points 1, 2, 3, &c., in r q, will give points in the curve.

XXXVII. To draw the section of a cuneoid made by a plane cutting both its sides.

Let A C B (No. 1, Fig. 47) be the vertical projection of the cuneoid, and A 5 B the plan of its base, and A B (No. 4) the length of the arris at C, and let D E be the line of section. Divide the semicircle of the base into any number of parts 1, 2, 3, 4, 5, &c., and through them draw perpendiculars to A B, cutting it in l, m, n, o, p, &c., and join Cl, C m, Cn, &c., by lines cutting the section line in 6, 7, 8, 9, &c. From these points draw lines perpendicular to D E, and make them equal to the corresponding ordinates of the semicircle, either by transferring the lengths by the compasses, or by proceeding as shown in the figure. The curve drawn through the points thus obtained will give the required section.

The section on the line D K is shown in No. 2, in which A B equals D K; and the divisions e f g h k in D K, &c., are transferred to the corresponding points on A B; and the ordinates e l, f m, g n, &c., are made equal to the corresponding ordinates l1, m2, n3, of the semicircle of the base. In like manner, the section of the line G H, shown in No. 3, is drawn.*

XXXVIII. To describe the section of a cylinder made by a curve cutting the cylinder.

Let A B D E (Fig. 48) be the projection of the cylinder, and C D the line of the section required. On A B

*A cuneoid is a solid ending in a straight line, in which, if any point be taken, a perpendicular from that point may be made to coincide with the surface. The base of the cuneoid may be of any form; but in architecture it is usually semicircular or semi-elliptical, and parallel to the straight line forming the other end.

describe a semicircle, and divide it into any number of parts. From the points of division draw ordinates 1 h, 2k, 3l, 4m, &c., and produce them to meet the line of section in o, p, q, r, s, t, u, v, w. Bend a rule or slip of paper to the line C D, and prick off on its points C, o, p, q, &c.; then draw any straight line F G, and, un-

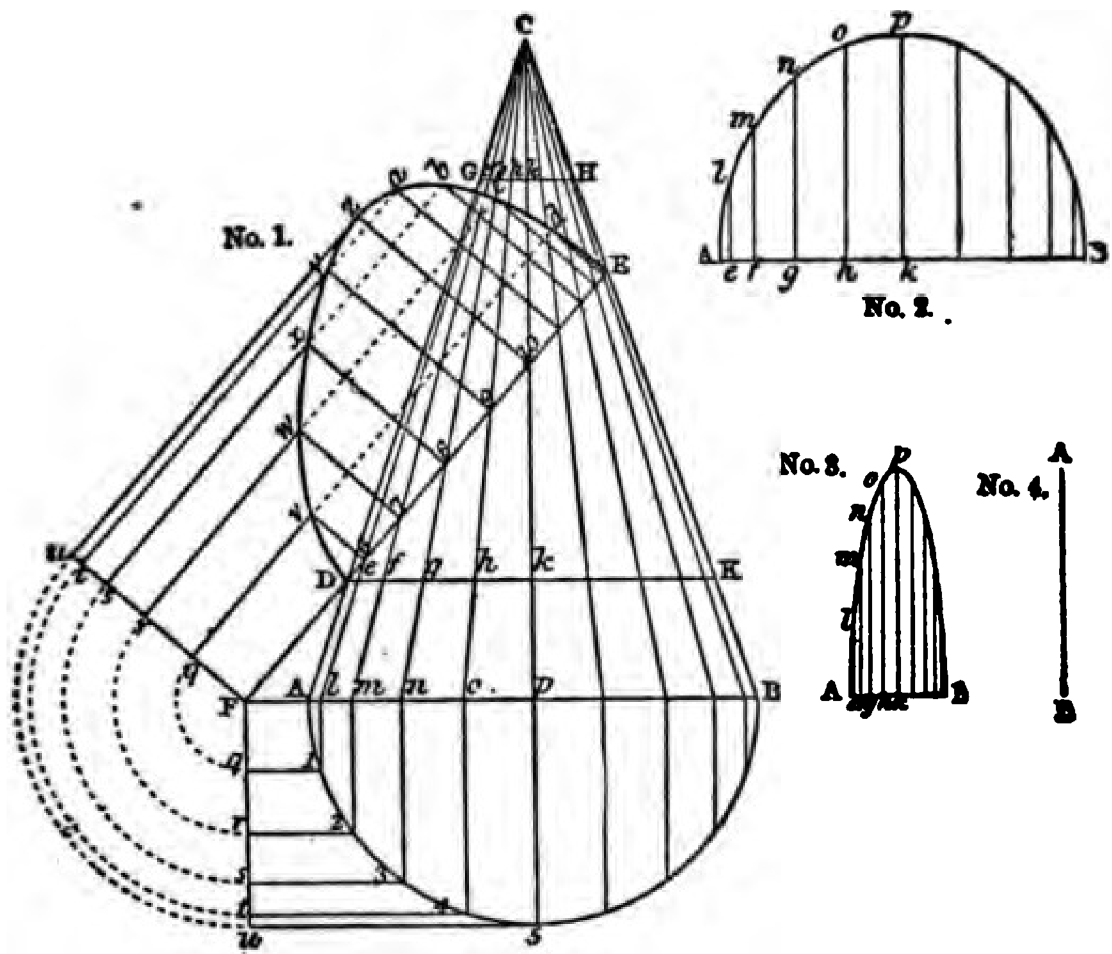


Fig. 47.

bending the rule, transfer the points C, o, p, q, &c., to F, a, b, c, d, &c. Draw the ordinates a1, b 2, c 3, &c., and make them respectively equal to the ordinates h 1, k2, 1 3, &c., and through the points found trace the curve.

XXXIX. To describe the section of a sphere.

Let A B D C (Fig. 49) be the great circle of a sphere, and F G the line of the section required. Then since all the sections of a globe or sphere are circles, on F G describe a semicircle F 4G, which will be the section required.

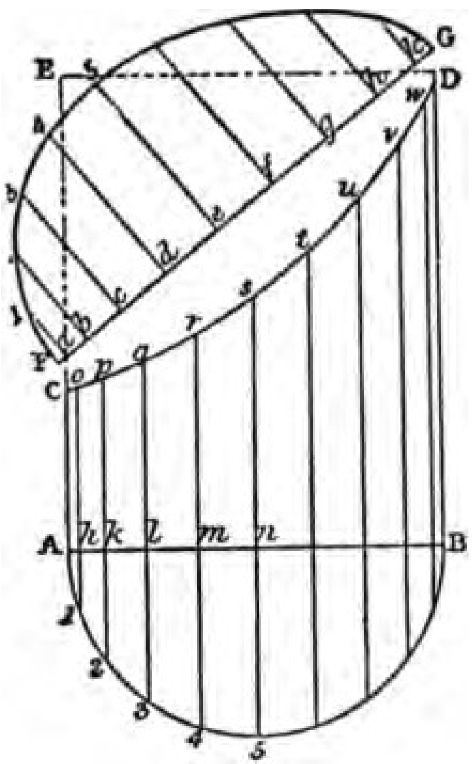


Fig. 48.

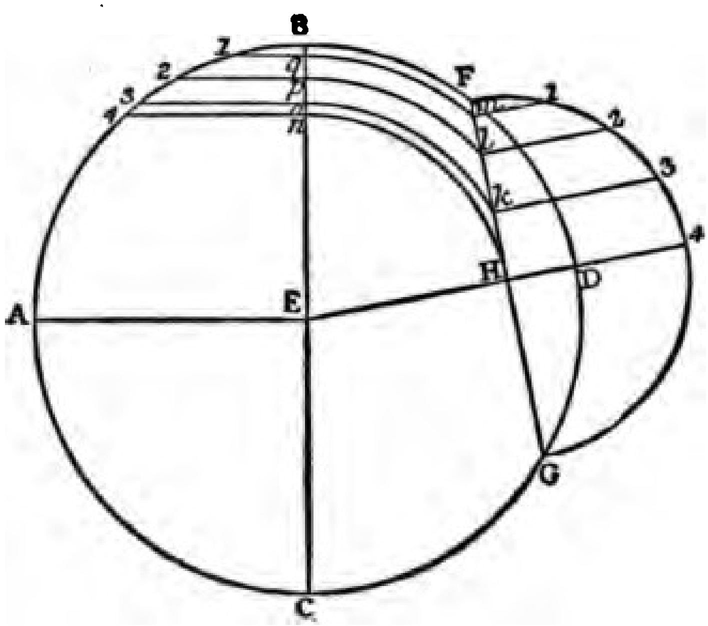


Fig. 49.

Or, in F G take any number of points, as m, l, k, H, and from the centre of the great circle E, describe the arcs H n, k o, l p, m q, and draw the ordinates H 4, k 3, l 2, ml, and n 4, o3, p2, q l; then make the ordinates on F G equal to those on B C, and the points so obtained will give the section required.*

*The projections of sections of spheres are, if the section panes are oblique, either straight lines or ellipses, and are found as follows:

Let *a b* (Fig. 50) be the horizontal projection of the section plane. On the line of section take any number of points, as *a*, *c*, *b*. and through each of them draw a line perpendicular to *y z*.

XL. To describe the section of an ellipsoid when a section through the fixed axis, and the position of the line of the required section are given.

Let A B C D (Fig. 51) be the section through the fixed axis of the ellipsoid, and F G the position of the line of the required section. Through the centre of

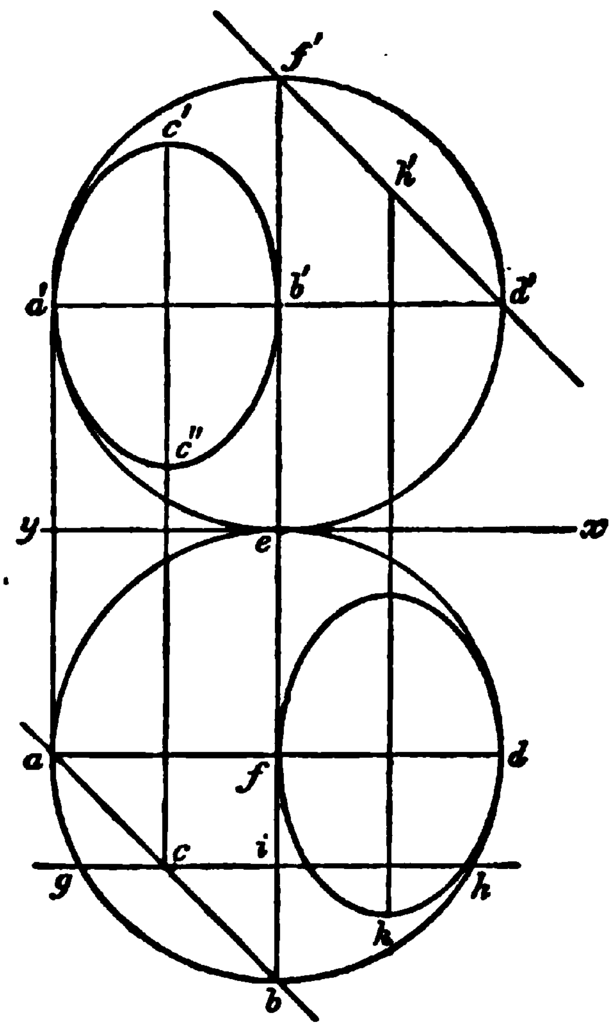


Fig. 50.

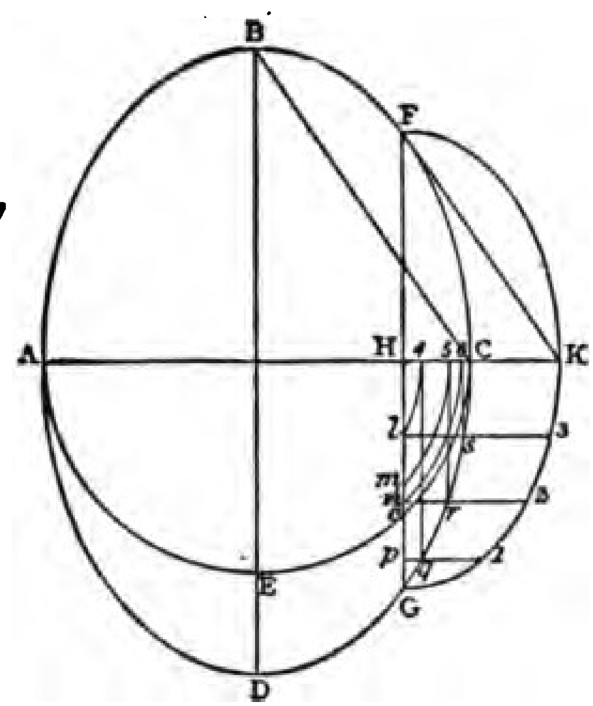


Fig. 51.

the ellipsoid draw B D parallel to F G; bisect F G in H and draw A C perpendicular to F G; join B C, and from F draw F K, parallel to B C, and cutting A C produced in K; and then will H K be the height of the semi-ellipse forming the section on F G.

Or, the section may be found by the method of ordinates, thus: As the section of the ellipsoid on the line A C is a circle, from the point of intersection of B D and A C describe a semicircle A E C. Then on H G, the line of section, take any number of points, l, m, p, and from them raise perpendiculars cutting the ellipse in q, r, s. From q, r, s draw lines perpendicular to A C, cutting it in the points 4, 5, 6; and again, from the intersection of B D and A C as centre draw the arcs 4 l, 5 m, 6n, C o, cutting H G in l, m, n, o; then H o, set off on the perpendicular from H to K, is the height of the section; and the heights H n, H m, Hl, set off on the perpendiculars from l to 3, n to 2 and p to 1, give the heights of the ordinates.

XLI. To find the section of a cylindrical ring perpendicular to the plane passing through the axis of the ring, the line of section being given.

Let A B E D (Fig. 52) be the section through the axis of the ring, A B a straight line passing through the concentric circles to the centre C, and D E be the line of section. On A B describe a semicircle; take in its circumference any points as 1, 2, 3, 4, 5, &c., and draw the ordinates 1 f, 2g, 3 h, 4 k, &c. Through the points f, g, h, k, l, &c., where the ordinates meet the line A B, and from the centre C, draw concentric circles, cutting the section line in m, n, o, p, q, &c. Through these points draw the lines m1, n2, o 3, &c., perpendicular to the section line, and transfer to them the heights of the ordinates of the semicircle f 1, g 2, &c.; then through the points 1, 2, 3, 4, &c., draw the curve D 5 E, which is the section required.

Again, let R S be the line of the required section; then from the points t, u, v, w, c, x, d, &c., where the concentric circles cut this line, draw the lines t 1, u2, v 3, &c., perpendicular to R S, and transfer to them the

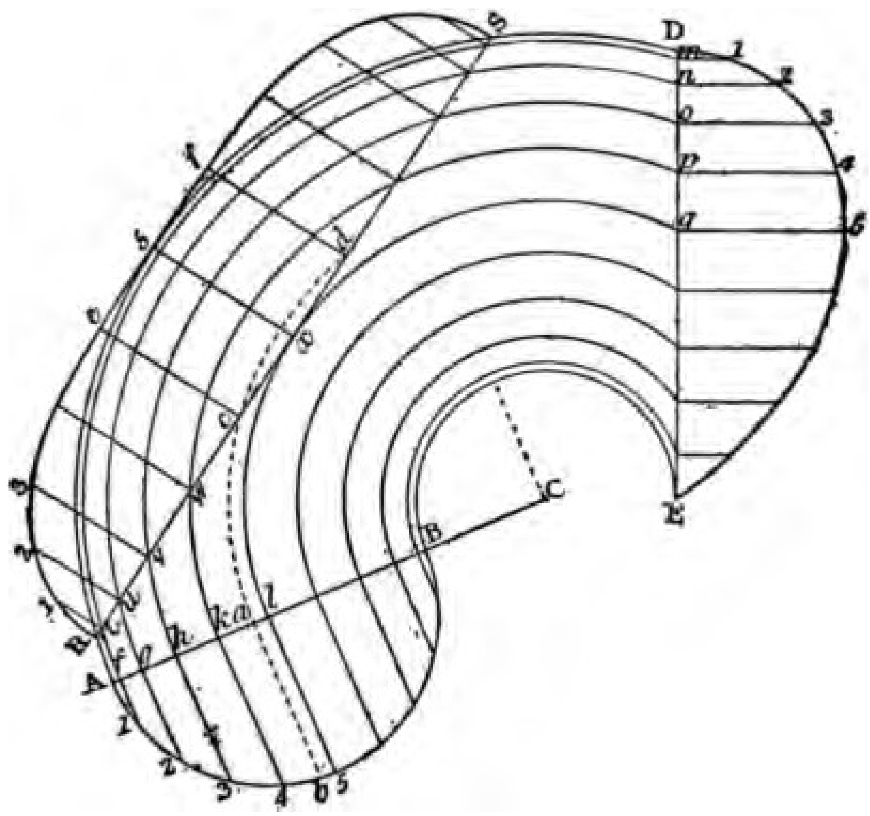


Fig. 52.

corresponding ordinates of the semicircle; and through the points 1, 2, 3, 4, e, 5, f, &c., draw the curve R e f S, which is the section required.

XLII. To describe the section of a solid of resolution the generating curve of which is an agee.

Let A D B (Fig. 53) be half the plan or base of the solid, A a b B the vertical section through its axis, and E F the line of section required. From G draw C 5 perpendicular to E F, and bisecting it in m. In E m take any number of points, g h k, &c., and through them draw the lines g 1, h 2, k 3, &c., perpendicular to E F. Then from C as a centre, through the points g, h, k, &c., draw concentric arcs cutting A B in r, s, t, u, v, and through these points draw the ordinates r 5, s 4, t 3, &c., perpendicular to A B. Transfer the heights of the ordinates on A B to the corresponding ordinates on each side of the centre of E F; and through the points 1, 2, 3, 4, 5, &c., draw the curve E 5 F which is the section required.

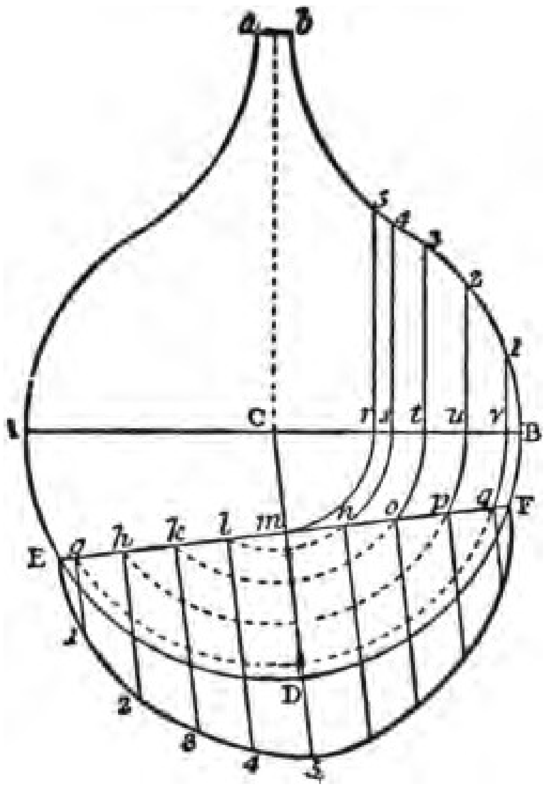


Fig. 53.

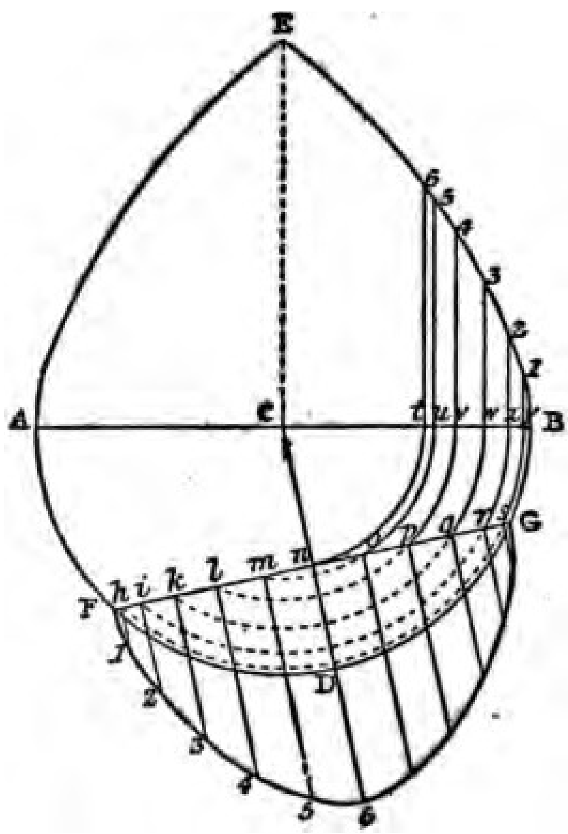


Fig. 54.

XLIII. To find the section of a solid of resolution, the generating curve of which is of a lancet form.

Let A D B (Fig. 54) be the plan of half the base, A E B the vertical section, and F G the line of the required section. The manner of finding the ordinates and transferring the heights is precisely the same as in the last problem.

XLIV. To find the section of an ogee pyramid with a hexagonal base.

Let A D E F B (Fig. 55) be the plan of the base of the pyramid, A a b B a vertical section through its axis, and G H the line of the required section. Draw

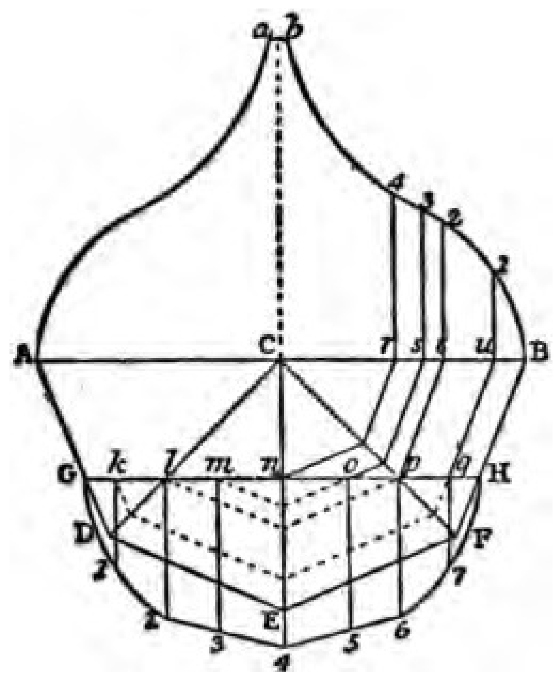


Fig. 55.

the arrises C D, C E, C F. On the line of section G H, at the points of intersection of the arrises with it, and at some intermediate points k, m, o, q (the corresponding points k and q, and m and o, being equidistant from n), raise indefinite perpendiculars. Through these points k, l, m, n, o, p, q, draw lines parallel to the sides of the base, as shown by dotted lines; and from the points where these parallels meet the line A B, draw r 4, s 3, t 2, u 1, perpendicular to A B. These perpendiculars transferred to the ordinates n 4, m 3, o 5, l 2, p 6, k 1, q 7, will give the points 1, 2, 3, 4, 4, 5, 6, 7, through which the section can be drawn.

6. INTERSECTIONS OF CURVED SURFACES.

When two solids having curved surfaces penetrate or intersect each other, the intersections of their surfaces form curved lines of various kinds. Some of these, as the circle, the ellipse, &c., can be obtained in the plane; but the others cannot, and are named curves of double curvature. The solution of the following problems depends chiefly on the knowledge of how to obtain, in the most advantageous manner, the projections of a point on a curved surface; and is in fact the application of the principles elucidated in the several previous problems. The manner of constructing the intersections of these curved surfaces which is the simplest and most general in its application, consists in conceiving the solids to which they belong as cut by planes according to certain conditions, more or less dependent on the nature of the surfaces. These section planes may be drawn parallel to one of the planes of the projection; and as all the points of intersection of the surfaces are found in the section planes, or on one of their projections, it is always easy to construct the curves by transferring these points to the other projection of the planes.

XLV. The projection of two equal cylinders which intersect at right angles being given, to find the projections of their intersections.

Conceive, in the horizontal projection (Fig. 56), a series of vertical planes cutting the cylinders parallel to their axis. The vertical projections of all the sections will be so many right-angled parallelograms, similar to e f e f, which is the result of the section of the cylinder from surface to surface. The circumference of the second cylinder, whose axis is vertical, is cut by the same plane, which meets its upper surface at the two points g, h, and its under surface at two corresponding points. The vertical projections of these points are on the lines perpendicular to a b, raised on

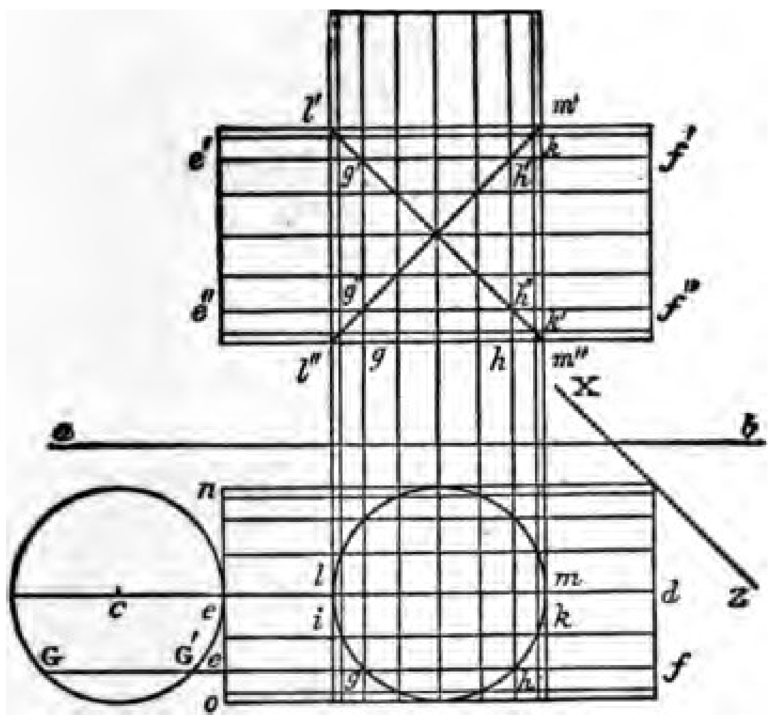


Fig. 56.

each of them, so that upon the lines e f, e f, will be situated the intersections of these lines at the points g, h, and g, h and the same with other points i, k, l, m. It is not necessary to draw a plan to find these projections. All that is actually required is to draw the circle representing one of the bases (as n o) of the cylinder laid flat on the horizontal plane. Then to produce g h till it cuts the circle at the superior and inferior points G, G, and to take the heights e G, e G, and carry them, upon a b, from g to g g and from h to h, h.

Fig. 57 is the vertical projection made on the line x z.

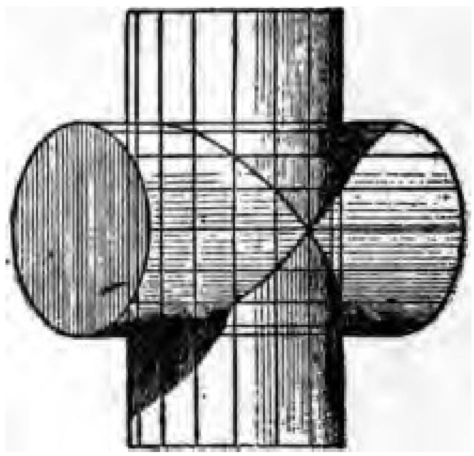


Fig. 57.

XLVI. To construct the projections of two unequal cylinders whose axis intersect each other obliquely.

Let A (Fig. 58) be the vertical projection of the two cylinders, and h S d e the horizontal projection of their axis. Conceive in the vertical projection, the cylinders cut by any number of horizontal planes; the horizontal projections of these planes will be rectangles, as in the previous example, and their sides will be parallel to the axis of the cylinders.

The points of intersection of these lines will be the points sought. Without any previous operation, six of those points of intersection can be obtained. For example the point c is situated on d e, the highest point of the smaller cylinder; consequently, the horizontal projection of c is on d e, the horizontal projection of

d e, and it is also on the perpendicular let fall from c, that is to say, on the line c f parallel to the axis of the cylinder S h. The point sought will therefore, be the intersection of those lines at c. In the same way i is

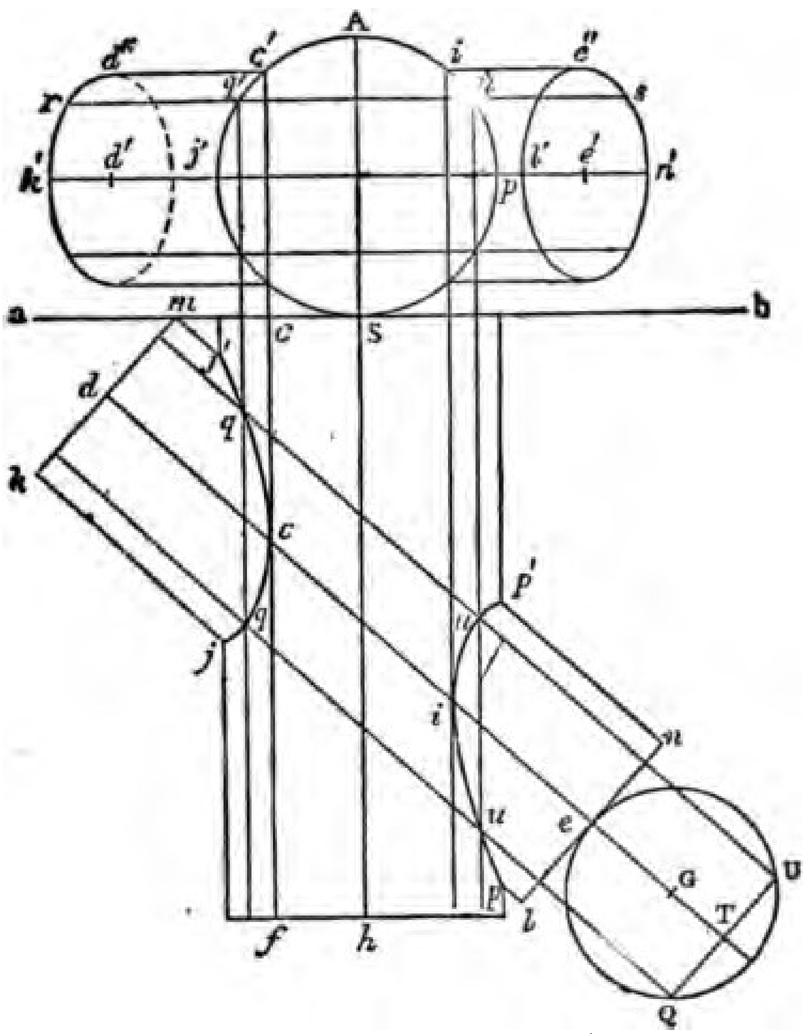


Fig. 58.

obtained. The point j is on the line k l, which is in the horizontal plane passing through the axis d e, the horizontal projections of k l are k l, and its opposite m n; therefore in letting fall perpendiculars from j p, the intersections of these with k l, m n, give the points j j, p p. Thus six points are obtained. Take at pleasure an intermediate point q; through this point draw a line r s parallel to a b, which will be the vertical projection of a horizontal plane cutting the cylinder in q. The horizontal projection of this section will be, as in the preceding examples, a rectangle which is obtained by taking, in the vertical projection, the height of the section plane above the axis d e, and carrying it on the base in the horizontal projection from G to T. Through T is then to be drawn the line Q U perpendicular to G T; and through Q and U the lines parallel to the axis; and the points in which these lines are intersected by the perpendiculars let fall from q u are the intermediate point required. Any number of intermediate points can thus be obtained; and the curve being drawn through them, the operation is completed.

XLVII. To find the intersections of a sphere and a cylinder.

Let e f c d and i k g h (Fig. 59) be the horizontal projections of the sphere and cylinder respectively. Draw parallel to A B, as many vertical section planes are considered necessary, as e f, c d. These planes cut at the same time both the sphere and the cylinder, and the result of each section will be a circle in the case of the sphere, and a rectangle in the case of the cylinder. Through each of the points of intersection g, h, i, k, and from the centre l, draw indefinite lines perpendicular to A B. Take the radius of the circles of the sphere proper to each of these sections and with them, from the centre l, cut the correspondent perpendiculars in g g, h h, i i, &c., and draw through these points the curves of intersection.

XLVIII. To construct the intersection of two right cones with circular bases.

The solution of this problem is founded on the knowledge of the means of obtaining on one of the projections of a cone a point given on the other.

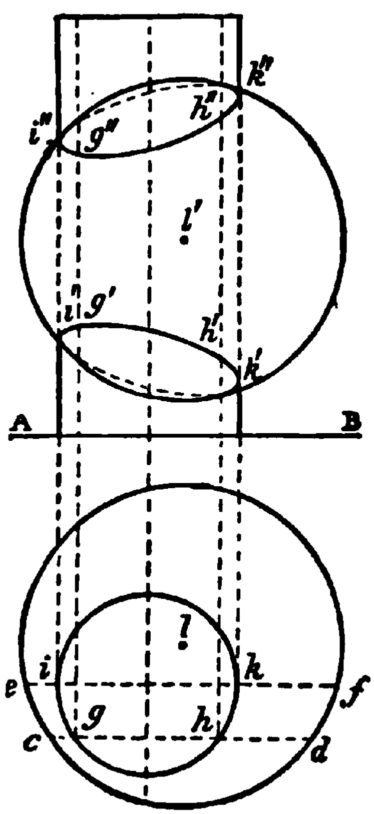


Fig. 59.

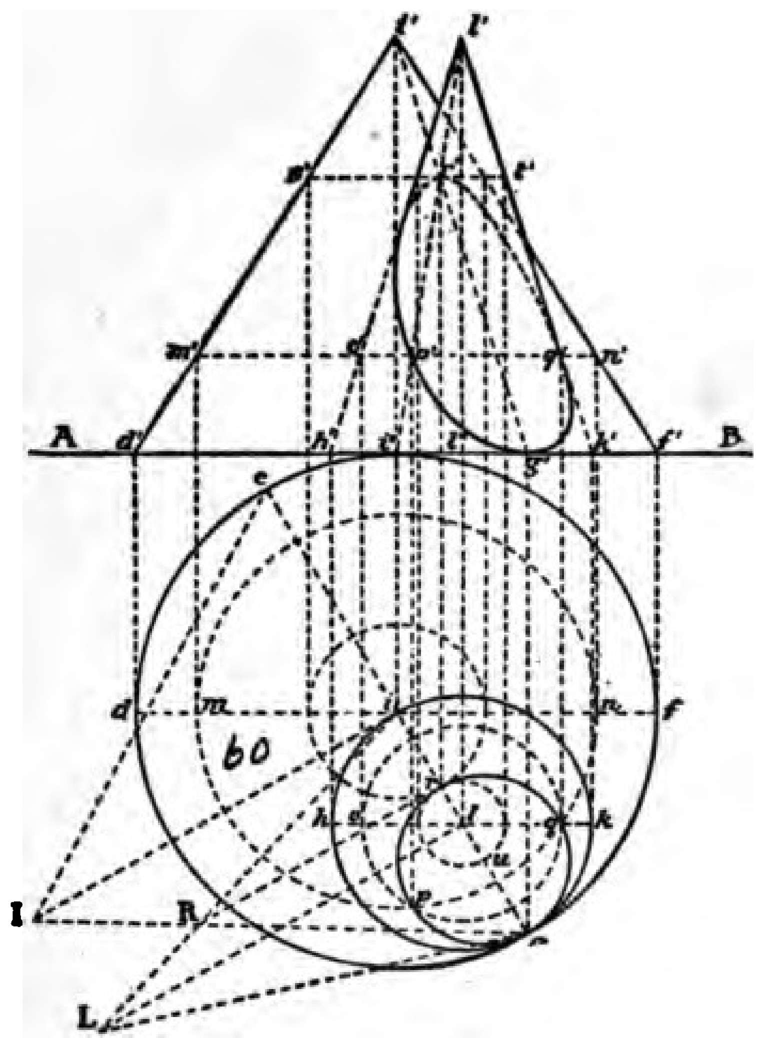


Fig. 60.

No. 1. Let A B (Fig. 60) be the common section of the two planes of projection, the circles g d e f and g h i k the horizontal projections of the given cones, and the triangles d i f and h l k their vertical projections. Suppose these cones cut by a series of horizontal planes:

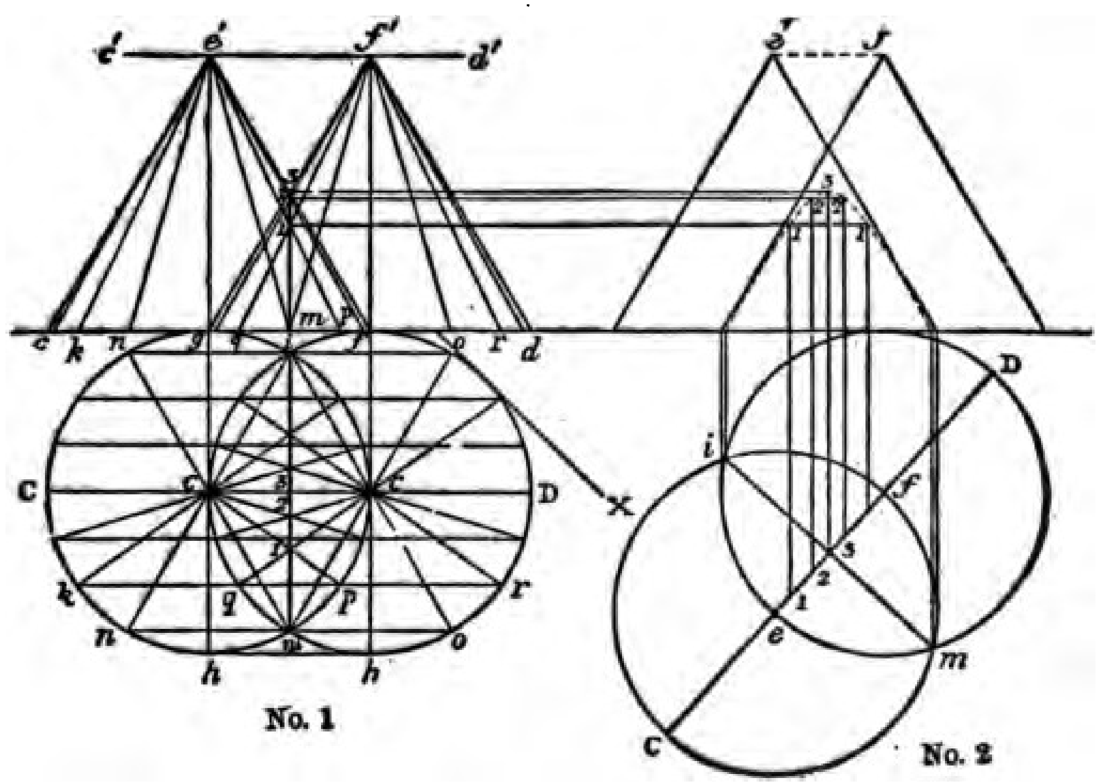
each section will consist of two circles, the intersections of which will be points of intersection of the conical surfaces. For example, the section made by a plane m n will have for its horizontal projections two circles of different diameters, the radius of the one being i m, and of the other l o. The intersecting points of these are p and q, and these points are common to the two circumferences; and their vertical projection on the plane m n will be p q. Thus, as many points may be found as is necessary to complete the curve.

But there are certain points of intersection which cannot be rigorously established by this method without a great deal of manipulation. The point r in the figure is one of those; for it will be seen that at that point the two circles must be tangents to each other, and it would be difficult to fix the place of the section plane s t so exactly by trial, that it would just pass through the point.

It will be seen that the point r must be situated in the horizontal projection of the line g i a perpendicular i l equal to the height of the cone. From one raise a perpendicular and make it equal to the height of the second cone, and draw its side L i; and from the point of intersection R let fall a perpendicular on g i meeting it in r; through r draw an indefinite line perpendicular to A B, and set up on it from A B to r the height r R. The point r can also be obtained directly in the vertical projection by joining i g and l i as shown.

Bisect r g in u, and from u as a centre, with the radius u r, describe a circle, the circumference of which will be the horizontal projection of the intersection of the two cones. The vertical projection of this circle will be g p r q, and can be found by the method indicated above.

No. 2. Conceive the horizontal projection (Fig. 61, No. 1) a vertical plane C D cutting both cones through their axis: the sections will be two triangles, having the diameters of the bases of the cones as their bases,



No. 1 No. 2

Fig. 61.

and the height of the cones as their height. And as in the example the cones are equal, the triangles will also be equal, as the triangles c e f, g f d, in the vertical projection. Conceive now a number of inclined planes, as c n m, c k p, &c., passing through the different points of the base, but still passing through the summits of the cones: the sections which result will still be triangles (as has already been demonstrated), whose bases diminish in proportion as the planes recede from the centres of the bases of the cones, until at length the plane becomes a tangent to both cones and the result is a tangent line whose projections are h c, h c, g e, f f. It will be observed that the circumferences of the bases cut each other at m and i, which are the first points of their intersections, whose vertical projections are the point m merely. If the projections of the other points of intersection on the lines of the section planes are found (an operation presenting no difficulty, and easily understood by the inspection of the figure), it will be seen that the triangles n c m, m c o, k e p, q e r, &c., in the horizontal projection, have for their vertical projections the triangles n e m, m o, k e p, &c., and that the intersections of the cones are in a plane perpendicular to both planes of projection, and the projections of the intersections are the right lines i m, m 3. From the known properties of the conic sections, the curve produced by this plane will be a hyperbola. Fig. 61, No. 2, gives the projections of the cones on the line o x.

No. 3. The next example (Fig. 62) differs from the last in the inequality of the size of the cones. Suppose an indefinite line C D to be the horizontal projection of the vertical section plane, cutting the two cones through their axis e f. Conceive in this plane an indefinite line e f D, passing through the summits of the cones, the vertical projection of this line will be e f d: from d let fall on C D a perpendicular meeting it in D; this will be the point in which the line passing through the summits of the cones will meet the horizontal plane; and it is through this point, and through the summits e and f, that the inclined section planes should be made to pass. The horizontal traces of these planes are O D, G D, &c.: O D is then the trace of a tangent plane to the two conical surfaces O e, P f; and the plane e G D cuts the greater cone, and forms by the section the triangles G e H in the horizontal, and g e h in the vertical projection; and it cuts the lesser cone, and forms the triangles I f J, i f j. In the horizontal projection it

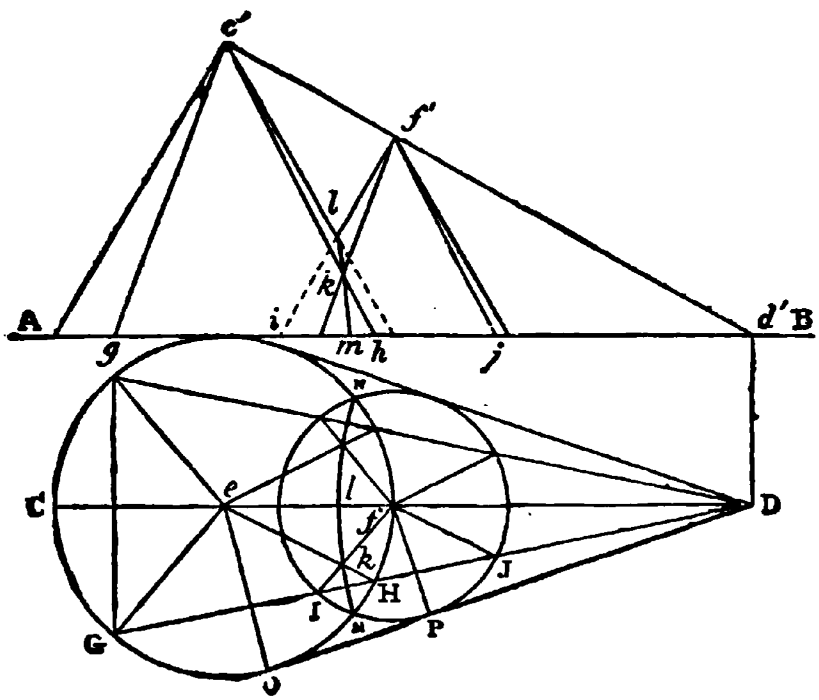


Fig. 62.

is seen that the sides H e, I f of the triangles intersect in k, which is therefore the horizontal projection of one of the points of intersection; and its vertical projection is k. In the same manner, other points can be found. It is seen at once that M, N, l, are also points in the intersection. The curves traced through the points M k l N in the horizontal, and m k l in the vertical projection, are the projections of the intersection of the two cones.

7. COVERINGS OF SOLIDS.

I. **Regular Polyhedrons.** A solid angle cannot be formed with fewer than three plane angles. The simplest solid is therefore the tetrahedron or pyramid having an equilateral triangle for its base, and its other three sides formed of similar triangles.

The development of this figure (Fig. 63) is made by drawing the triangler base A B C, and then drawing around it the triangles forming the inclined sides. If the diagram is on flexible material, such as paper, then cut out, and the triangles folded on the lines A B, B C, C A, the solid figure will be constructed.

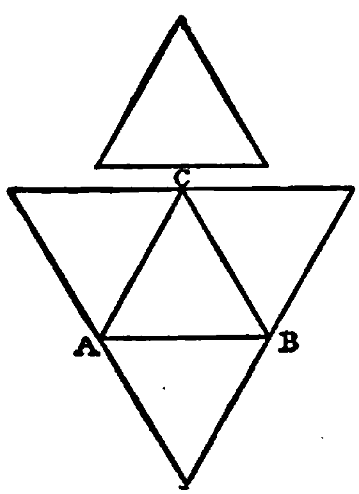


Fig. 63.

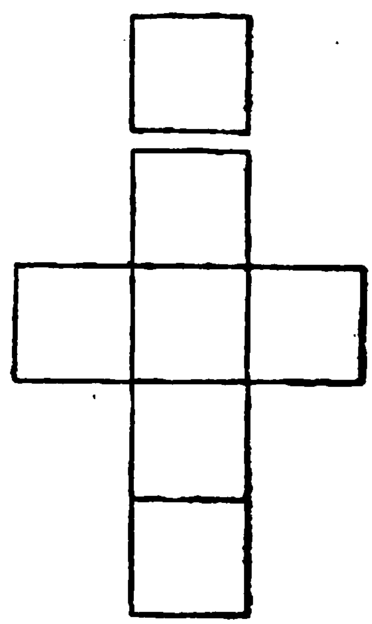


Fig. 64.

The **hexahedron,** or cube is composed of **six equal** squares (Fig. 64); the **octahedron** (Fig. 65) of eight equilateral triangles; the **dodecahedron** (Fig. 66) of twelve pentagons; the **icosahedron** (Fig. 67) of twenty equilateral triangles. In these figures, A is the elevation, and B the development.

The elements of these solids are the equilateral triangle, the square, and the pentagon. The irregular polyhedrons may be formed from those named, by cutting off the solid angles. Thus, in cutting off the

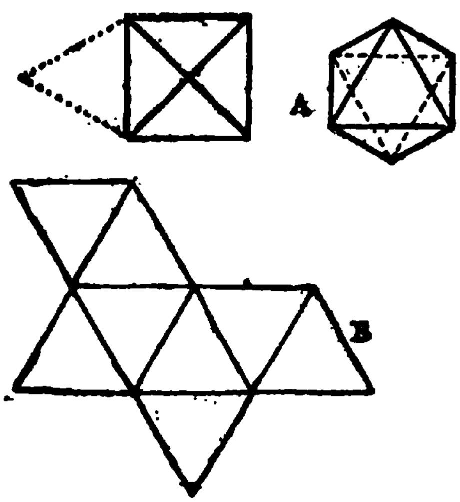


Fig. 65.

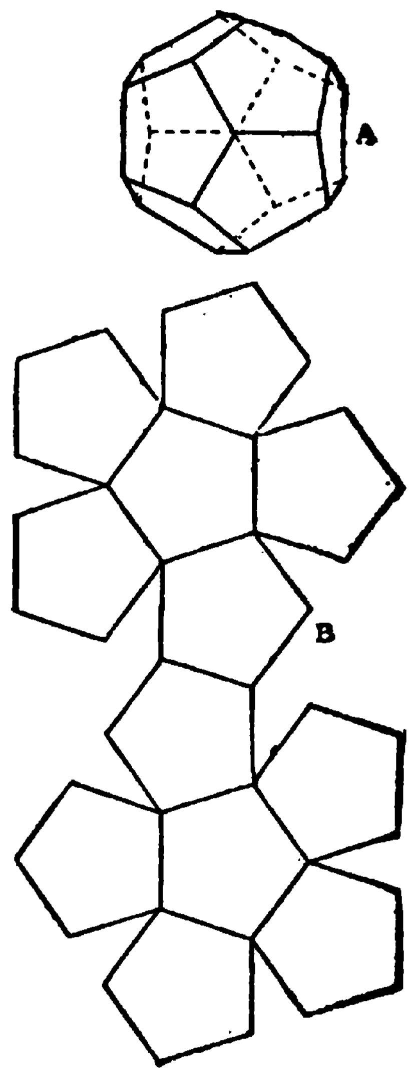


Fig. 66.

angles of the tetrahedron, there results a polyhedron of eight faces, composed of four hexagons and four equilateral triangles. The cutting off the angles of the cube, in the same manner gives polyhedron of fourteen faces, composed of six octagons, united by eight equilateral triangles.

The same operation performed on the octahedron produces fourteen faces, of which eight are hexagonal and six square; on the dodecahedron it gives thirty-two sides, namely, twelve decagons, and twenty triangles; on the icosahedron it gives thirty-two sides—twelve tentagons and twenty hexagons. This last approaches almost to the globular form and can be rolled like a ball.

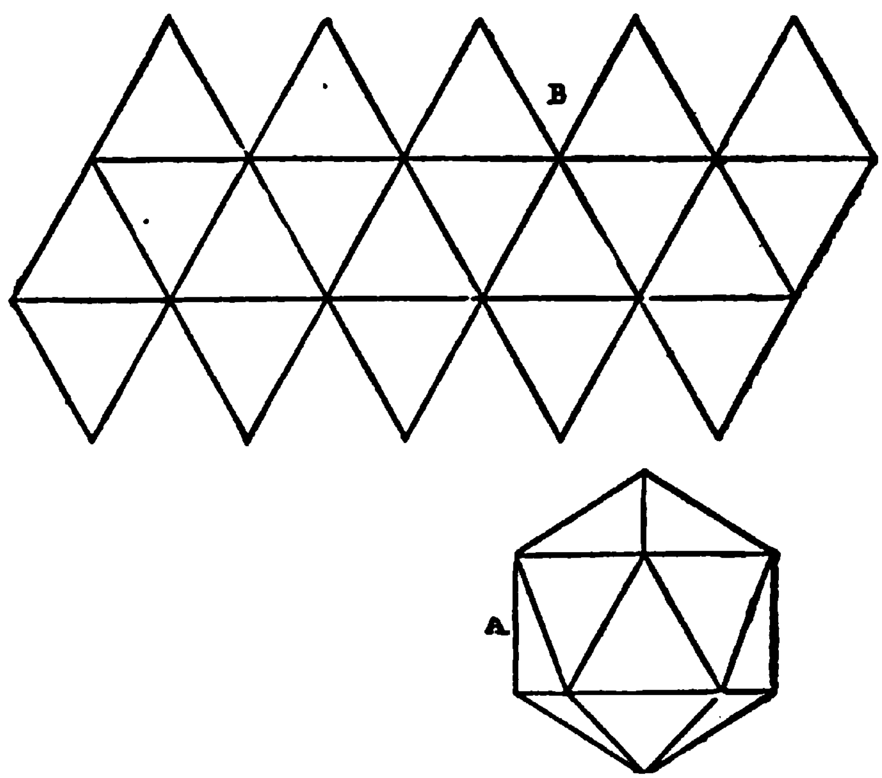


Fig. 67.

The other solids which have plane surfaces are the pyramids and prisms. These may be regular and irregular: they may have their axis perpendicular or inclined: they may be truncated or cut with a section, parallel or oblique, to their base.

II. Pyramids. The development of a right pyramid, of which the base and the height are given, offers no difficulty. The operation consists (Fig. 68) in elevating on each side of the base, a triangle having its height equal to the inclined height of each side, or, otherwise, connecting the sides, together as shown by the dotted lines.

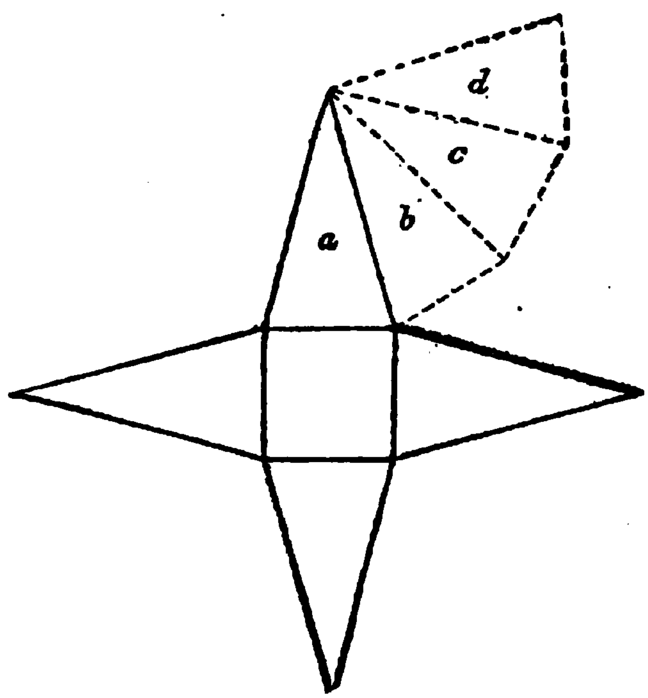


Fig. 68.

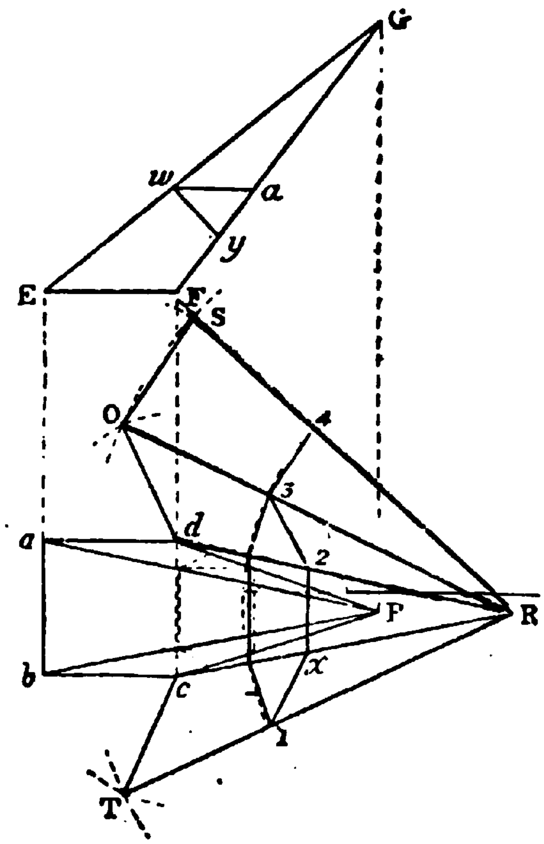


Fig. 69.

In an oblique pyramid the development is found as follows: Let a b c d (Fig. 69) be the plan of the base of the pyramid, a b c d its horizontal projection, and E F G its vertical projection. Then on the side d c construct the triangle c R d, making its height equal to the sloping side of the pyramid F G. This triangle is the development of the side d P c of the pyramid.

Then from d, with the radius E F, describe an arc O; and from R, with the radius equal to the true length e G of the arris E G, describe another arc intersecting the last at O. Join R O, d O; the triangle d R O will be the development of the side a P d. In the same way, describe the triangle c R T, for the development of the side b P c. From R, again, with the same radius R O, describe an arc S, which intersect by an arc described from O with the radius a b; and the triangle O R S will be the development of the side a P b.

If the pyramid is truncated by a plane w a parallel to the base, the development of that line is obtained by setting off from R on R c, and R d, the true length of the arris G a in x and 2, and on R S, R O, and R T, the true length of the arris G w in 4, 3, 1; and drawing the lines 1 x, x2, 2 3, 3 4, parallel to the base of the respective triangles T R c, c R d, d R O, O R S. If it is truncated by a plane w y, perpendicular to the axis, then from the point R, with the radius equal to the true length of the arris G w, or G y, describe an arc 1 4, and inscribe in it the sides of the polygon forming the pyramid.

III. Prisms. In a right prism, the faces being all perpendicular to the bases which truncate the solid, it results that their development is a rectangle composed of all the faces joined together, and bounded by two parallel lines equal in length to the contour of the bases. Thus, in Fig. 70, a b c d is the base, and b e the height of the prism; the four sides will form the rectangle b e f g, and e h i k will be the top of the prism. The full lines show another method of development.

When a prism is inclined, the faces form different angles with the lines of the contours of the bases; whence there results a development, the extremities of which are bounded by lines forming parts of polygons.

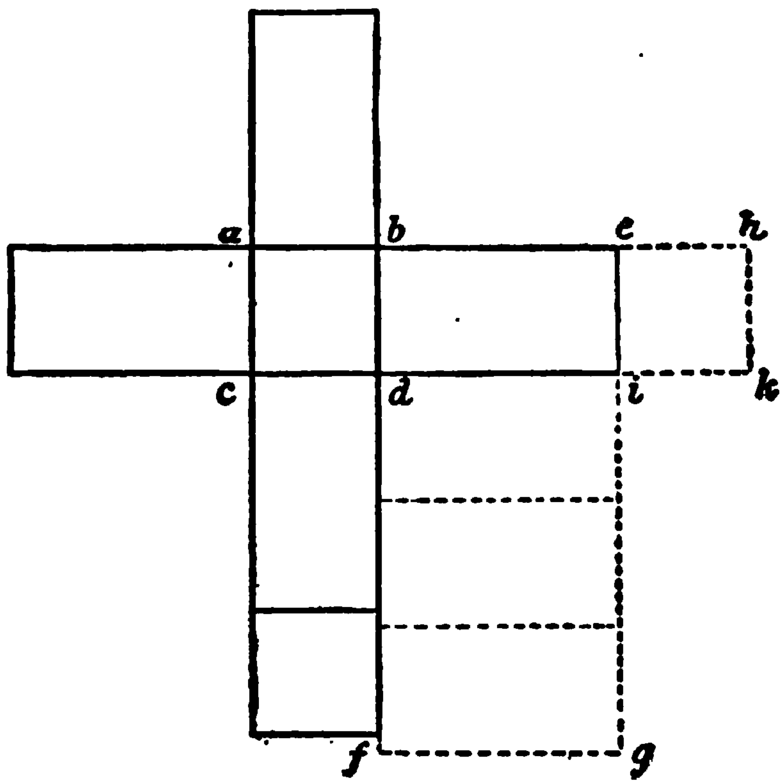


Fig. 70.

After having drawn the line C C (Fig. 71), which indicates the axis of the prism and the lines A B, D E, the surfaces which terminate it, describe on the middle of the axis the polygon forming the plan of the prism, taken perpendicularly to the axis, and indicated by the figures 1 and 8. Produce the sides 1 2, 6 5, parallel to the axis, until they meet the lines A B, D E. These lines then indicate the four arrises of the prism, corresponding to the angles 1 2 5 6. Through the points 8 3 7 4 draw lines parallel to the axis meeting A B, D E in F H, G L. These lines represent the four arrises 8 3 7 4.*

In this profile the sides of the plan of the polygon 1 2 3 4 5 6 7 8 give the width of the faces of the prism,

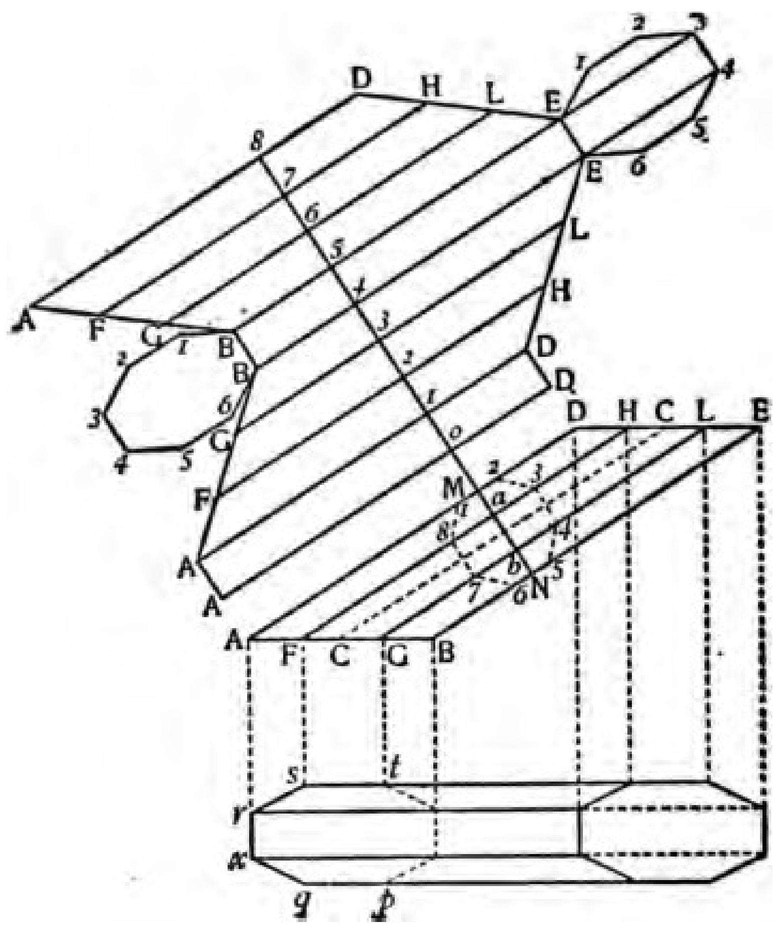


Fig. 71.

and the lines A D, F H, G L, B E their length. From this profile can be drawn the horizontal projection, in the manner shown below. To trace the development of this prism on a sheet of paper, so that it can be folded

*In Fig. 75 another example is given, but as the method of procedure is the same as in Fig 71, detailed description is unnecessary.

together to form the solid, proceed thus: On the middle of C C raise an indefinite perpendicular M N. On that line set off the width of the faces of the prism, indicated by the polygon, in the points 0 1 2 3 4 5 6 7 8. Through these points draw lines parallel to the axis, and upon them set off the lengths of the lines in profile, thus: From the points O, 1, and 8, set of the length M D in the points D D D; from 2 and 7, set off a H in H from 3 and 6, set off b L in L and L; and so on. Draw the lines DD, D H L E, EE, E L H D, for the contour of the upper part of the prism. To obtain the contour of the lower portion, set off the length M A from O, 1, and 8 to A A A, the length a F from 2 and 7 to F and F, the length b G from 3 and 6 to G and G, and so on; and draw A A, A F G B, B B, B G F A, to complete the contour. The development is completed by making on B B and E E the polygons 1 2 3 4 5 6 BB, 1 2 3 4 5 6 EE, similar to the polygons of the horizontal projection.

IV. Cylinders. Cylinders may be considered as prisms, of which the base is composed of an infinite number of sides. Thus we shall obtain graphically the development of a right cylinder by a rectangle of the same height, and of a length equal to the circumference of the circle, which serves as its base.

To find the covering of a right cylinder.

Let A B C D (Fig. 72) be the seat or generating section. On A D describe the semicircle A 5 D, representing the vertical section of half the cylinder, and divide its circumference into any number of parts, 1, 2, 3, 4, 5, &c., and transfer those divisions to the lines A D and B C produced; then the parallelogram D C G F will be the covering of one half the cylinder.

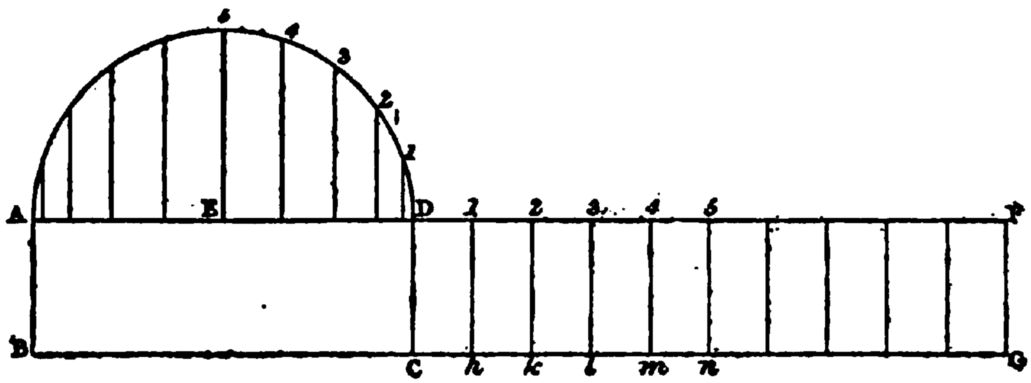


Fig. 72.

To find the edge of the covering when it is oblique in regard to the sides of the cylinder.

Let A B C D (Fig. 73) be the seat of the generating section the edge B C being oblique to the sides A B, D C. Draw the semicircle A 5 D, and divide it into any

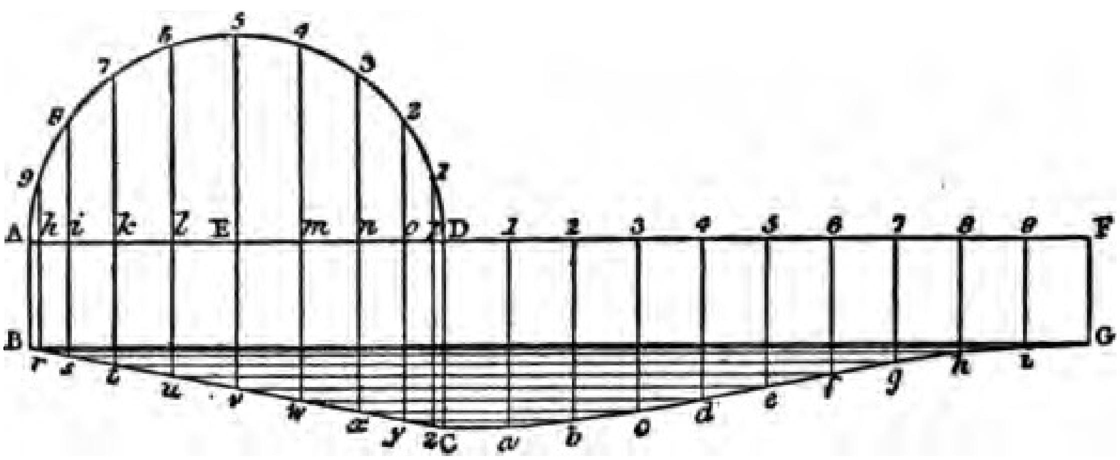


Fig. 73.

number of parts as before; and through the divisions draw lines at right angles to A D, producing them to meet B C in r s, t, u, v, &c. produce A D, and the lines 1 a, 2 b, 3 c, &c., perpendicular to D F. To these lines transfer the length of the corresponding lines intercepted between A D and B C, that is, to 1a transfer the length p z, to 2 b transfer o y, and so on, by drawing the lines z a, y b, x c, &c., parallel to A F. Through the points thus obtained, draw the curved line C a b c, &c., to G; then shall D F C G be the development of the covering of the semi-cylinder A B C D.

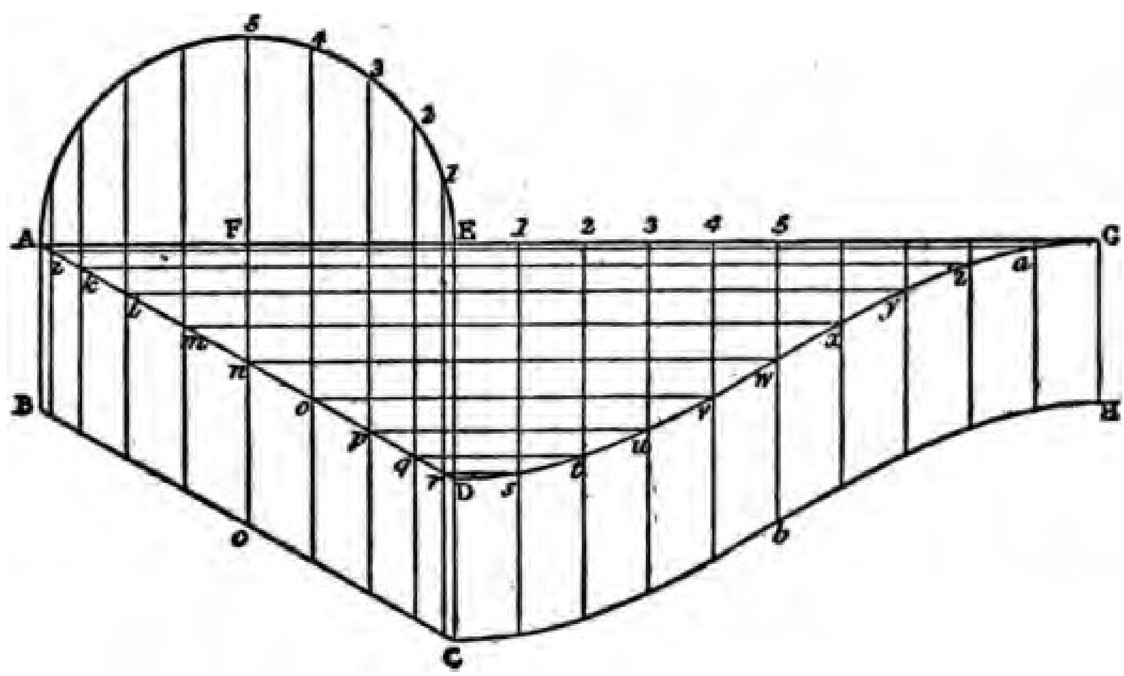


Fig. 74.

To find the covering of a cylinder contained between two oblique parallel planes.

Let A B C D (Fig. 74) be the seat of the generating section. From A draw A G perpendicular to A B, and produce C D to meet it in E. On A E describe the semicircle, and transfer its perimeter to E G, by dividing it into equal parts, and setting off corresponding divisions on E G. Through the divisions of the semicircle draw lines at right angles to A E, producing them to meet the lines A D and B C, in i, k, l, m, &c. Through the divisions on E G draw lines perpendicular to it; then through the intersections of the ordinates of the semicircle, with the line A D, draw the lines i a, k z, l y,

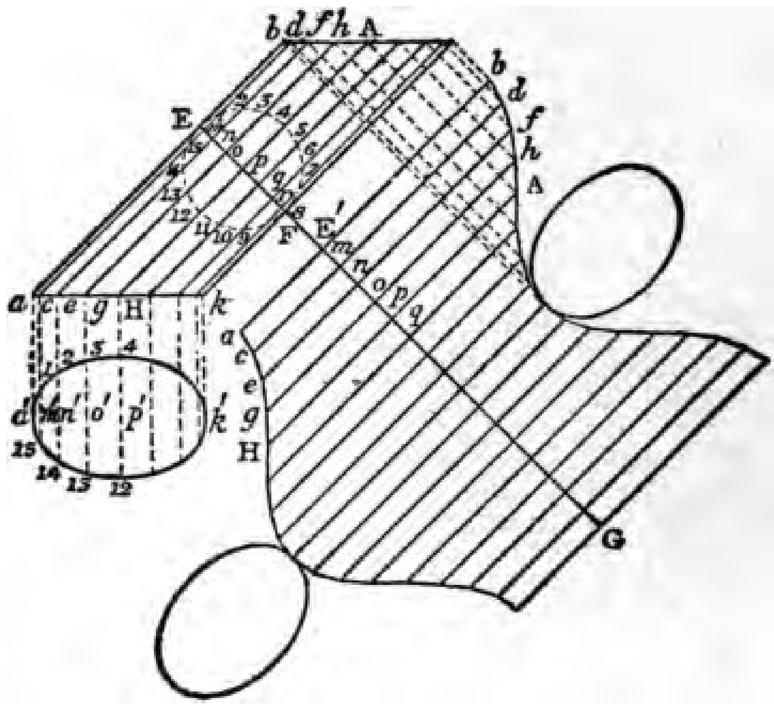


Fig. 75.

&c., parallel to A G and where these intersect the perpendiculars from E G, in the points a, z, y, x, w, u, &c., trace a curved line G D, and draw parallel to it the curved line H C; then will D C H G be the development of the covering of the semi-cylinder A B C D.

To find the covering of a semi-cylindric surface bounded by two curved lines.

The construction to obtain the developments of these coverings (Figs. 76 and 77) is precisely similar to that described in Fig. 74.

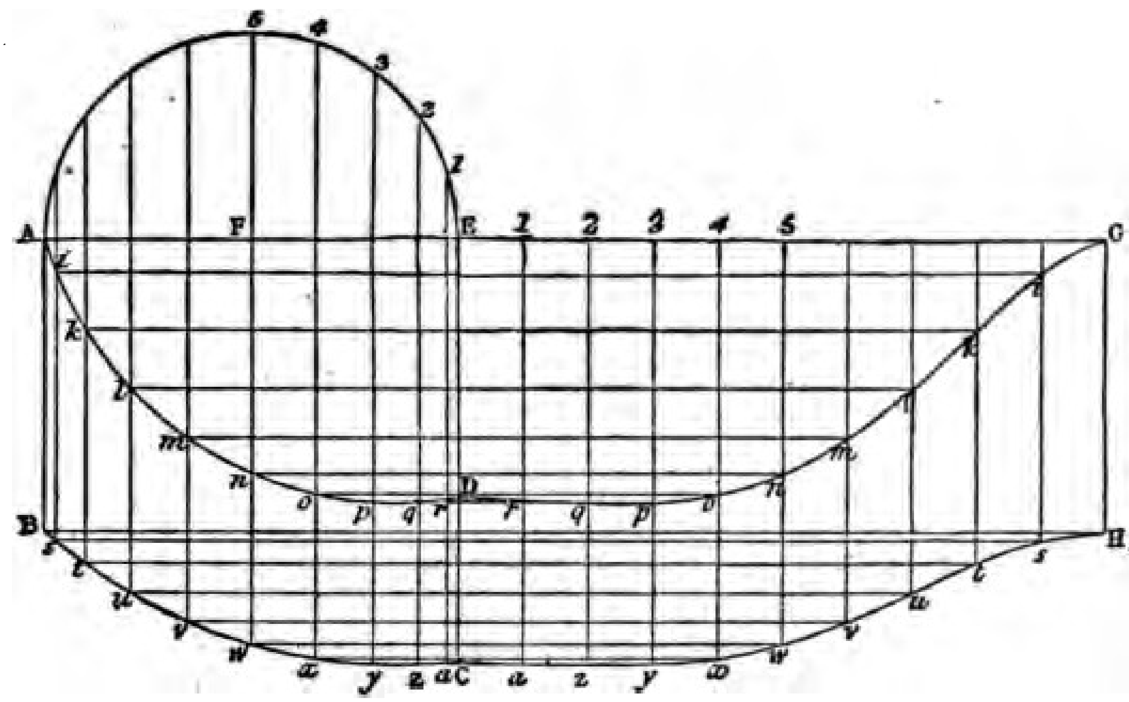


Fig. 76.

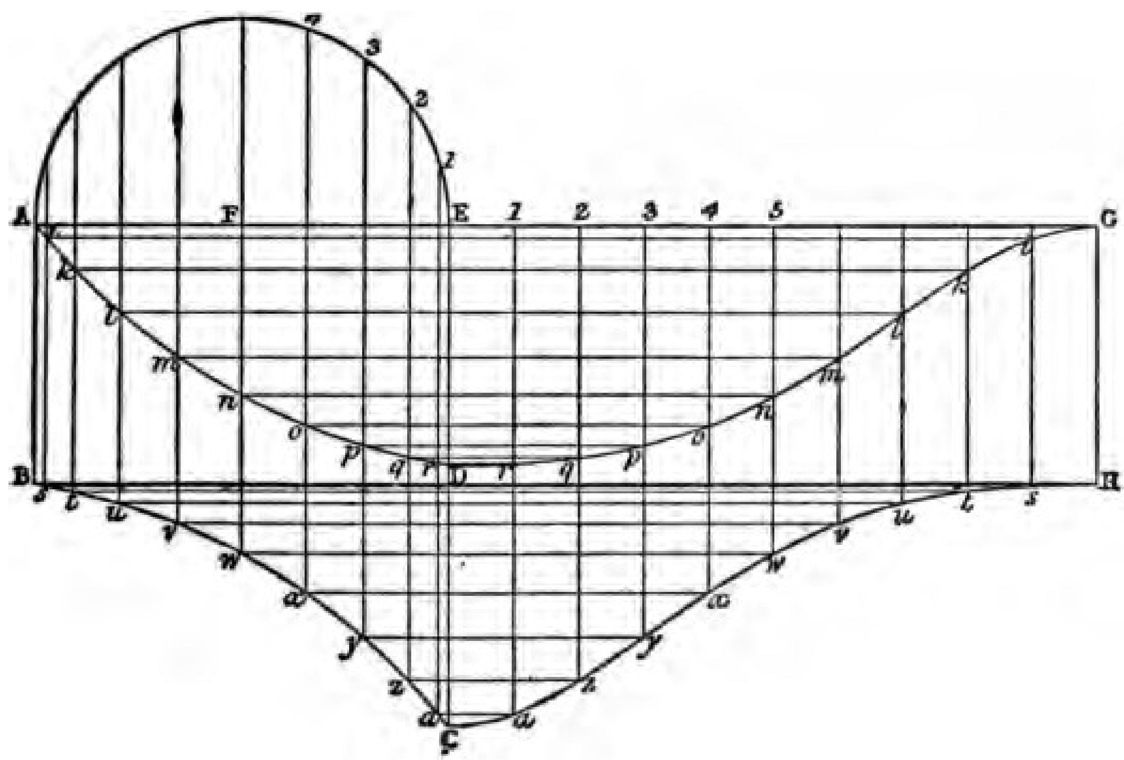


Fig. 77.

V. Cones. We have considered cylinders as prisms with polygonal bases. In a similar manner we may regard cones as pyramids.

In right pyramids, with regular symmetrical bases,

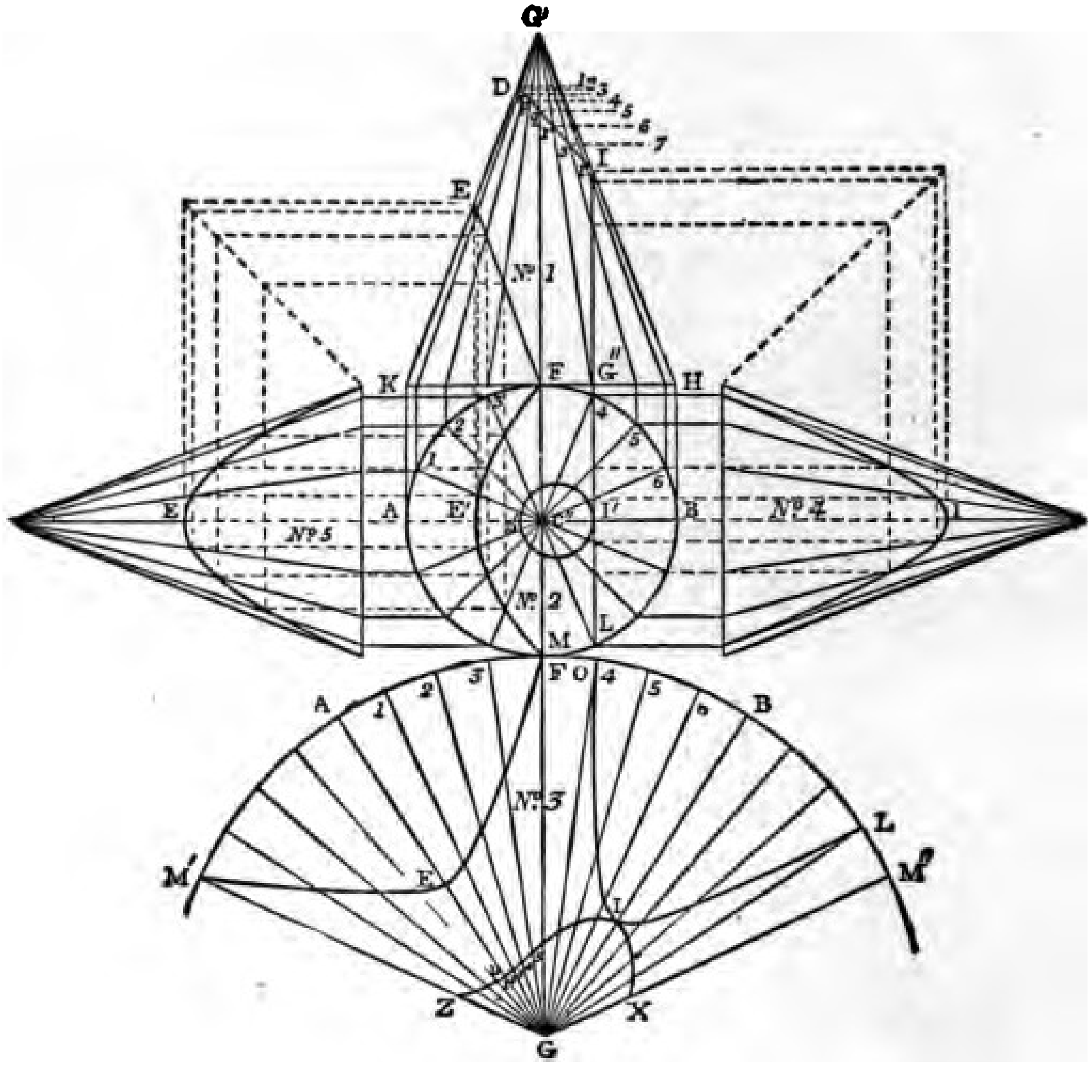


Fig. 78.

as the lines of the arrises extending from the summit to the base are equal, and as the sides of the polygons forming the base are also equal, their developed surfaces will be composed of similar and equal isosceles triangles, which, as we have seen (Fig. 78, a, b, c, d), will, when united, form a part of a regular polygon inscribed in a circle, of which the inclined sides of the polygon form the radii. Thus in considering the base of the cone K H (Fig. 78) as a regular polygon of an infinite number of sides, its development will be found in the sector of a circle, M A F B M (No. 3), of which the radius equals the side of the cone K G (No. 1), and the arc equals the circumference of the circle forming its base (No. 2).

To trace on the development of the covering, the curves of the ellipse, parabola, and hyperbola, which are the result of the sections of the cone by the lines D I, E F, I G, it is necessary to divide the circumference of the base A F B M (No. 2) into equal parts, as 1, 2, 3, &c., and from these to draw radii to the centre C, which is the horizontal projection of the vertex of the cone; then to carry these divisions to the common intersection line K H, and from their terminations there to draw lines to the vertex G, in the vertical projection No. 1. These lines cut the intersecting planes, forming the ellipse, parabola, and hyperbola, and by the aid of the intersections we obtain the horizontal projection of these figures in No. 2—the parabola passing through M E F, the hyperbola through G I L, and the ellipse through D I.

To obtain points in the circumference of the ellipse upon the development, through the points of intersection o, p, q, r, &c., draw lines parallel to K H, carrying the heights to the side of the cone G H, in the points 1, 2, 3, 4, 5, 6, 7, and transfer the lengths G 1, G 2, G 3, &c., to G 1, G 2, G 3, G 4, &c., on the radii of the development in No. 3; and through the points thus obtained draw the curve z D l X.

To obtain the parabola and hyperbola, proceed in the same manner, by drawing parallels to the base K H, through the points of intersection; and transferring the lengths thus obtained on the sides of the cone G K, G H, to the radii in the development.

Nos. 4 and 5 give the vertical projections of the hyperbola and parabola respectively.

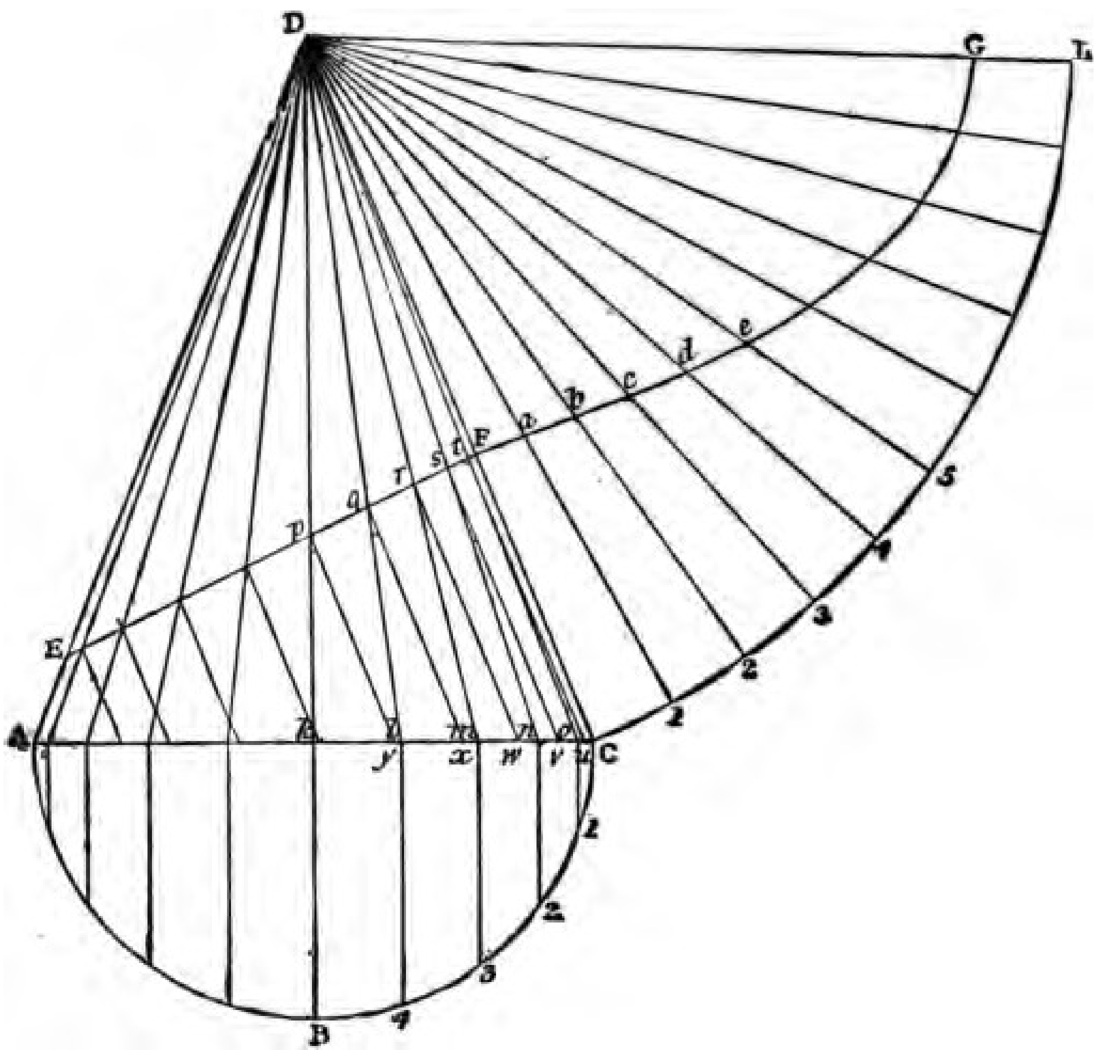


Fig. 79.

To find the covering of the frustum of a cone, the section being made by a plane perpendicular to the axis.

Let A C E F (Fig. 79) be the generating section of the frustum. On A C describe the semicircle A B C, and produce the sides A E and C F to D. From the centre D, with the radius D C, describe the arc C H; and from the same centre with the radius D F, describe

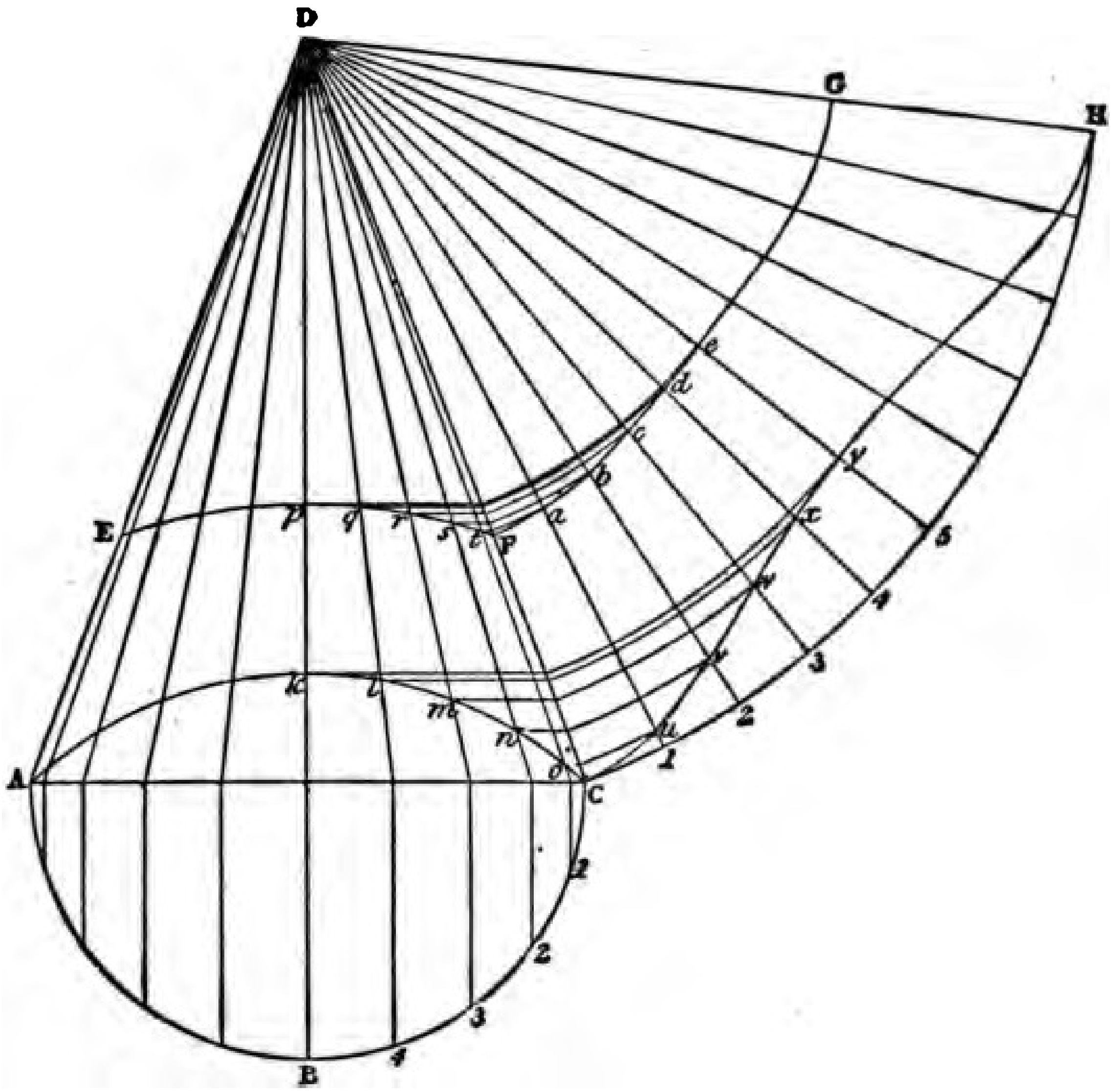


Fig. 80.

the arc F G. Divide the semicircle A B C into any number of equal parts, and run the same divisions along the arc C H; draw the line H D, cutting E G in G; then shall C H G F be half the development of the covering of the frustum A C F E.

To find the covering of the frustum of a cone, the section being made by a plane not perpendicular to the axis.

Let A C F E (Fig. 80) be the frustum. Proceed as in the last problem to find the development of the covering of the semi-cone. Then—to determine the edge of the covering of the line E F—from the points P, q, r, s, t, &c., draw lines perpendicular to E F, cutting A C in y, x, w, v, u; and the length u t transferred from 1 to a, v s, transferred from 2 to b, and so on, will give a, b, c, d, e, &c., points on the edge of the covering.

To find the covering of the frustum of a cone, when cut by two cylindrical surfaces perpendicular to the generating section.

Let A E F C (Fig. 81) be the given frustum, and A k C, E p F, the given cylindrical surfaces. Produce A E, C F, till they meet in the point D. Describe the semicircle A B C, and divide it into any number of equal parts, and transfer the divisions to the arc C H, described from D, with the radius D C. Through the divisions in the semicircle 1, 2, 3, 4, &c., draw lines perpendicular to A C, and through the points where they intersect A C draw lines to the summit D. Draw lines also through the points 1, 2, 3, 4, 5, &c., of the arc C H, to the summit D; then through the intersections of the lines from A C to D, with the seats of the cylindrical surfaces k, l, m, n, o, and p, q, r, s, t, draw lines parallel to A C, cutting C D; and from the points of intersection in C D, and from the centre D, describe arcs cutting the radial lines in the sector D C H in u, v, w, x, y,

&c., and a, b, c, d, e, &c.; and curves traced through the intersections will give the form of the covering.

VI. Spheres, Ellipsoids, &c. The development of the sphere, and of other surfaces of double curvature,

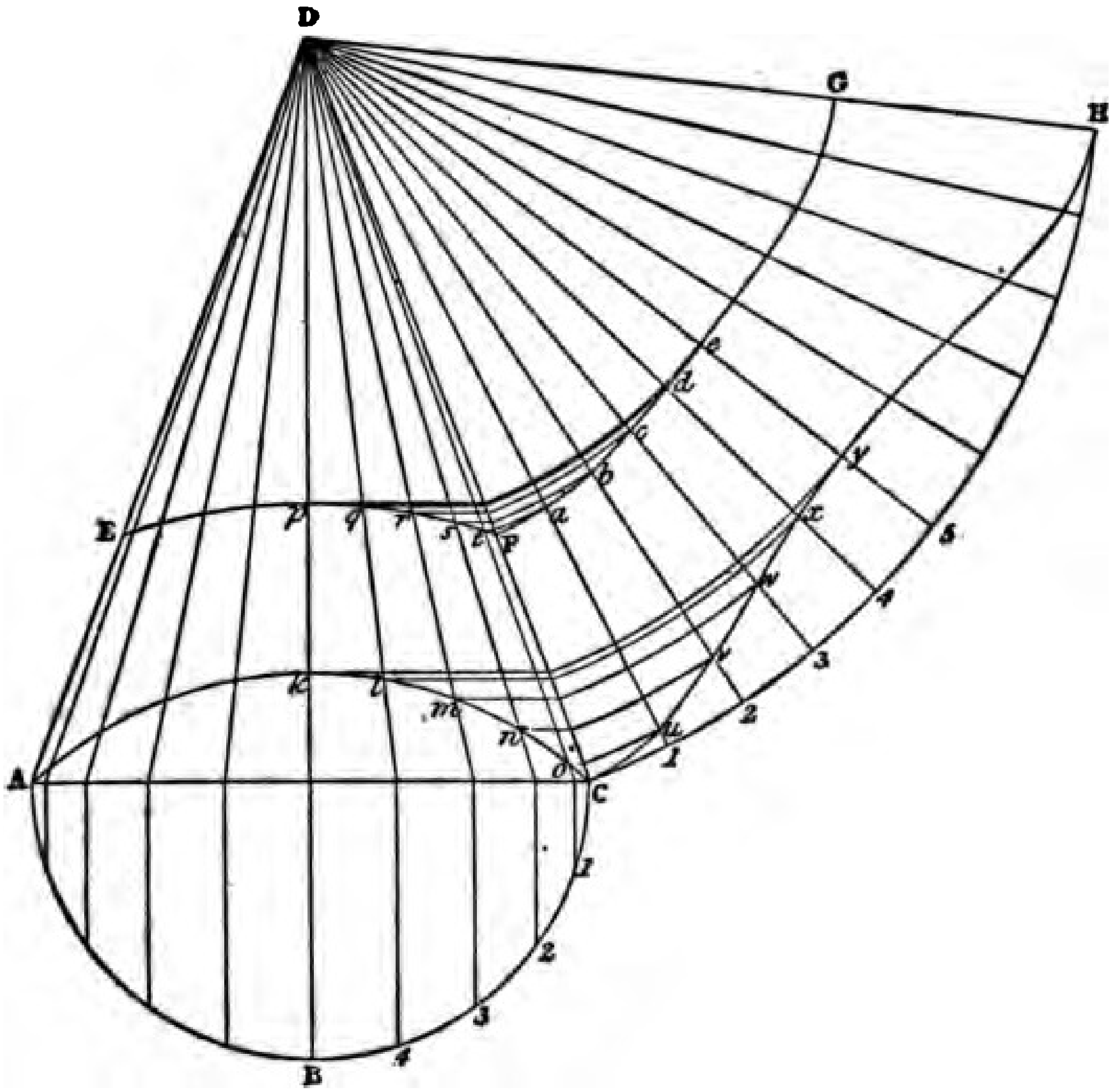


Fig. 81.

is impossible, except on the supposition of their being composed of a great number of small faces, either plane, or of a simple curvature, as the cylinder and the

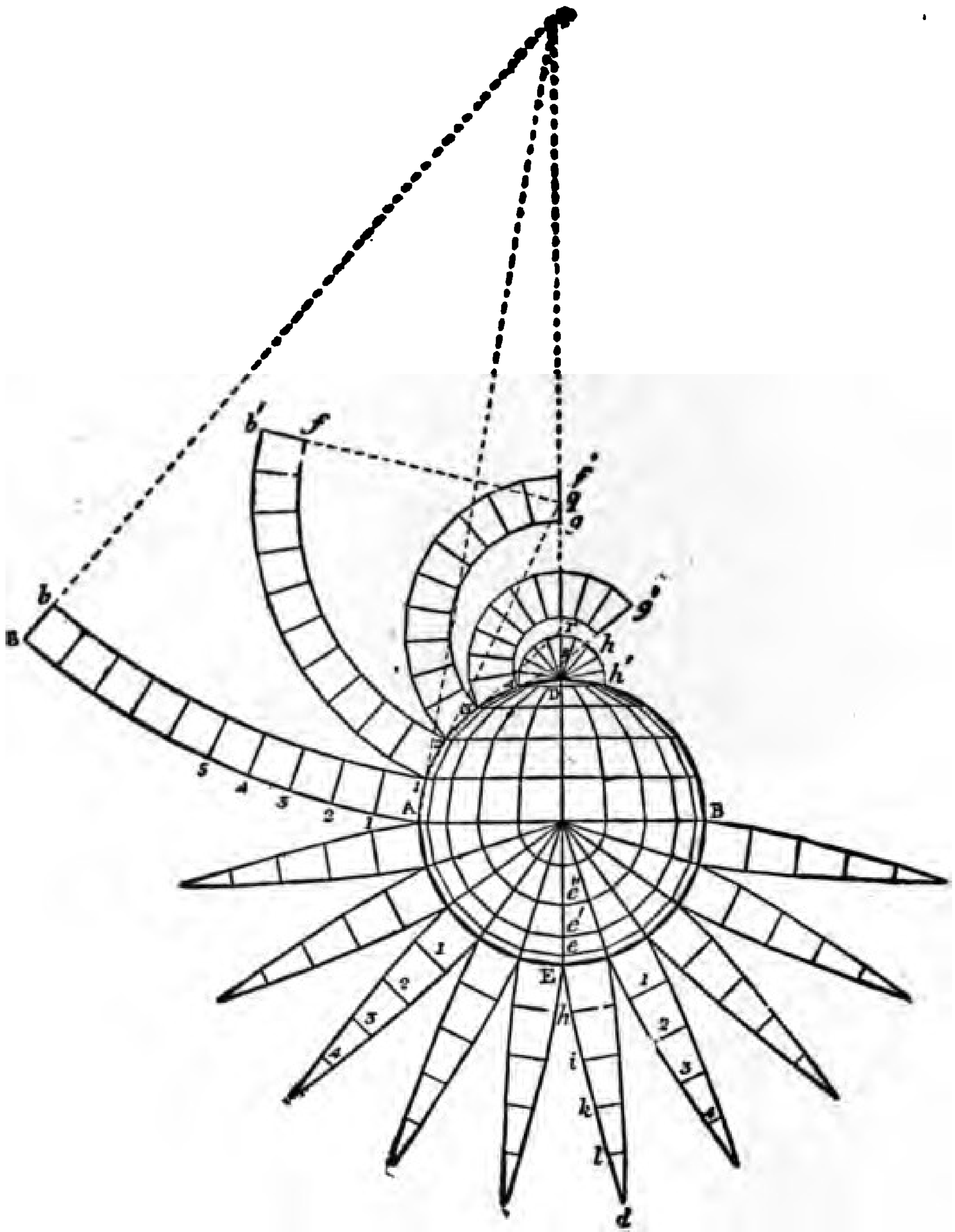


Fig. 82.

cone. Thus, the sphere or spheroid may be considered as a polyhedron, terminated, 1st, by a great number of plane faces, formed by truncated pyramids, of which the base is a polygon, as in Fig. 82; 2nd, by parts of truncated cones forming zones, as in Fig. 83, the part above A B being the vertical projection, and the part below A B the horizontal projection; 3rd, by parts of cylinders cut in gores, forming flat sides, which diminish in width, as in Fig. 84.

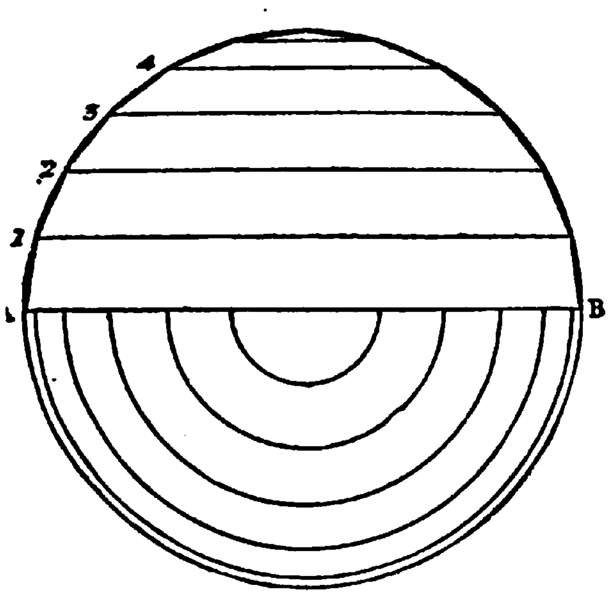


Fig. 83.

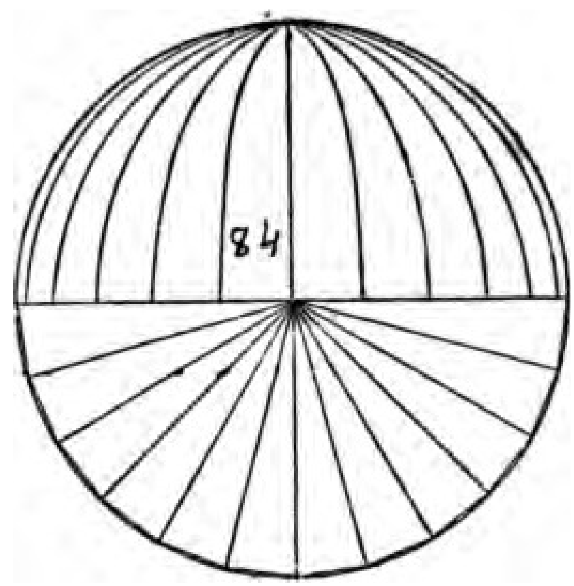


Fig. 84.

In reducing the spheres, or spheroid, to a polyhedron with flat sides, two methods may be adopted, which differ only in the manner of arranging the developed faces.

The most simple method is by parallel circles, and others perpendicular to them, which cut them in two opposite points, as in the lines on a terrestrial globe. If we suppose that these divisions, in place of being circles, are polygons of the same number of sides, there will result a polyhedron, like that represented in Fig. 82, of which the half, A D B, shows the geometrical elevation, and the other half, A E B, the plan.

To find the development, first obtain the summits

P, q, r, s, of the truncated pyramids, which from the demi-polyhedron A D B, by producing the sides A1, 12, 23, 34, until they meet the axis E D produced; then from the points p, q, r, s, and with the radii P A, P I, q I q 2, r 2, r 3, and s 3, s 4, describe the indefinite arcs A B, 1 b, 1 b, 2 f, 2 f, e q, 3 g, 4 h, and from D describe the arc 4 h; upon all these arcs set off the divisions of the demi-polygons A E B, and draw the lines to the summits p, q, r, s, and D, from all the points so set out, as A, 1, 2, 3, 4, &c., from each truncated pyramid. These lines will represent for every band or zona the faces of the truncated pyramids of which they constitute a part.

The development can also be made by drawing through the centre of each side of the polygon A E B, indefinite perpendiculars, and setting out upon them the heights of the faces in the elevation, A1 2 3 4 D, and through the points thus obtained drawing parallels to the base. On each of these parallels then set out the widths h, i, k, l, d, of the corresponding faces (e, e, e, &c.) in the plan, and there will be thus formed trapeziums and triangles, as in the first development, but arranged differently. This method is used in constructing geographical globes, the other is more convenient in finding the stones of a spherical dome.

The development of the sphere by reducing it to conical zones (Fig. 83) is accomplished in the same manner as the reduction to truncated pyramids, with this difference, that the developments of the arrises, indicated by A 1 2 3 4 5 B in Fig. 82, are arcs of circles described from the summits of cones, in place of being polygons.

The development of the sphere reduced into parts of cylinders, cut in gores (Fig. 84), is produced by the second method described, but in place of joining, by straight lines, the points E, h, i, k, l, d (Fig. 82), we

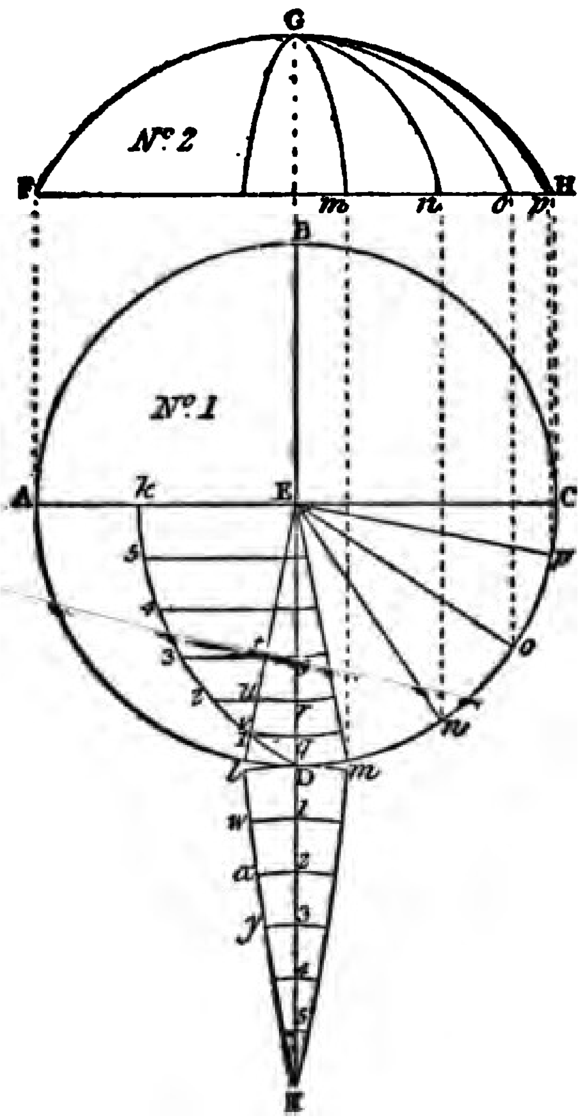


Fig. 85.

unite them by curves. This last method is used in tracing the development of caissons in spherical or spheroidal vaults.

To find the covering of a segmental dome.

In Fig. 85, No. 1 is the plan, and No. 2 the elevation of a segmental dome. Through the centre of the plan

E draw the diameter A C, and the diameter B D perpendicular to A C, and produce B D ti I. Let D E represent the base of semi-section of the dome; upon D E describe the arc D k with the same radius as the arc F G H (No. 2); divide the arc D k into any number of equal parts, 1, 2, 3, 4, 5, and extend the division upon the right line D I, making the right line D I equal in length, and similar in its divisions, to the arc D k: from the points of division, 1, 2, 3 4, 5, in the arc D k, draw lines perpendicular to D E, cutting it in the points q, r, s, &c. Upon the circumference of the plan No. 1, set off the breadth of the gores or boards l m, m n, n o, o p, &c.; and from the points l, m, n, o, p, draw right lines through the centre E: from E describe concentric arcs q v, r u, st, &c., and from l describe concentric arcs through the points D, 1, 2, 3, 4, 5,: l m, being the given breadth of the base, make l w equal to q v, 2x equal to r u, 3 y equal to st, &c.; draw the curved line through the points l, w, x, y, &c., to l, which will give one edge of the board or gore to coincide with the line l E. The other edge being similar, it will be found by making the distances from the centre line D 1 respectively equal. The seats of the different boards or gores on the elevation are found by the perpendicular dotted lines, pp, oo, nn, m m, &c.

To find the covering of a semicircular dome.

Fig. 86, Nos. 1 and 2.—The procedure here is more simple than in the case of the segmental dome, as, the horizontal and vertical sections being alike, the ordinates are obtained at once.

To find the covering of a semicircular dome when it is required to cover the dome horizontally.

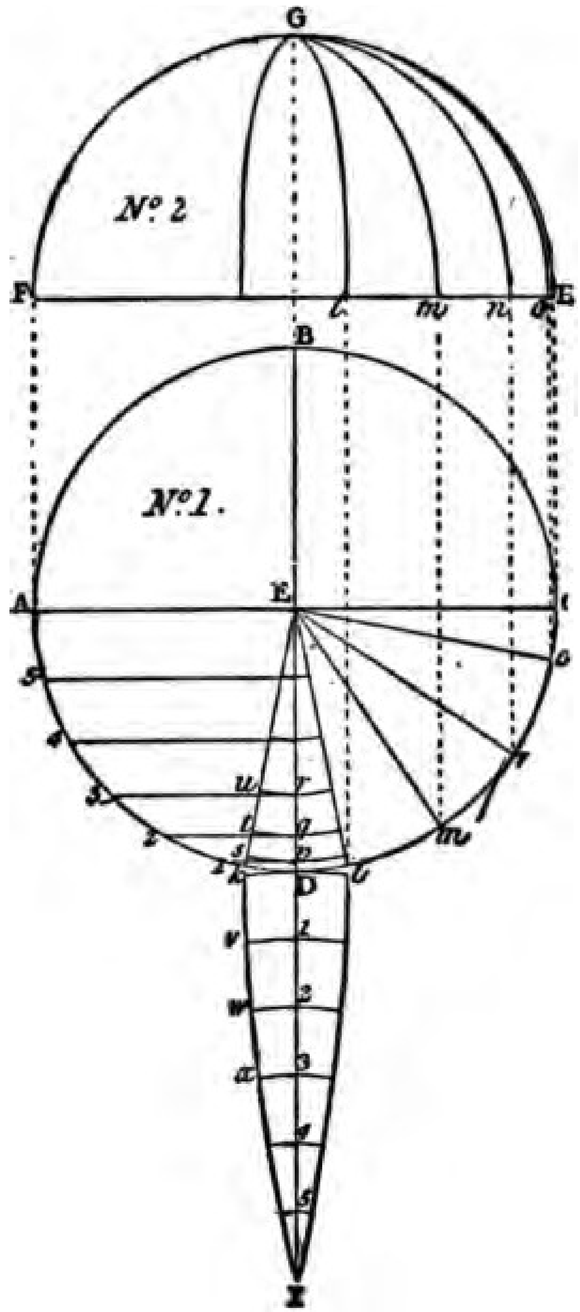


Fig. 86.

Let A B C (Fig. 87) be a vertical section through the axis of a circular dome, and let it be required to cover this dome horizontally. Bisect the base in the point

D, and draw D B E perpendicular to A C, cutting the circumference in B. Now divide the arc B C into equal parts, so that each part will be rather less than

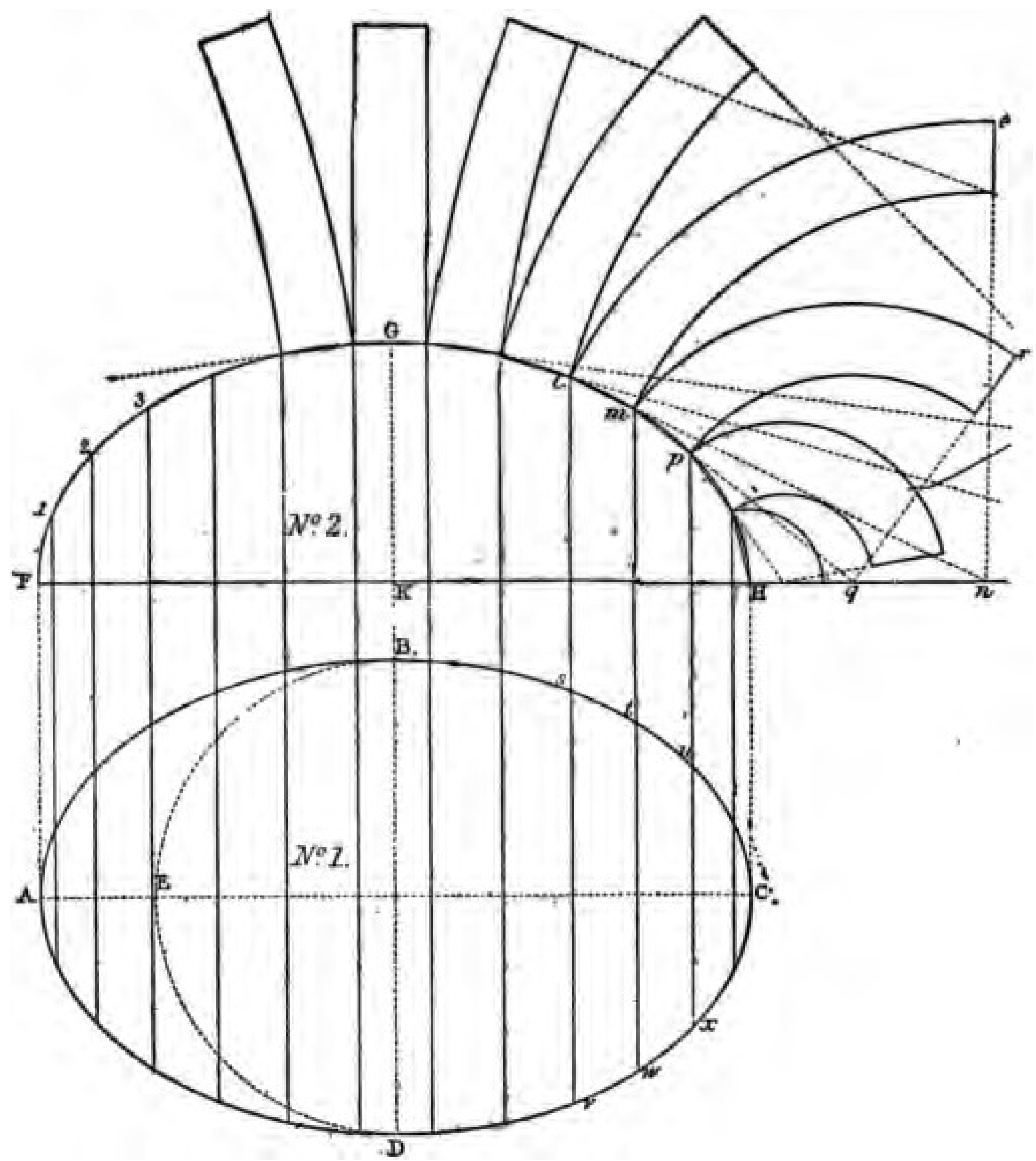


Fig. 87.

the width of the a board; and join the points division by straight lines, which will form an inscribed polygon of so many sides; and through these points draw line parallel to the base A C, meeting the opposite sides of the circumference. The trapezoids formed by the sides of the polygon and the horizontal lines may then be regarded as the sections of so many frustums of cones; whence results the following mode of procedure, in

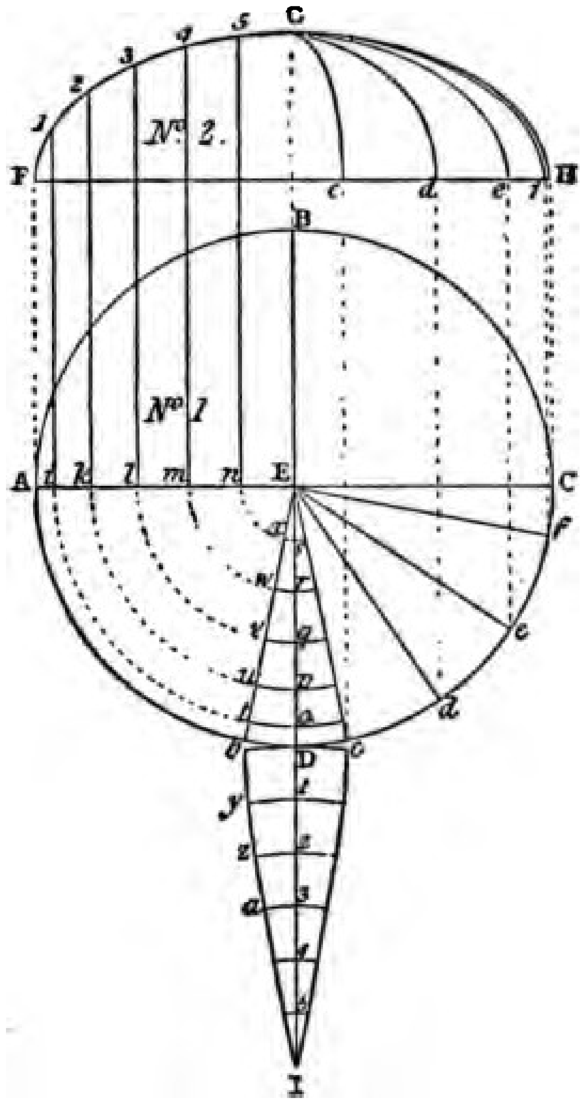


Fig. 88.

accordance with the introductory illustration Fig. 82; —Produce, until they meet the line D E, the lines g f, f, n, &c., forming the sides of the polygon. Then, to describe a board which corresponds to the surface of one of the zones, as f g, of which the trapezoid m l f g is a section,—from the point h, where the line f g produced meets D E, with the radii h f, hg, describe two arcs, and cut off the end of the board k on the line of a radius h k.

To obtain the true length of the board, proceed as in Fig. 89. The other boards are described in the same manner.

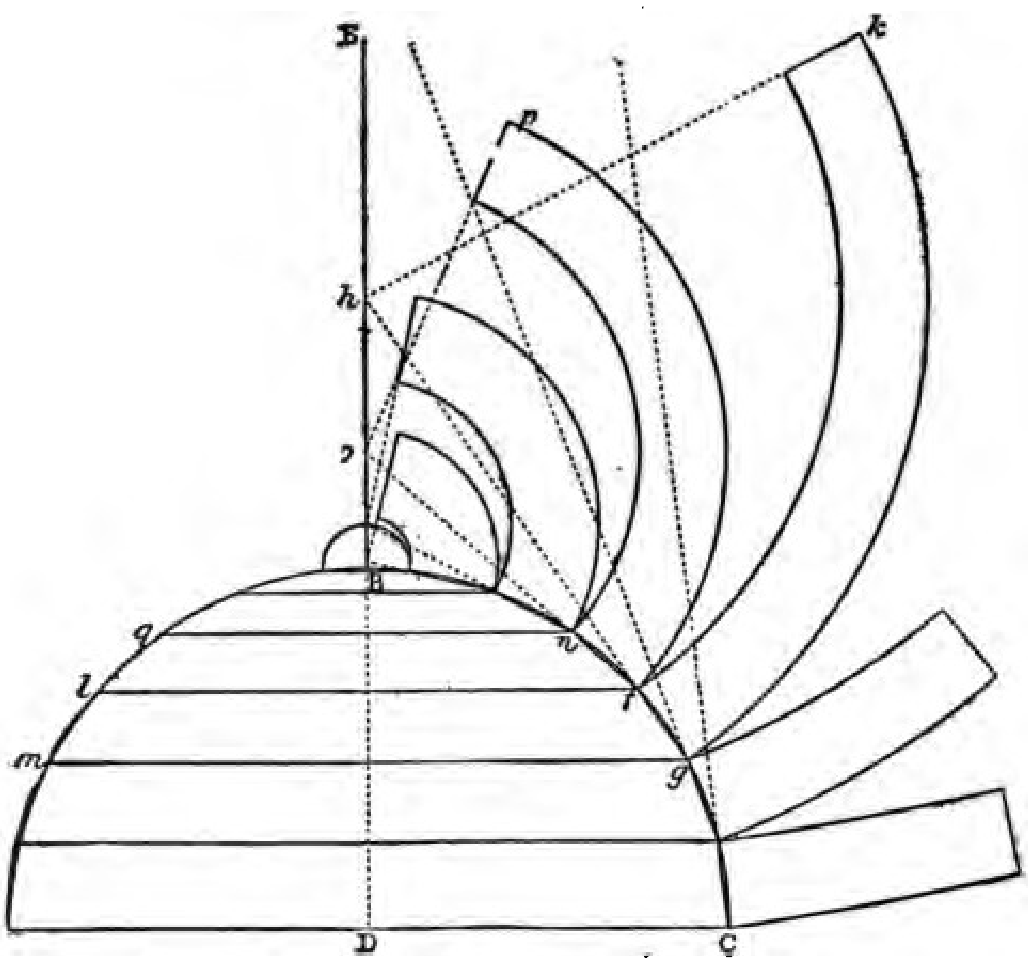


Fig. 89.

To find the covering of an elliptic dome.

Let A B C D (Fig. 90) be the plan, and F G H the elevation of the dome. Divide the elliptical quadrant F G (No. 2) into any number of equal parts in 1, 2, 3, 4, 5, and draw through the points of division lines perpendicular to F H, and produced to A C (No. 1), meeting it in i, k, m, n,: these divisions are transferred by the dotted arcs to the gore b E c and the remainder of the process is as in Figs. 85 and 86.

To find the covering boards of an ellipsoidal dome.

Let A B C D (No. 1, Fig. 89) be the plan of the dome, and F G H (No. 2) the vertical section through its major axis. Produce F H indefinitely to n; divide the circumference, as before, into any number of equal parts, and join the divisions by straight lines, as p m, ml, &c. Then, describe on a board, produce the line forming one of the sides of the polygon, such as l m, to meet F n in n; and from n, with the radii n m, n l describe two arcs forming the sides of the board, and cut off the board on the line of the radius n o. Lines drawn through the points of divisions at right angles to the axis, until they meet the circumference A D C of the plan, will give the plan of the boarding.

To find the covering of an ellipsoidal dome in gores.

Let the ellipse A B C D (Fig. 90, No. 1) be the plan of the dome, A C its major axis and B D its minor axis; and let A B C (No. 2) be its elevation. Then, first, to describe on the plan and elevation the lines of the gores, proceed thus:—Through the line A C (No. 1) produced at H, draw the line E G perpendicular to it, and draw B E, D G, parallel to the axis A C, cutting E G; then will E G be the length of the axis minor, on which is to be described the semi-circle E F G, representing a section of the dome on a vertical plane passing through the axis minor.

Divide the circumference of the semi-circle into any number of equal parts, representing the widths of the covering boards on the line B D; and through the points of division 1, 2, 3, 4, 5, draw lines parallel to the axis A C, cutting the line B D in 1, 2, 3, 4, 5. Divide the quadrant of the ellipse C D (No. 1) into any number of equal parts in e, f, g, h, and through these points draw the lines e a, f b, g c, h d in both diagrams, per-

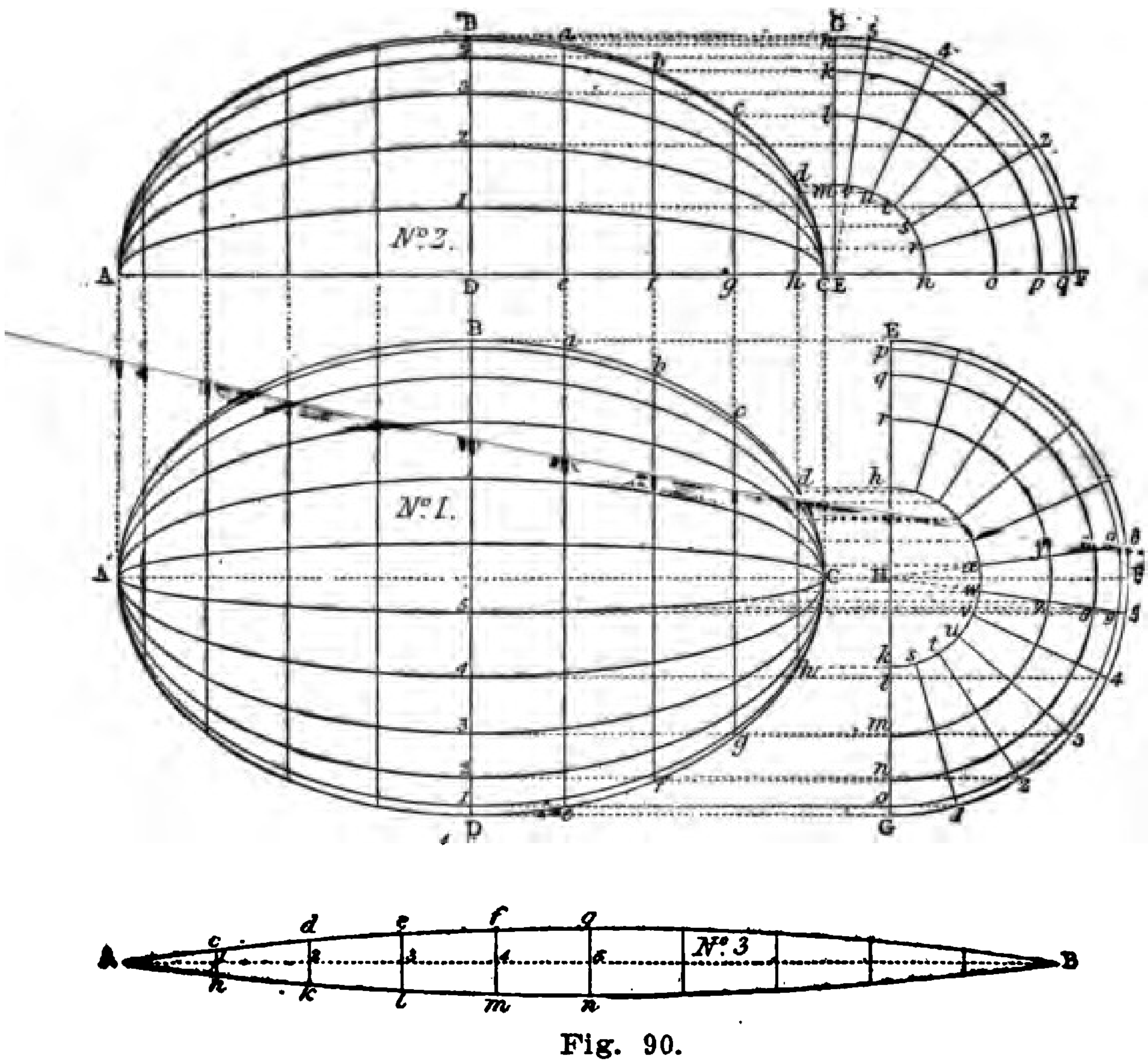


Fig. 90.

pendicular to A C, and these lines will then be the seats of vertical sections through the dome, parallel to E F G. Through the points e, f, g, h (No. 1) draw lines parallel to the axis A C, cutting E G in o, n, m, k;

and from H, with the radii H o, H n, H m, H k, describe concentric circles o 9 9 p, n 8 8q, m z y r, &c. To find the diminished width of each gore at the sections a, e, b f, c g, d h:—Through the divisions of the semicircle, 1, 2, 3, 4, 5, draw the radii ls, 2t, 3u, 4v, 5 w, 6x; then by drawing through the intersections of these radii with the concentric circles, lines parallel to H F, to meet the section lines corresponding to the circles, the width of the gores at each section will be obtained; and curves through these points will give the representation of the lines of the gores of the plan.

In No. 2 the intersections of the lines are more clearly shown. The quadrant E G F is half the end elevation of the dome, and is divided as in No. 1. The parallel lines 5 5, 4 4, 3 3, show how the divisions of the arc of the quadrant are transferred to the line D B, and the other parallels a h, b k, cl, d m, are drawn from the divisions in the circumference of the ellipse to the line E G, and give the radii of the arcs m, l o, k p, h q.

To describe one of the gores draw any line A B (No. 3), and make it equal in length to the circumference of the semi-ellipse A D C, by setting out on it the divisions 1, 2, 3, 4, 5, &c., corresponding to the divisions C h, h g, g f, &c., of the ellipse: draw through those divisions lines perpendicular to A B. Then from the semi-circle (No. 1) transfer to these perpendiculars the widths 6 5 to g n, 9 9 to f m, 8 8 to e l, y z to d k, and x w to c h, and join A c, d d, de, e f, f g, A h, h k, k l, l m, and m n,; which will give the boundary lines of one-half of the gore, and the other half is obtained in the same manner.

To describe the covering of an ellipsoidal dome with horizontal boards of equal width.

Let A B C D (No. 1, Fig. 91) be the plan of the dome, A B C (No. 2) the section on its major axis, and L M N the section on its minor axis. Draw the circumscribing parallelogram of the ellipse, namely, F G H K (No. 1), and its diagonals F H G K. In No. 2 divide the circumference into equal parts, 1, 2, 3, 4, representing the number of covering boards, and through the points of division draw lines 1 8, 2 7, &c., parallel to A C. Through the points of divisions draw 1 p, 2 t, 3 x, &c., perpendicular to A C, cutting the diagonals of the circumscribing parallelogram of the ellipse (No. 1), and meeting its major axis in p, t, x, &c. Complete the parallelograms, and inscribe ellipses therein corresponding to the lines of the covering. Produce the sides of the parallelograms to intersect the circumference of the section on the minor axis of the ellipse in 1, 2, 3, 4, and lines drawn through these parallel to L N, will give the representation of the covering boards in that section. To find the development of the covering, produce the axis D B, in No. 2, indefinitely. Join by a straight line the divisions 1 2 in the circumference, and produce the line indefinitely, making e k equal to e 2, and k g equal to 1 2; 1 2e k g will be the axis major of the ellipses of the covering 1 2 7 8. Join also the corresponding divisions in the circumference of the section on the minor axis, and produce the line 1 2 b to meet the axis produced; and the length of this line will be the semi-axis minor, e h, of the ellipse, 2 h k, while the width f h will be equal to the division 12 in N M L.

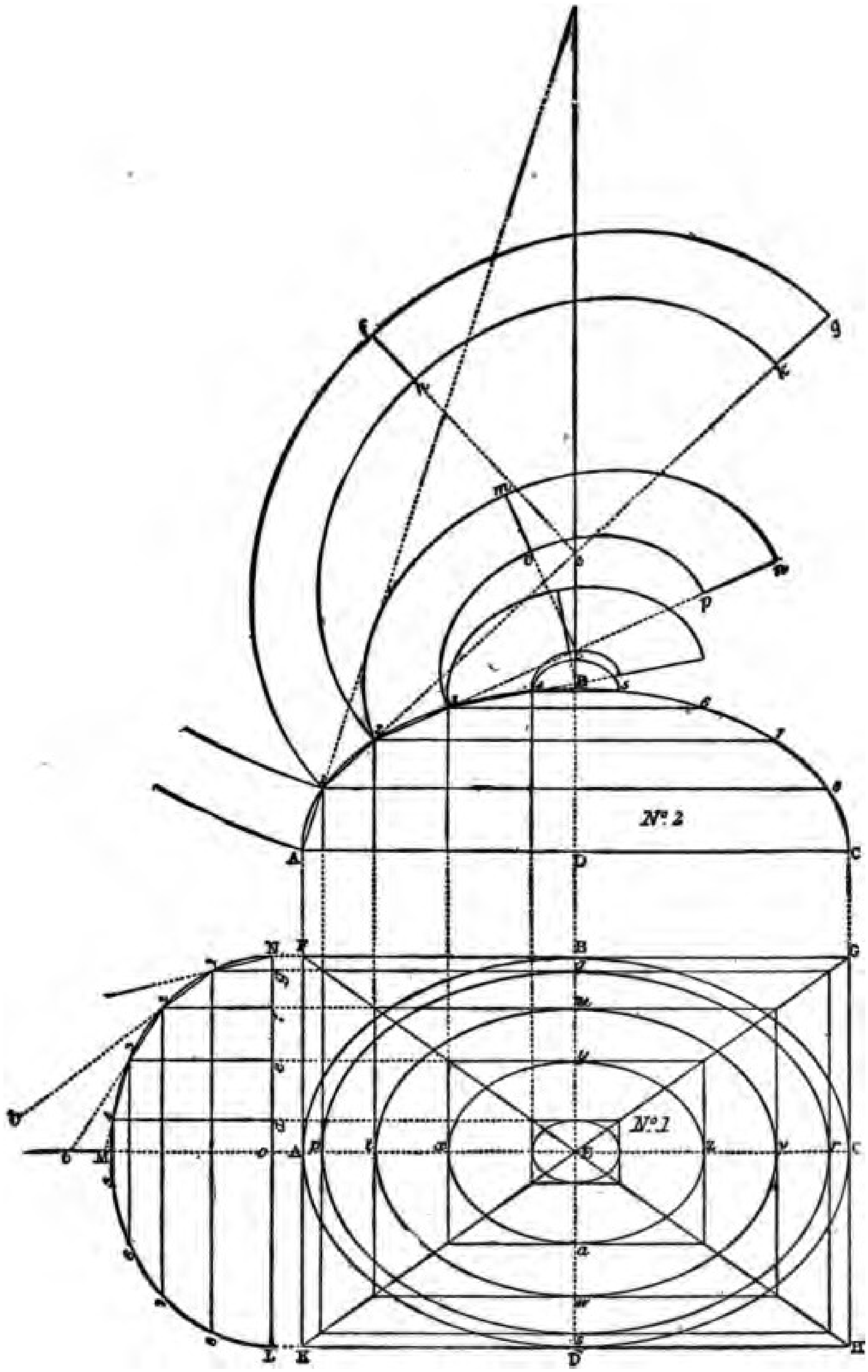


Fig. 91.

To find the covering of an annular vault.

Let A C K G E F A (Fig. 92) be the generating section of the vault. On A C describe a semi-circle A B C, and divide its circumference into equal parts, representing the boards of the covering. From the divisions of the semi-circle, b, m, t, &c., from the centre D of

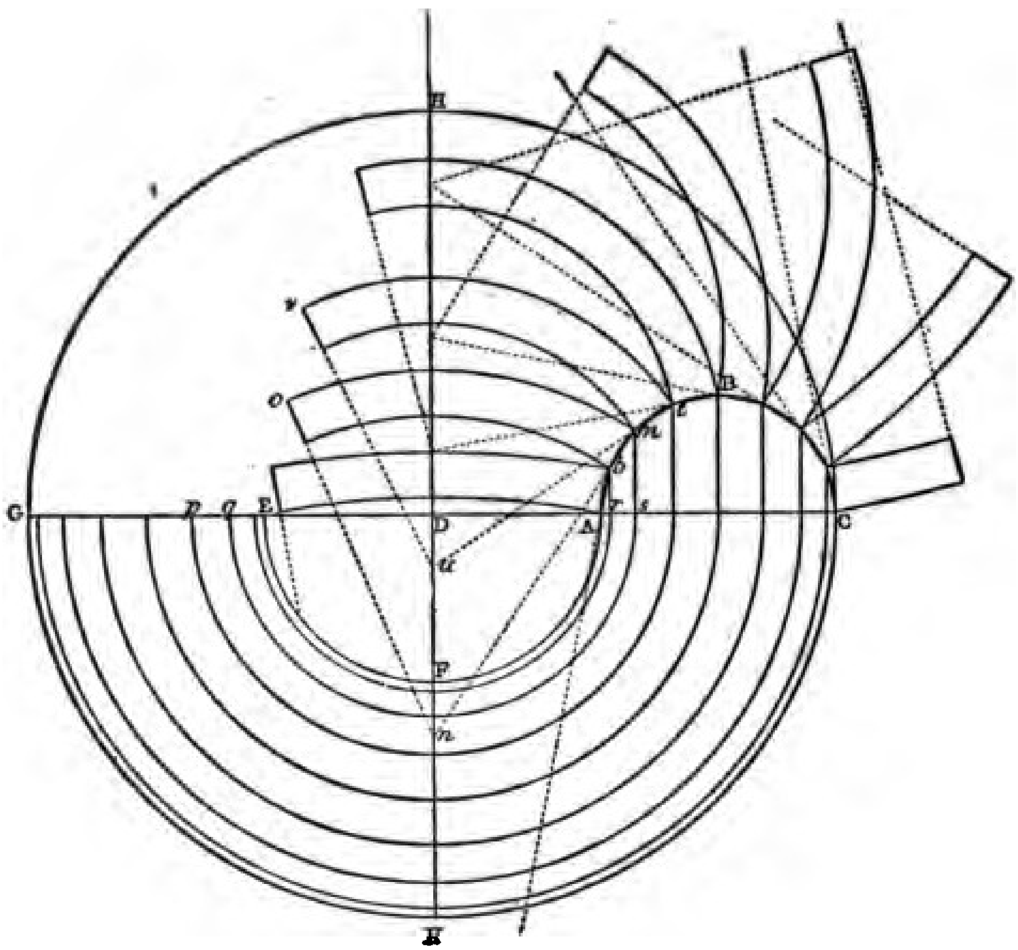


Fig. 92.

the annulus, with the radii D r, D s, &c., describe the concentric circles, s q, &c., representing the covering boards in plan. Through the centre D draw H K perpendicular to G C, indefinitely extending it through K. Join the points of division of the semi-circle, A b, b m, m t, by straight lines, and produce them until they cut the line K H as m b n, t m u, when the points n, u, &c., are the centres from which the curves of the covering boards m o, t v, &c., are described.

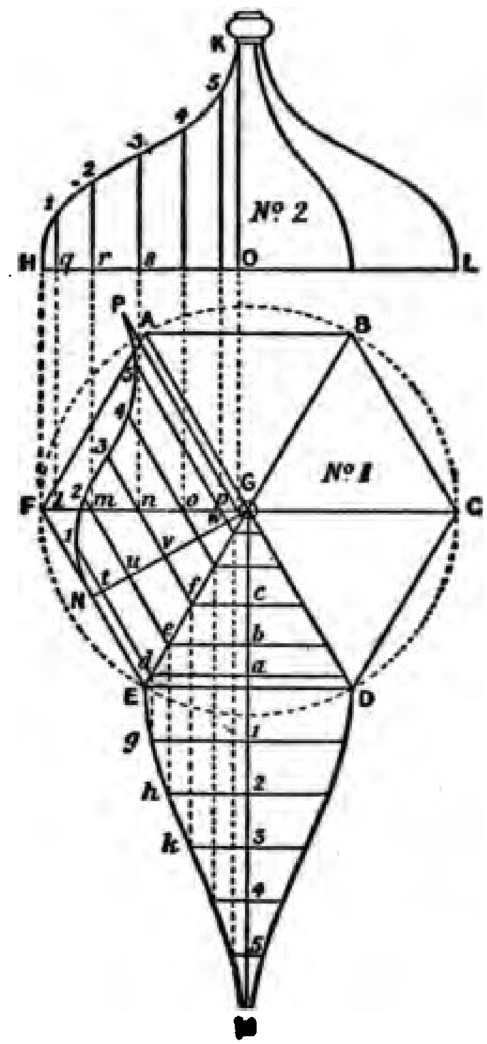


Fig. 93.

To find the covering of an ogee dome, hexagonal in plan.

Let A B C D E F (No. 1, Fig. 93) be the plan of the dome, and H K′L (No. 2) the elevation, on the diameter F C. Divide H K into any number of equal parts in 1, 2, 3, 4, 5, k, and through these draw perpendiculars to H L, and produce them to meet F C

(No. 1) in l, mm, n, o, p, G. Through these points draw lines l d, me, n f, &c., parallel to the side F E of the hexagon: bisect the side F E in N, and draw G N, which will be the seat of a section of the dome, at right angles to the side E F. To find this section nothing more is required than to set up on N G from the points t, u, v, &c., the heights of the corresponding ordinates q 1, r 2, s 3, &c., of the elevation (No. 2) to draw the ogee curve N 1 2 3 4 5 p, and then to use the divisions in this curve to form the gore or covering of one side E g h k M D.

PART II.

PRACTICAL SOLUTIONS.

Having taken a thorough course in Solid Geometry, the student should be now prepared to solve almost any problem in practical construction, almost as soon as they present themselves. The various problems in construction, however, are so numerous, and in many cases, so intricate, that the student will often be confronted by problems which will require so many applications of the rules he has been taught, in different forms, that without some helping guidance, he will fail to see exactly what to do.

The following examples, with their explanations, are intended to give him the aid necessary to solve many difficult problems, and equip him with the means of still further investigation and sure results.

It is often necessary for the workman to find the exact stretchout or length of a straight line that shall equal the quadrant or a semi-circle.

To accomplish this:—Take A B radius, and A centre; intersect the circle at C; join it and B; draw from D, parallel with C B, cutting at H; then A H will be found equal to curve A D. Fig. 1.

This method is somewhat different to that already given; both, however, are practical.

Fig. 2. To find a straight line which is equal to the circumference of a circle. Draw from the centre, O,

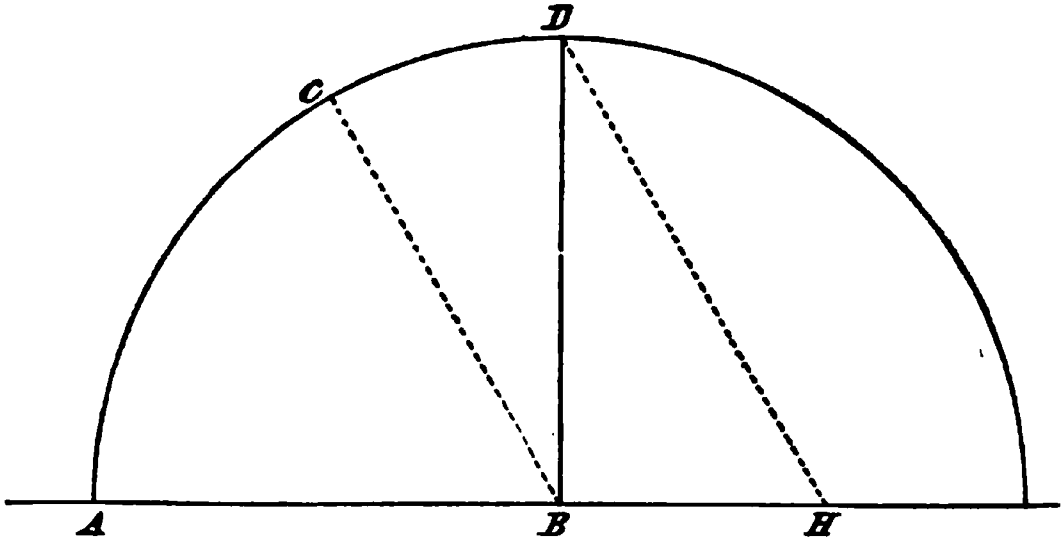


Fig. 1.

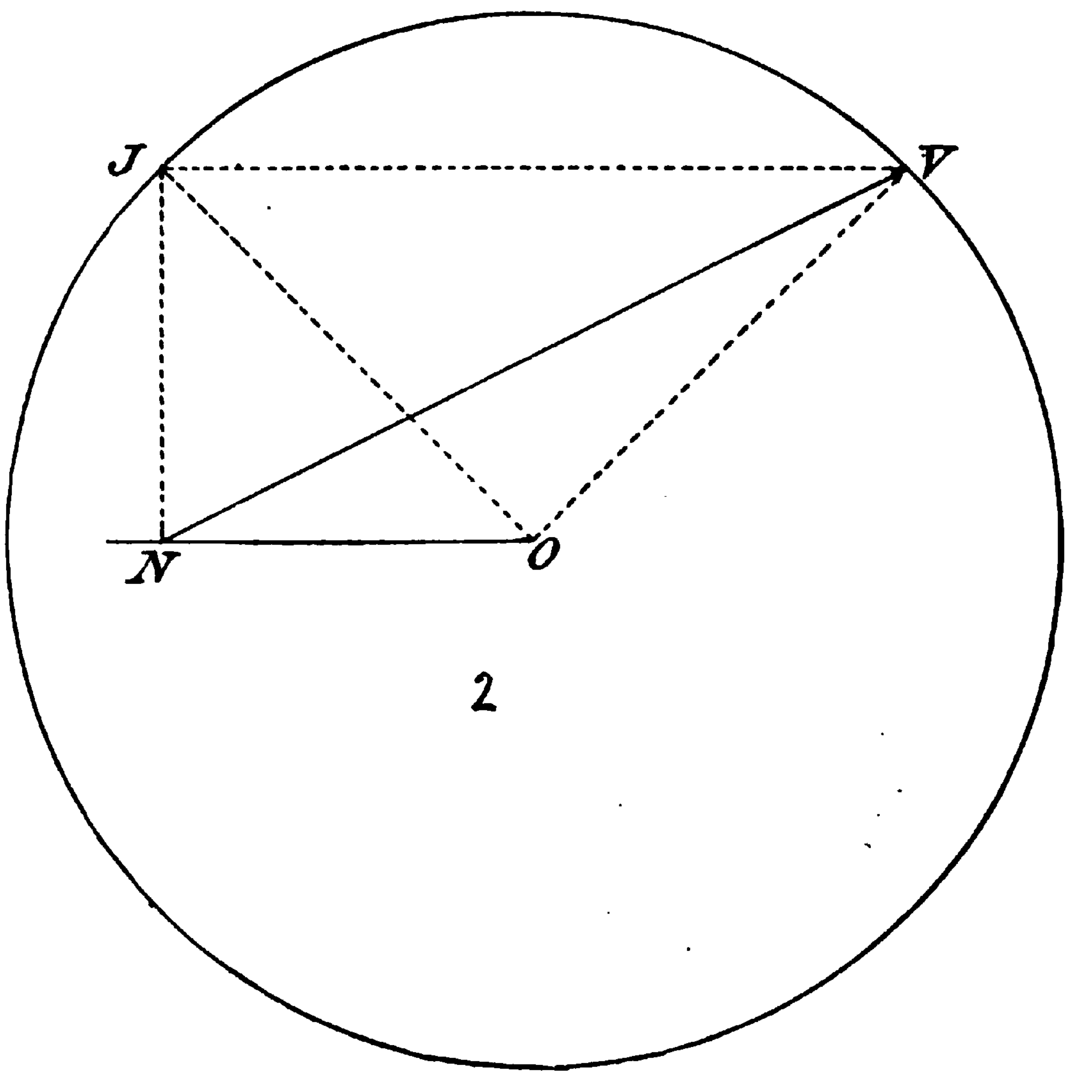


Fig. 2.

any right angle, cutting at J and V; join J V; draw from O parallel with J V; square down from J, cutting at N; join it and V; then four times N V will be found to equal the circumference.

How to find the mitres for intersecting straight and circular mouldings.

Figure 3 shows the form of an irregular piece of framing or other work, which requires to have mouldings mitre and properly intersect.

The usual way of doing this is to bisect each angle, or to lay two pieces of moulding against the sides of framing, and mark along the edge of each piece, thus making an intersectic or point, so that by drawing through it to the next point, which is the angle of framing, the direction of mitre is obtained. This process, however, is not the quickest and best by any means. The most simple and correct method is to extend the sides A L and P H.

Now suppose we wish to find a mitre from L; take it as centre, and with any radius, as K, draw the circle, cutting at J; join it and K; draw from L parallel with J K, and we have the mitre at once.

Now come to angle on the right; here take H as centre, and with any radius, as E, draw the circle, cutting at F; join it and E; draw from H parallel with E F, and you will find a correct mitre.

The next question is the intersection of straight and circular mouldings.

In the present case an extreme curve is given, in order to show the direction of mitre here, which is simply on the principle of finding a centre, for three points not on a straight line. For example, A B C are

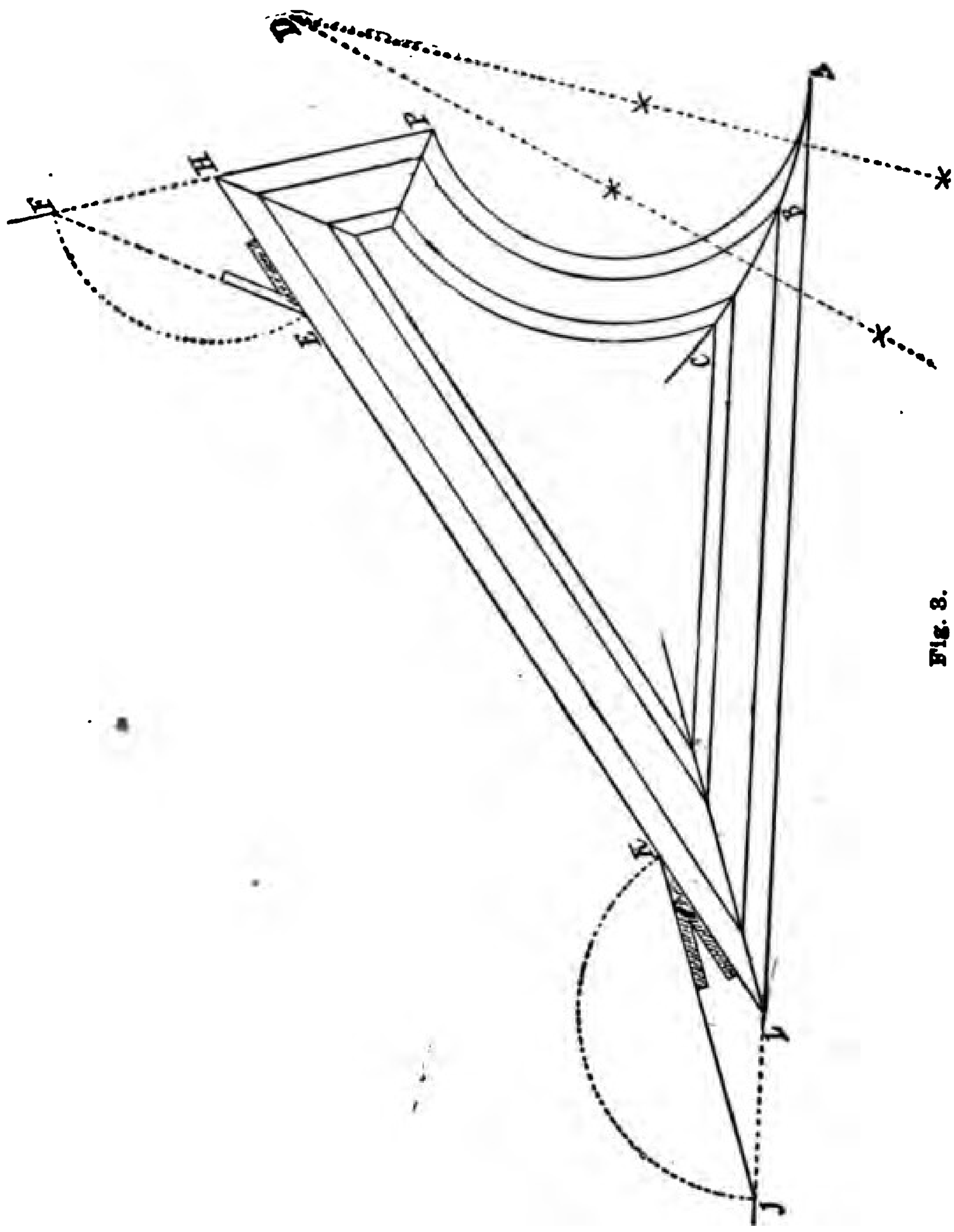


Fig. 8.

points; bisect A B and B C; draw through intersections thus made, and lines meeting in point D give a centre, from which strike the circular mitre as shown.

Here it may be stated that in some cases a straight line for mitres will answer; this means when the curve is a quadrant or less.

Fig. 4 shows the intersections of rake and level mouldings for pediments.

The moulding on the rake, increases in width, and is entirely different from that on the level, yet both mitre ,and intersect, the rake moulding being worked to suit the level. If the curves of Fig. 4 are struck from centres as shown, then by the same rule, the rake moulding is also struck from centres.

Take any point in the curve, as C; square up from it, cutting at B; draw from C parallel to S L; join L K, which bisect at N; make E D equal to A B on the right; join L D and D N; bisect L D, also D N; draw through intersections thus made, and the lines meeting in F, give a point from which draw through N; make N J equal H F; then F and J are centres, from which strike the curve, and it will be found to exactly intersect with that of Fig. 4.

Both mouldings here are shown as solid, and of the same thickness. This is done for the purpose of making the drawing more plain and easily understood; but bear in mind that all crown mouldings are generally sprung.

To find the form of a sprung or solid moulding on any rake without the use of either ordinates or centres.

It may not be generally known, that if a level moulding is cut to a mitre, that the extreme parts of mitre,

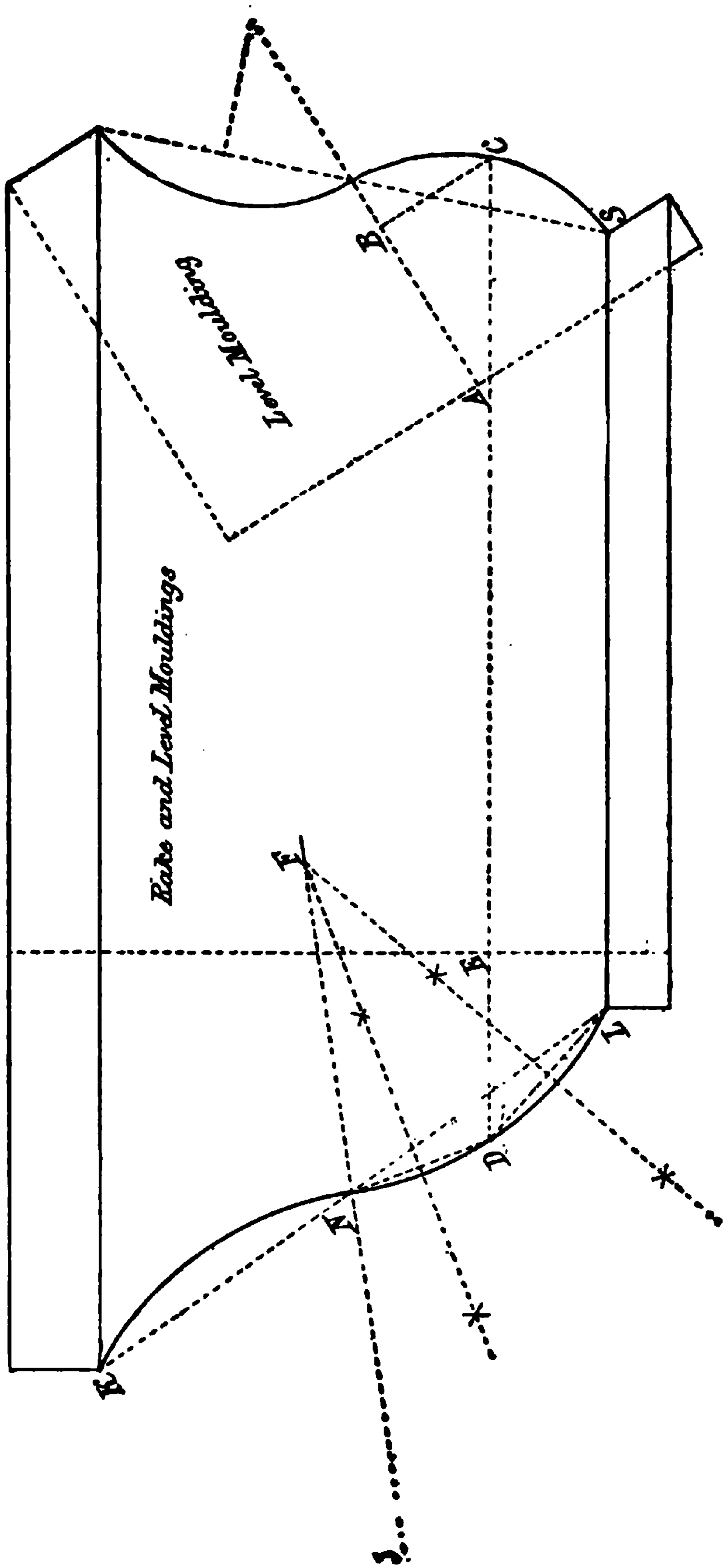


Fig. 4.

when in a certain position, will instantly give the exact form of a rake moulding, and it will intersect, and mitre correctly with that of level moulding. To do this, take the level piece which has been mitred; lay its flat surface on the drawing; make its point P at Fig. 5, stand opposite point P at Fig. 6; keep the outer edge fair with line N L. The piece being in this position, take a marker, hold it plumb against the mitre, and in this way, prick off any number of points as shown, through which trace the curve line, and the result is a correct pattern by which the rake moulding is worked.

A moment's consideration will convince us that this simple method must give the exact form of any rake moulding to intersect with one on the level.

To cut the mitres and dispense with the use of a box, this method will be found quick and off-hand. Take, for example, the back level moulding, and square over on its top edge any line, as that of F N; continue it across the back to H; make H V equal T L above, and from V, square over lower edge H K. Now take bevel 2 from above, and apply it on top edge, as shown; mark F L; then join L V; cut through these lines from the back, and the mitre is complete.

To cut the mitre on the rake moulding, square over any line on its back, as that of H J; continue it across the top and lower edge; take bevel X, shown above Fig. 5, apply it here on top edge, and mark D A; take the same bevel, and apply it on small square at E, and mark E 2.

We now want the plumb cut on lower edge J K, and the same cut on front edge N P, shown at Fig. 6. Take

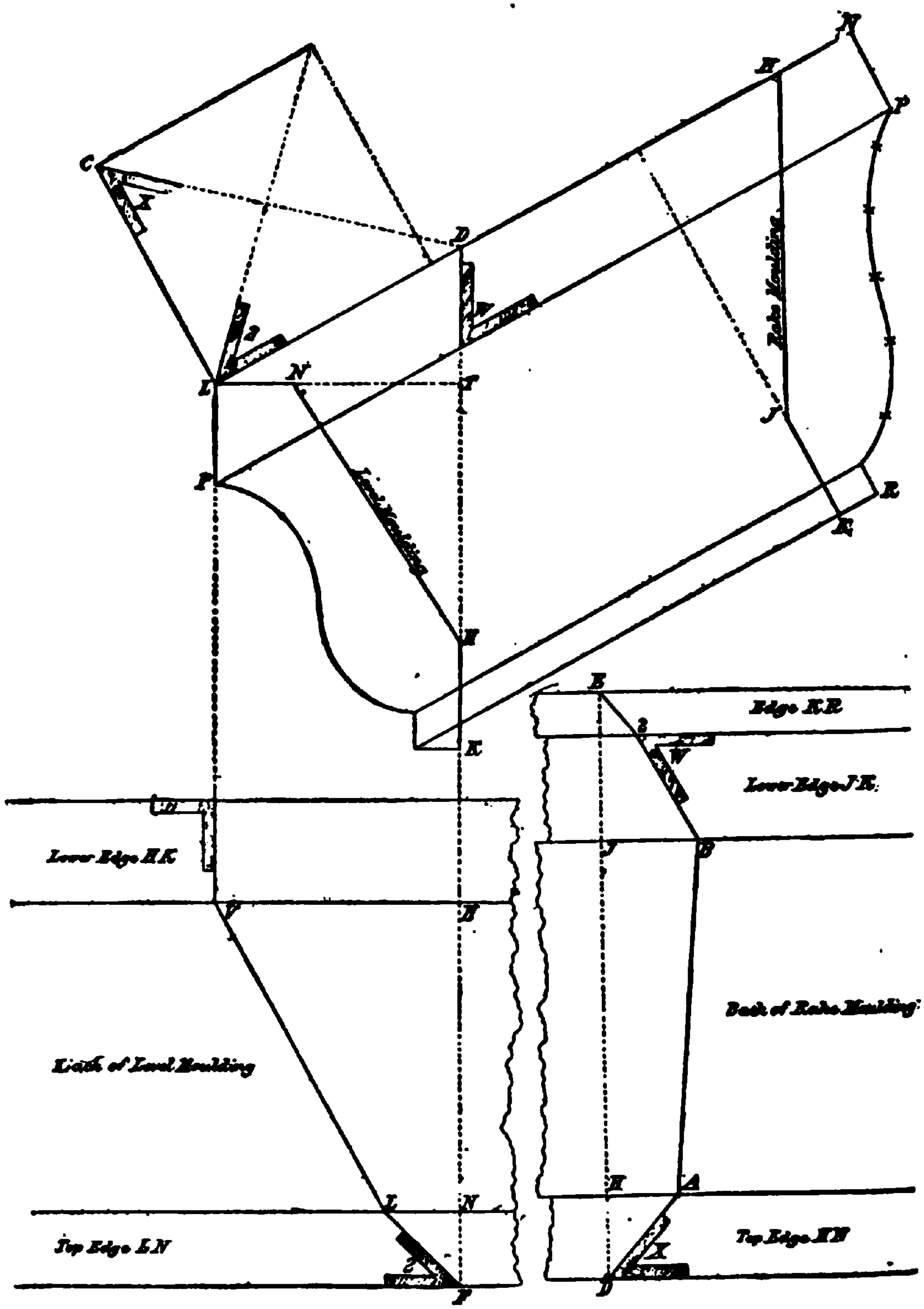

Rake Moulding
Level Moulding
Edge KR
Lower Edge JK
Lower Edge HK
Back of Rake Moulding
Top Edge LN

Fig. 5. Fig. 6.

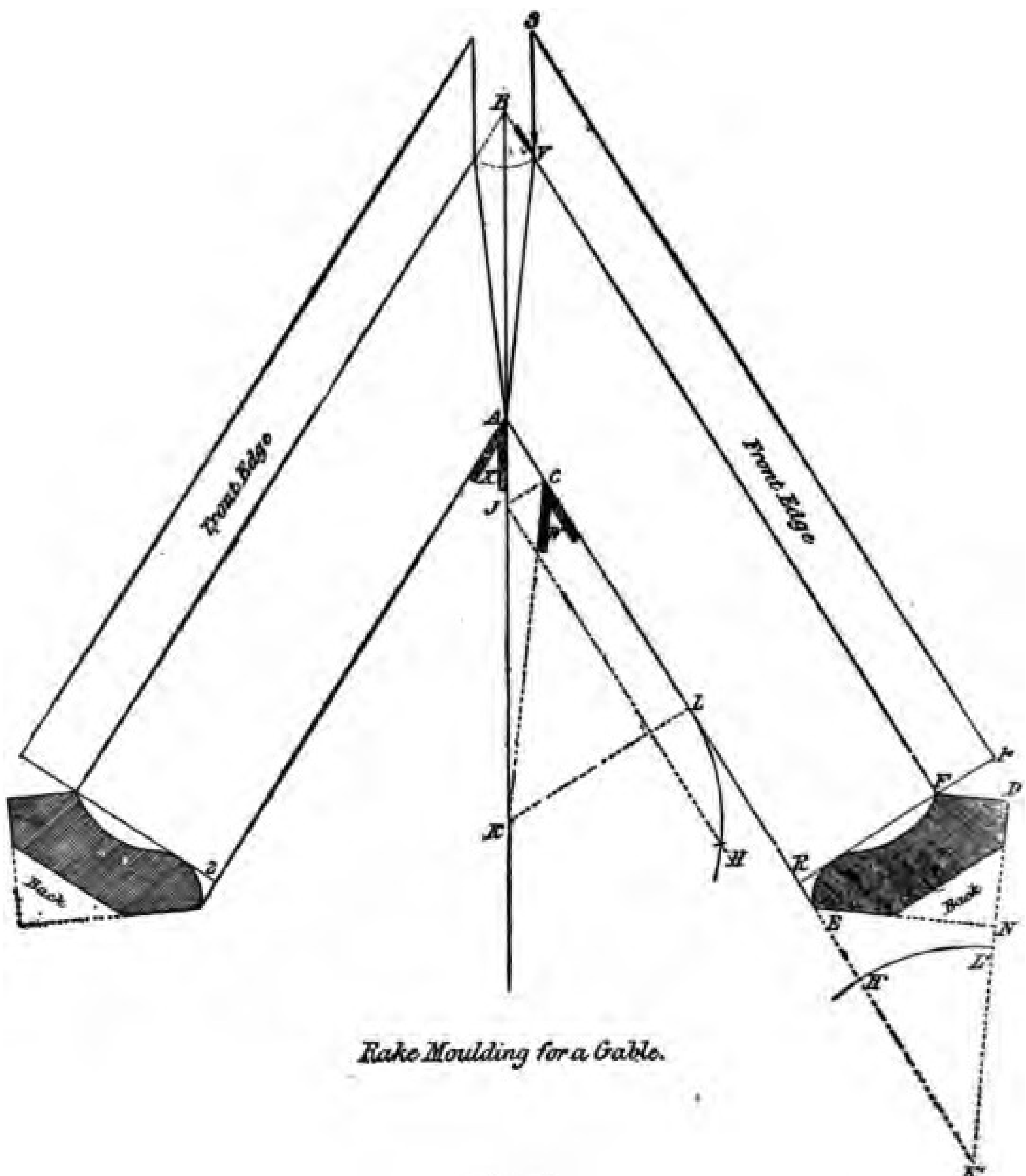
bevel W above Fig. 5, apply it here and mark 2B; join B A; this done, apply the same bevel on front edge N P, and mark the plumb cut, it being parallel with that of 2B here, or K J, Fig. 5; now cut through lines on the back, and the mitre is complete.

It has already been shown, that we dispense with making or using a box for mitreing sprung mouldings.

In this case, the front edge or upper member, stands parallel with face of wall, so that bevel X being applied, gives the plumb cut; then the cut on top edge is square with face of wall. This shows, that we have only to find the direction of a cut on the back of moulding to make the mitre.

To do this, take any point as R; draw from it square with rake of gable. Now mark sections of moulding, as shown, its back parallel with R F; draw from D square with E N; extend the rake to cut line from D at K; this done, take any point on the rake, say L; draw from it parallel with R F, cutting at K; take it as centre and L as radius, and draw the arc of a circle; with same radius return to K on the right; take it as centre, and draw the arc L H; make the first arc equal it; then draw from H parallel with L C, cutting at C J; draw from it square with rake, cutting at C, and join C K. This gives bevel W for cut on back of moulding.

A most perfect illustration of this may be had by having the drawing on card-board, and cutting it clear through all the outer lines, including that of the moulding on lines F D N E, making a hinge by a slight cut on line R F; also make a hinge of line R A, by a slight cut on the back, and in like manner make front edge work on a hinge by a slight cut on line F V. This cut is made on top surface. Perform the same operation



Rake Moulding for a Gable.

Fig. 7.

on the left. All the cuts being made, raise both sides on hinges A R and A 2; push the sections of mouldings on right and left from you; make front edge rest on F D. Now bring mitres together, and we have a practical illustration of mitreing sprung mouldings on the rake. (See Fig. 7.)

PROJECTION OF SOLIDS.

The following illustrations will be found by the student almost indispensable in the construction of various objects, and they will open the door to many more.

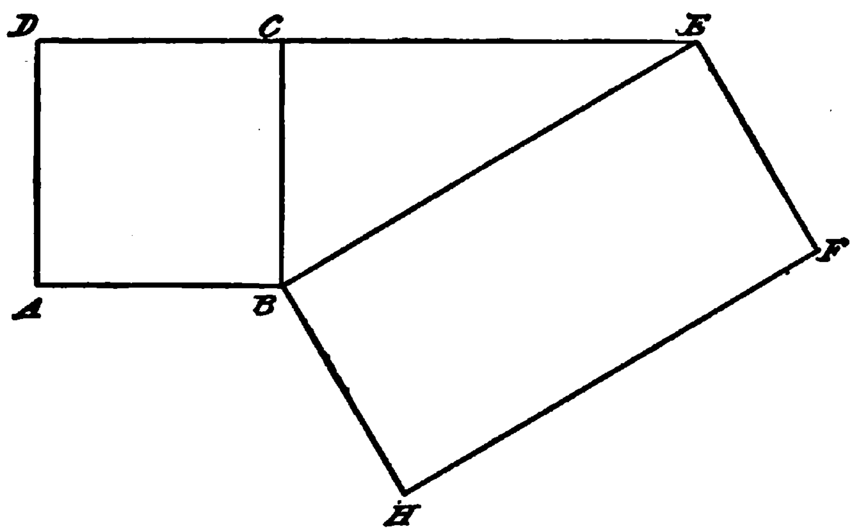


Fig. 8.

Fig. 8 shows the projection of a solid. This means the section of anything that is cut by a plane not parallel to its base; or, to put this in a more practical way —take a square bar of wood and cut it in the direction of B E; the section it makes is shown by B E F H; simple as this is, it still gives the idea of what is meant by projection.

Fig. 9 shows the section of a square bar which has been cut by two unequal pitches, say in the direction of bevels J and H; the line C B is called the seat; from it all measurements are taken and transferred to lines that are square with the pitch A B; this pitch may be called a diameter, because it and the ordinate A E are at right angles.

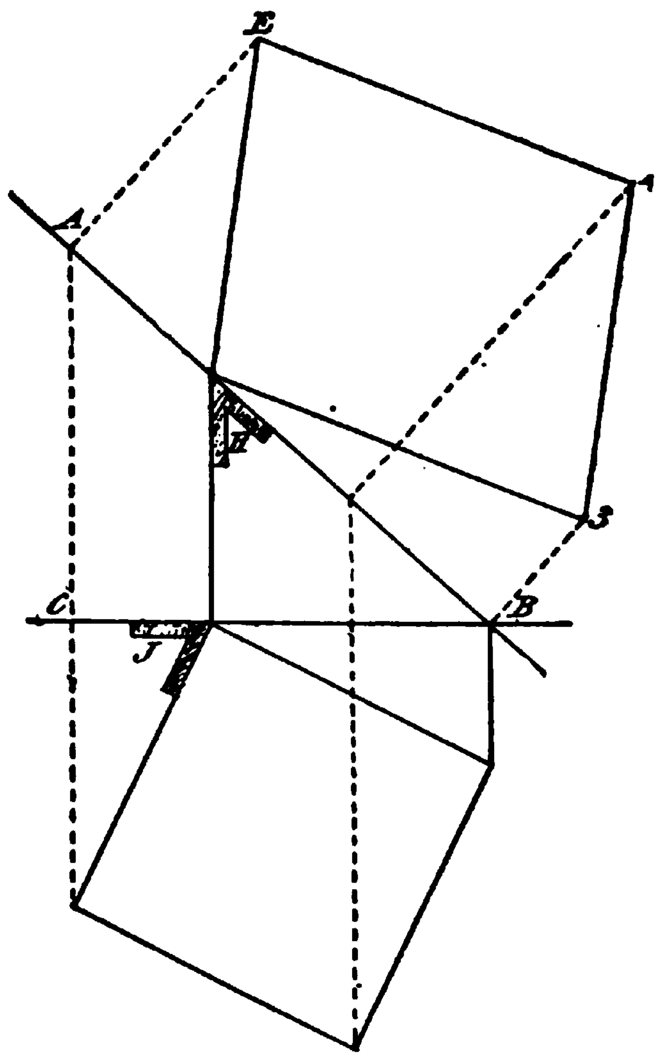


Fig. 9.

Fig. 10 shows the sides of a square bar which may be any length. The bar is to stand perpendicular, and pass through a plank that inclines at A B; the learner is now required to show on the surface of plank the shape of a mortise that shall exactly fit the bar.

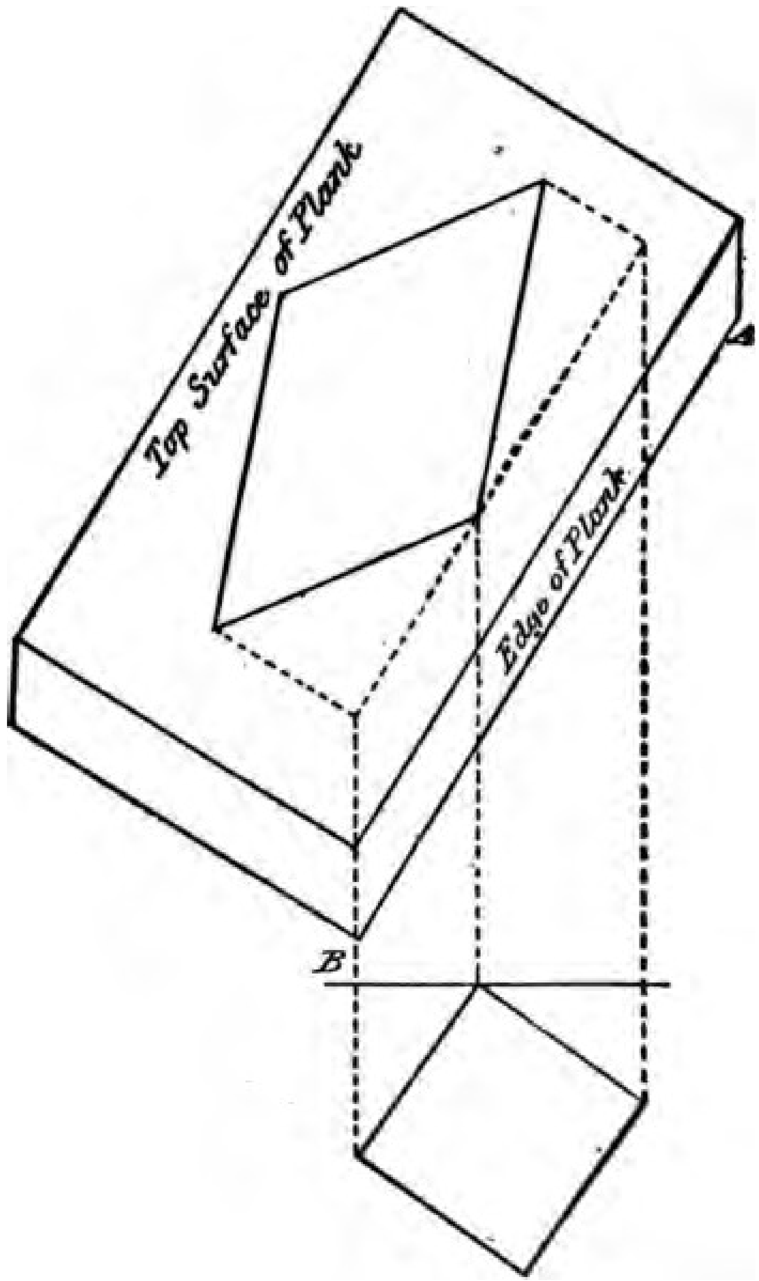


Fig. 10.

To solve this the student is left to exercise his own intelligence.

The problem may be clearly demonstrated by a card-board model; it being cut, and the parts projected from the flat surface will represent the plank, and show the mortise in it standing directly over the square, Fig. 10.

Fig. 11. It is here required to mark two unequal pitches on two sides of a square bar—then to have a piece of plank or board cut so that it shall exactly fit both pitches on the bar. The inclination of plank may be assumed as A B, and height of both pitches as H A; let E F be the seat from which all measurements are taken, and transferred to lines that are on the surfaces of plank and square with A B; thus giving points to direct in drawing line C D; and D E produced.

Could we apply the bevel J to points C and K, and have plumb lines on the edge of plank, then by cutting through these and those already marked on the surface, the problem would at once be solved, by making both upper and under surface of plank fit the two pitches as required. But in practice this would be inadmissible on account of the great waste of material.

The proper method is to take any point, E, and cut through the plank square with E D, and at C, cut through the plank square with C D; here it will be noticed that line A D on the surface makes a very different pitch to that of A B on the edge of plank, and that C D differs from both.

To understand these points thoroughly is the true secret of the nicest element in the joiner's art—hand railing. It being clear that two bevels are required one for each pitch—proceed to find them. Make N L equal one side of the square. Take N as centre, and strike an arc touching the line E D; with same radius, and L centre, make the intersection in S; join it at L,

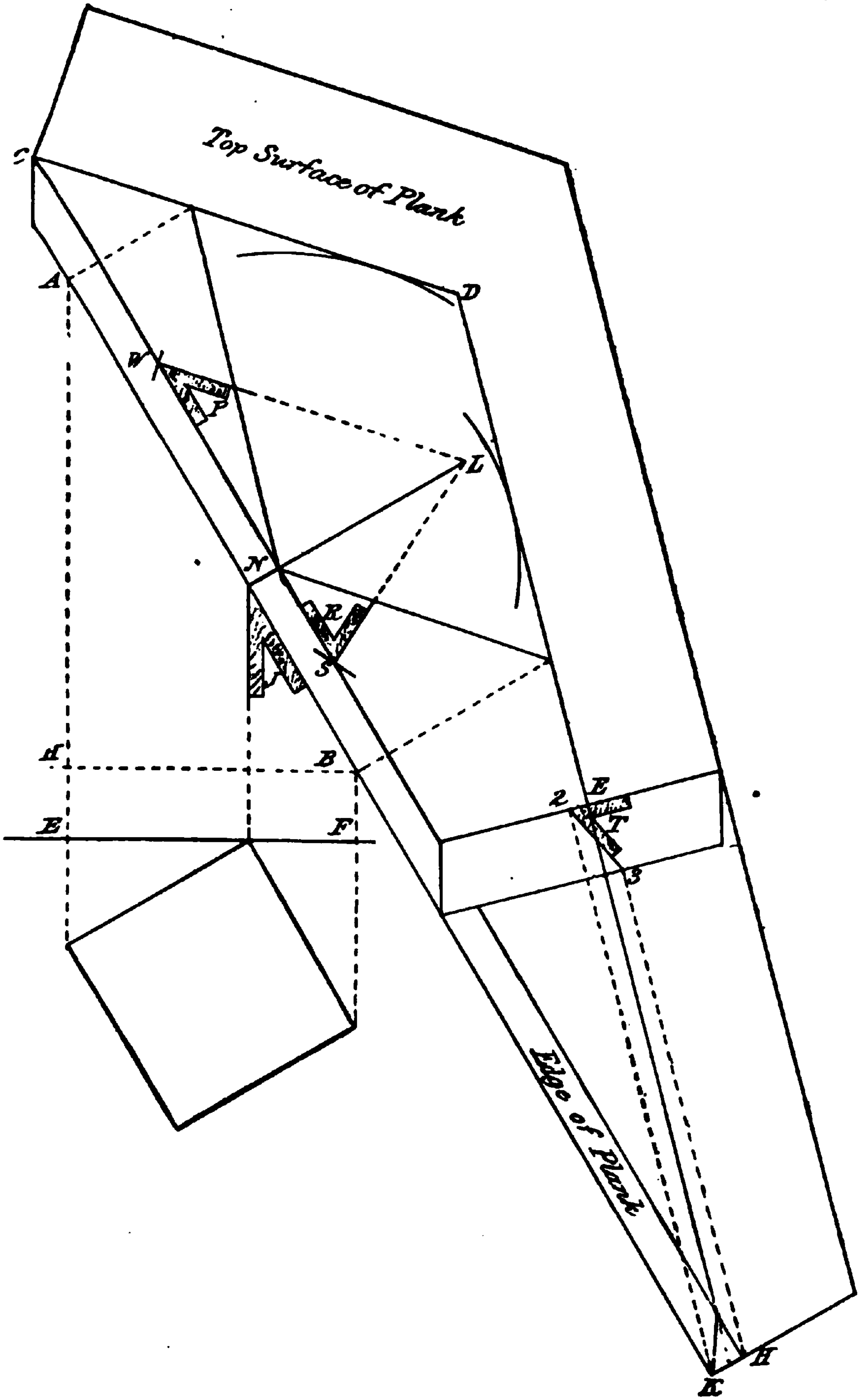


Fig. 11.

which gives bevel R for joint at E. Again take N as centre, and strike an arc touching line D C; with same radius, and L centre, make the intersection in W; join it and L, which gives bevel P for joint at C; the cuts being made by these bevels.

PANELLED CEILINGS IN WOOD AND STUCCO BRACKETS, AND SIMILAR WORK.

It is no part of the duty of this work to show designs for wooden ceilings, but in order to illustrate the method or methods, of constructing a wooden ceiling I deem it proper to show a design in this style the better to convey to the student the reason for the various steps taken to reach the desired result.

Fig. 12 shows a section of a roof and the mode of constructing a bracketed ceiling under it, but with slight modification the same arrangement can be adopted for ceilings under floors. The rafters A A are 18 inches from centre to centre. On these, straps a a, 3X1½ inches, are nailed at 16 inches apart, and similar straps a a, 1¼X1 inch, are nailed to the ceiling joists. To the straps are nailed the brackets b b b, shaped to the general lines of the intended ceiling, and also placed 14 inches apart. The laths are nailed to the brackets for the moulded parts, and to straps for the flat parts of the ceiling. The brackets and the straps for ceiling should not be more than one inch in thickness, for where the brackets and straps occur the plaster cannot be pressed between the laths to form a key. If the brackets and straps are made thicker,

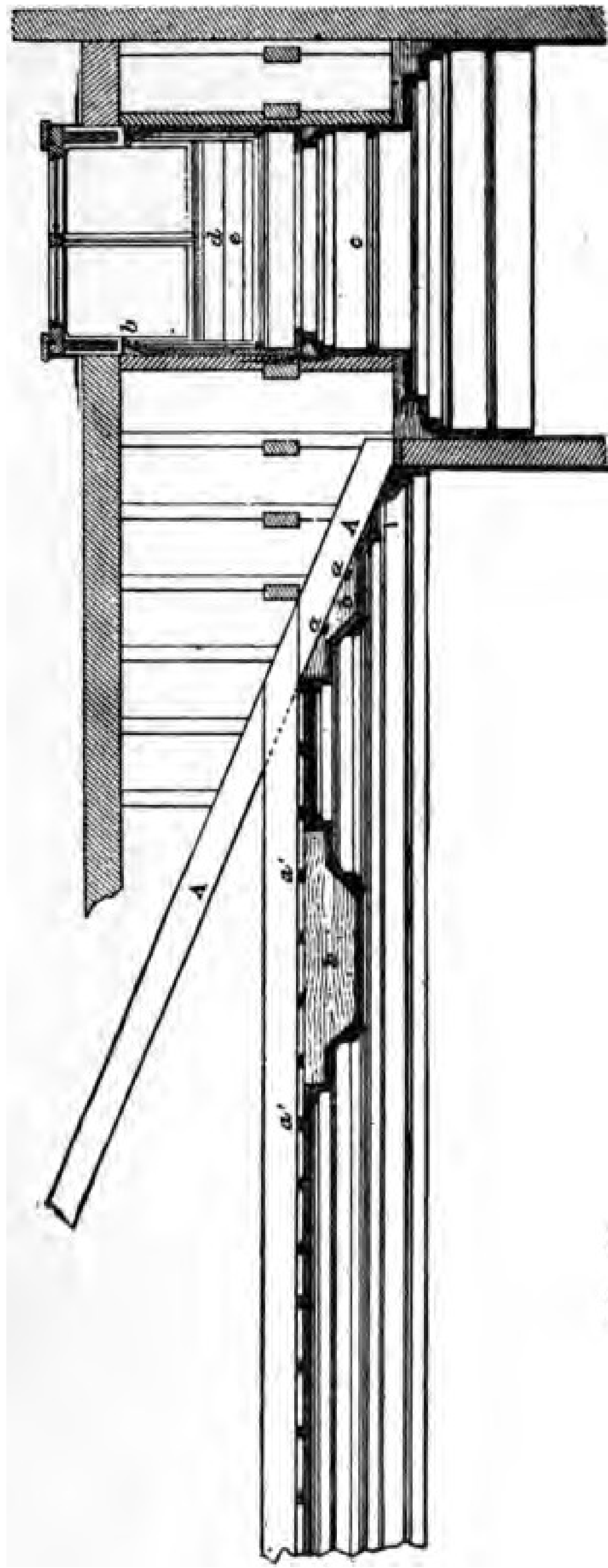


Fig. 15.

Fig. 12.

parts of the ceiling are apt to be weak and irregular. The lathing should be well bonded and have no end joints longer than twelve inches on the ceiling, and twenty-four inches on the walls and partitions. No joints of laths should be overlapped, as the plaster would thereby be made thinner at a part where it forms a no key, and would thus be liable to crack from the vibration of roof, floor or other causes.

Fig. 13 is a section on a larger scale at right angles to that shown in Fig. 12. Here the ceiling bracket b is shown affixed by hooks to the wall, and to the ceiling-joists by the strap a a. The ceiling having been plastered, and the mouldings of the cornice run on the lathed brackets prepared to receive them, the plaster enrichments marked c d e f g are then applied. No. 3 on Fig. 2 shows a curtain-box in section with its curtain-rod.

Fig. 14 is a section through a window-head, meeting-rails and sill of a window. The safe lintel is placed about 10 inches above the daylight of the window, for the purpose of allowing venetian blinds to be drawn up clear of the window, and leave the light unobstructed. The framing of the window is carried up to the lintel, and between it and the upper sash a panel is set in, and the blinds hang in front of it.

Fig. 15 is a cross vertical section, and Fig. 16 a plan, looking up, of a skylight on the ridge of a roof, suitable for a staircase or corridor. The design can be adopted to suit various widths. The skylight is bracketed for plaster finishings, in the same manner as the ceiling already described. The framing and mouldings at b are carried down the side of the light at the same slope

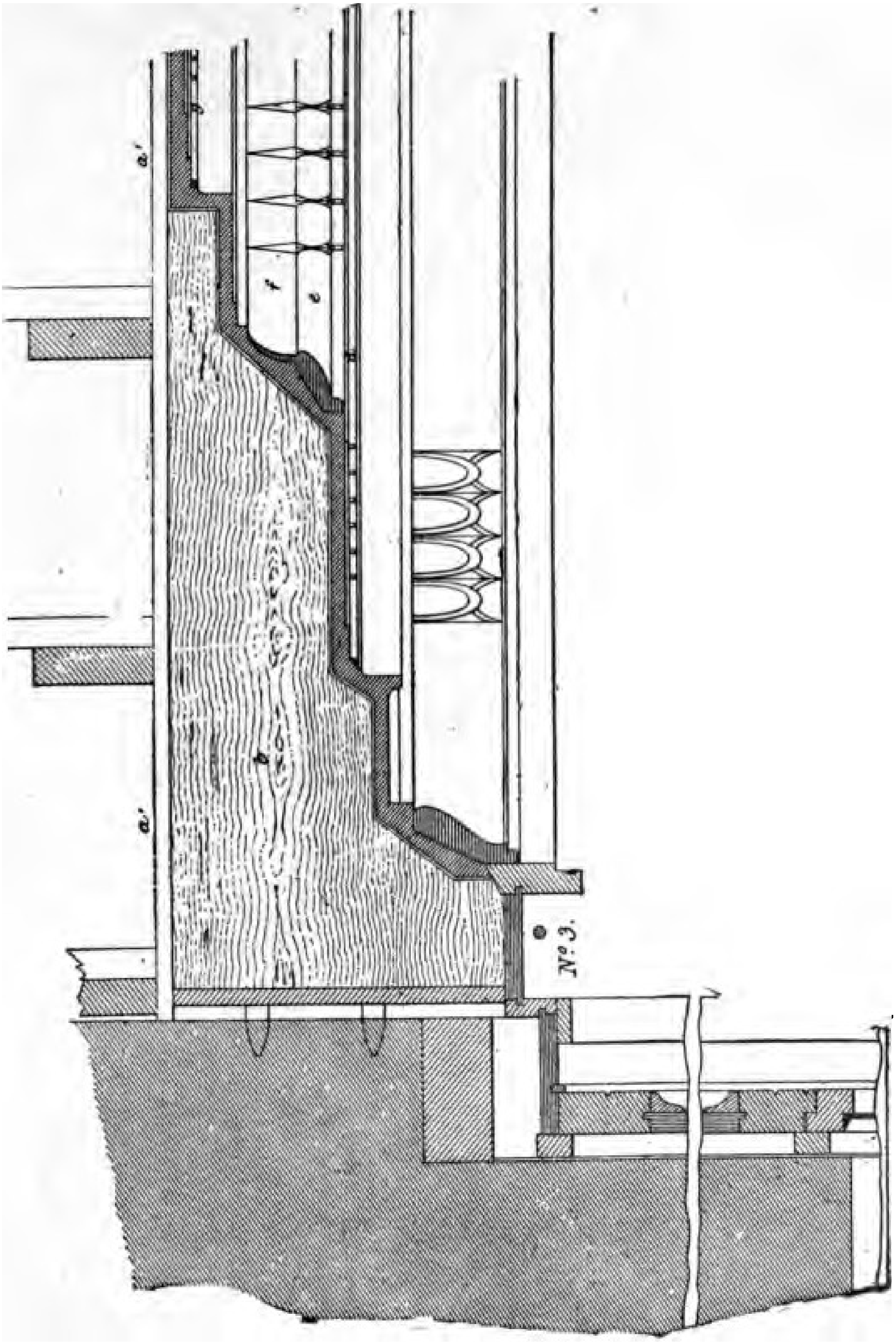


Fig. 13.

Fig. 14.

as the sash, till they butt against the sill and bridle d and e, forming a triangular panel having for its base the cornice c, which is carried round the aperture horizontally, and finishing flush with the ceiling, permits the cornice on the corridor to be continued with-

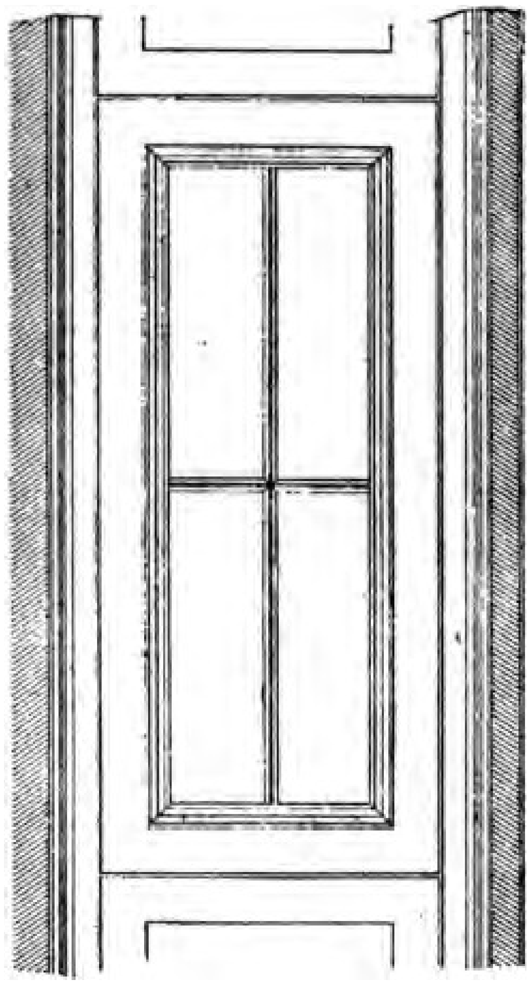


Fig. 16.

out interruption. Observe in Fig. 16 the lower cornice finishes the walls, and the upper mouldings 0 marked c in Fig. 15 are carried round the well of the skylight.

Fig. 17 shows a plan of a panelled ceiling. The lower members or bed mouldings are carried round the walls of the room, and tend to build together, and give an appearance of support to the several parts of the ceiling. The best way of making panelled ceilings is to cover the floor with boarding, and lay down the lines of the ceiling on the temporary floor thus formed. Then build and lath the ceiling on these lines. When it is completed it will be quite firm, and can be cut into sections suitable for being lifted up and attached to the joisting. The same lines will serve as guides for the plasterer setting the moulds for running the cornices and for preparing the circular ornament.

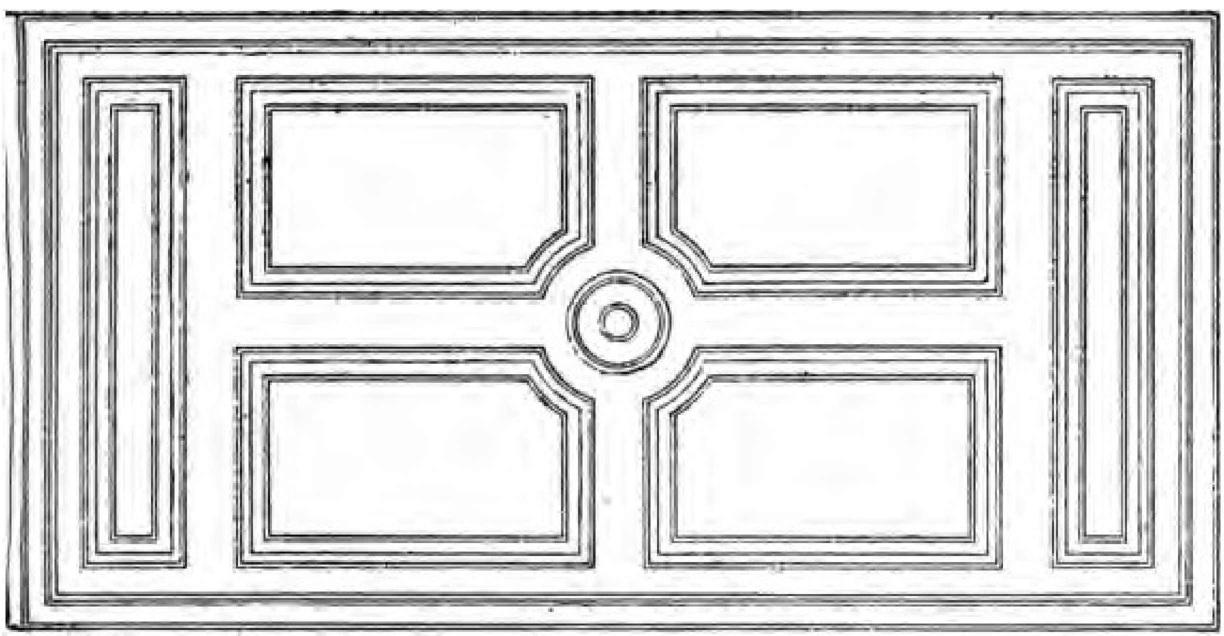


Fig. 17.

Fig. 18 shows in plan and section a centre suited for this ceiling. By fixing the central portion of it one inch from the surface of the finished ceiling and connecting it by small plaster blocks placed about one inch apart, it forms an excellent ventilator. A zinc tube can be led from this centre into a vent, and an air tight valve put on the tube to prevent a down draft when the vent is not in use. F is an iron rod fixed to

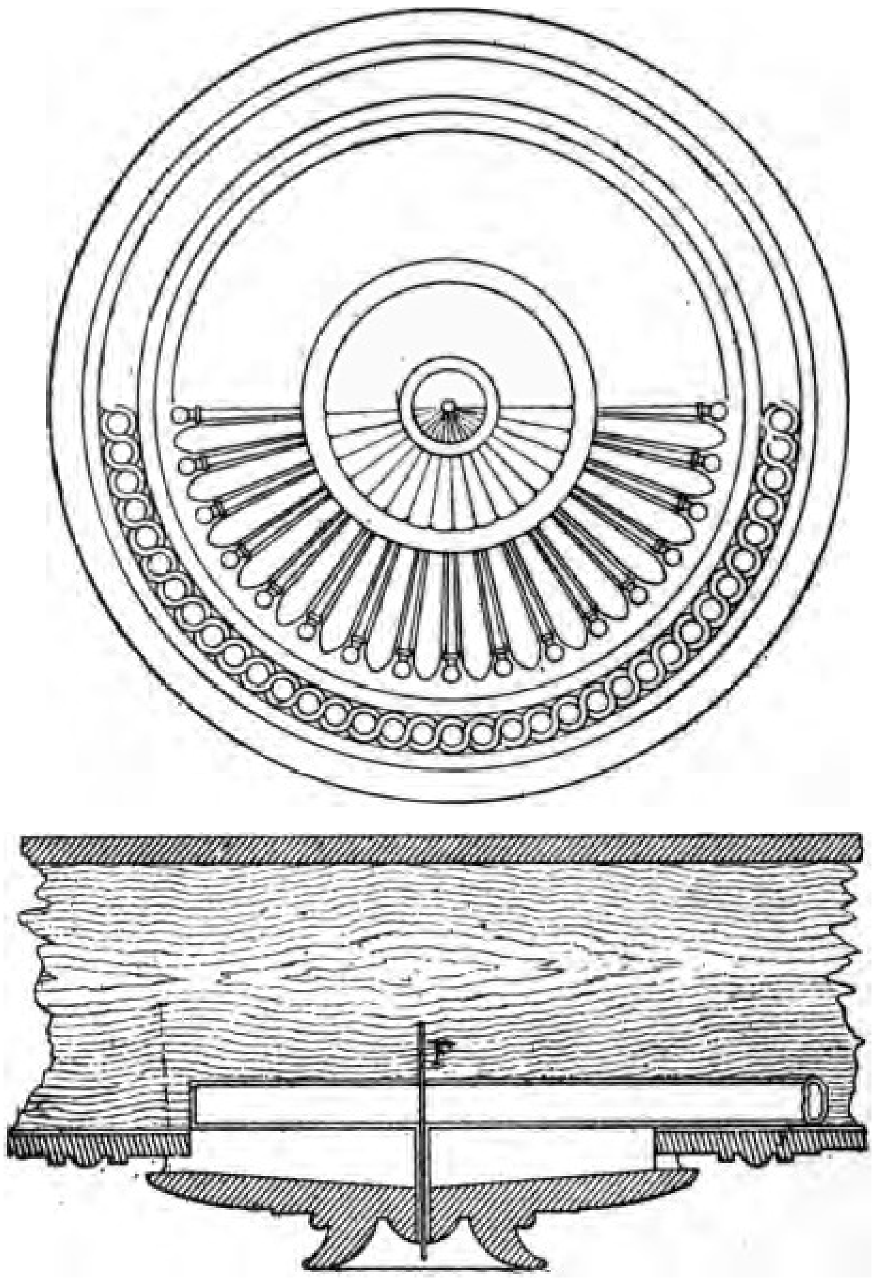


Fig. 18.

a strut or dwang between the joists for the purpose of securing a gas pendant or electrolier.

Let C A B (Fig. 19) be the elevation or the bracket of a core, to find the angle-bracket.

First, when it is a mitre-bracket in an interior angle, the angle being 45°: divide the curve C B into any

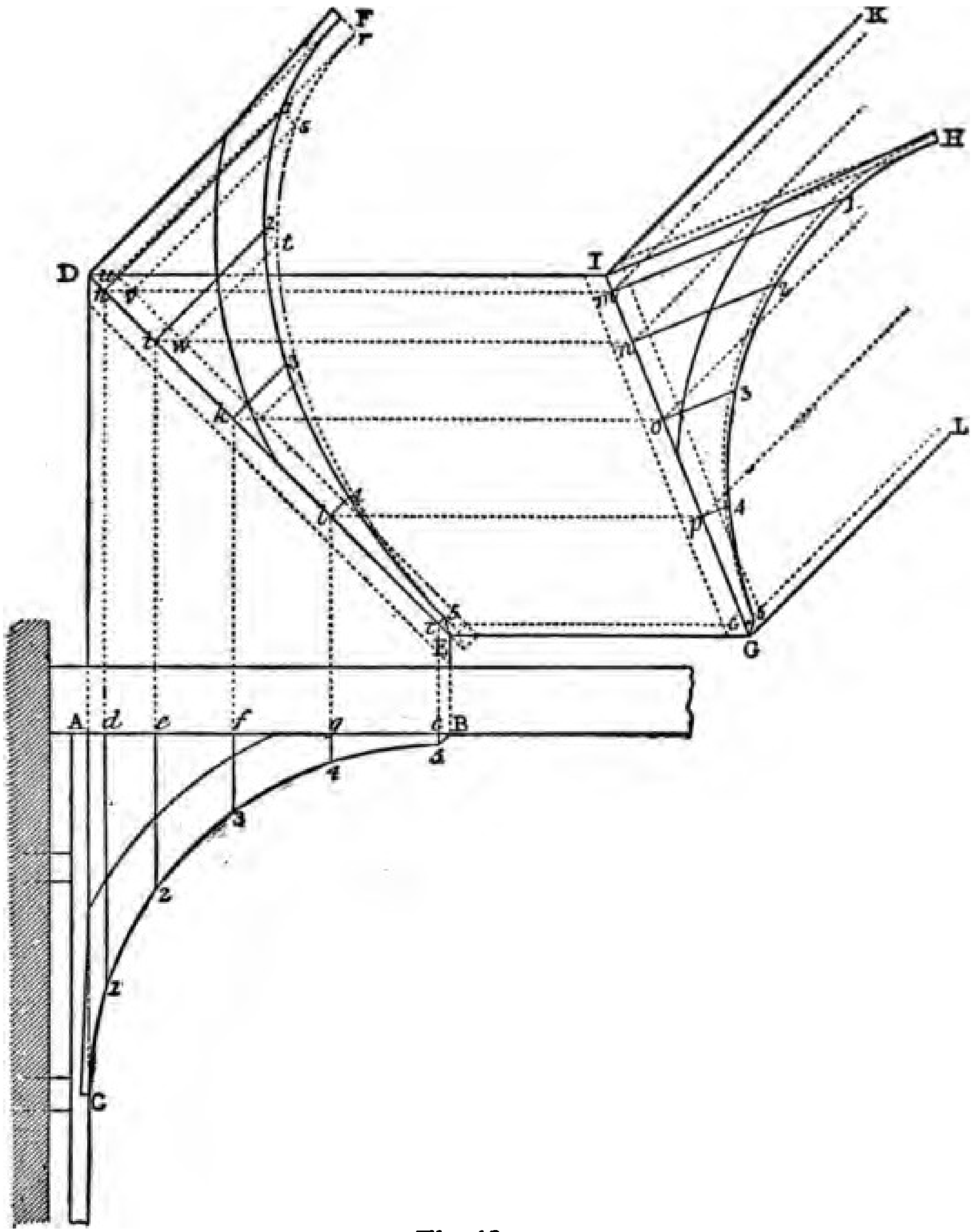


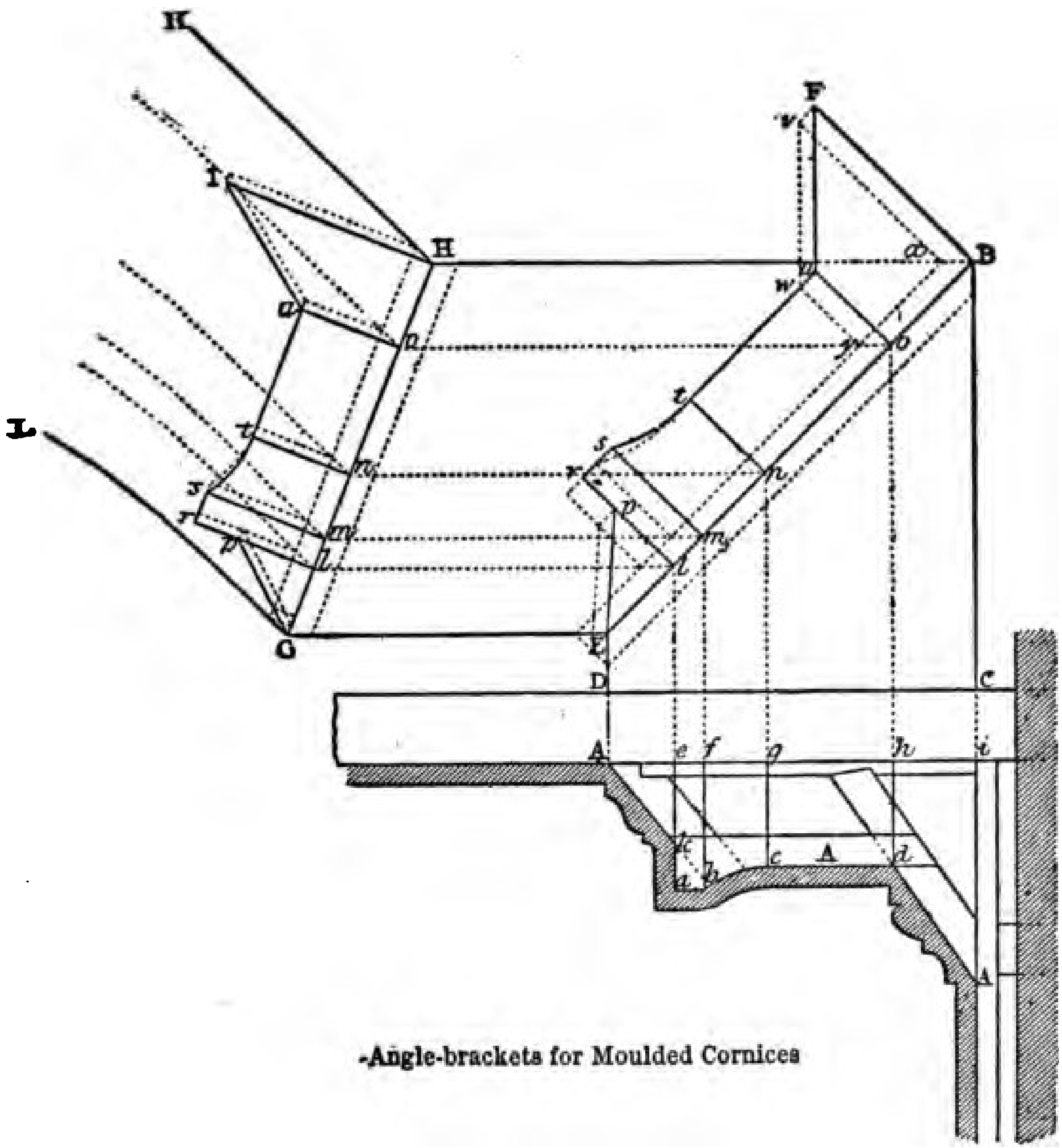
Fig. 19.

number of equal parts 1 2 3 4 5, and draw through the divisions the lines 1d, 2e, 3f, 4g, 5c, perpendicular to

A B, and cutting it in d e f g c; and produce them to meet the line D E, representing the centre of the seat of the angle-bracket: and from the points of intersection h i k l c draw lines h 1, i 2, k 3, l 4, at right angles to D E, and make them equal—h 1 to d 1, i 2 to e 2, &c.; and through F 1 2 3 4 5 draw the curve of the edge on the bracket. The dotted lines on each side of D E on the plan show the thickness of the bracket, and the dotted lines u r, v s w t, show the manner of finding the bevel of the face. In the same figure is shown the manner of finding the bracket for an obtuse exterior angle. Let D I K be the exterior angle: bisect it by the line I G, which will represent the seat of the centre of the bracket. The lines I H, m 1, n 2, o 3, p 4, c 5, are drawn perpendicular to I G, and their lengths are found as in the former case.

To find the angle-bracket of a cornice for interior and exterior, otherwise reentrant and salient, angles.

Let A A A (Fig. 20) be the elevation of the cornice-bracket, E B the seat of the mitre-bracket of the interior angle, and H G that of the mitre-bracket of the exterior angle. From the points A k a b c d A, or wherever a change in the form of the contour of the bracket occurs, draw lines perpendicular to A i or D C, cutting A i in e f g h i and cutting the line E B in E l m n o B. Draw the lines E G, G L B H, and H K, representing the plan of the bracketing, and the parallel lines from the intersection l m n o, as shown dotted in the engraving; then make B F and H I perpendicular to E B and G H respectively, and each equal to i A, o u to h d, n t to g c, m s to f b, l r to e a, l p to e k, and join the points so found to give the contours of the brackets required. The bevels of the face are found as shown by the dotted lines x v y w, &c.



-Angle-brackets for Moulded Cornices

Fig. 20.

To find the angle-bracket at the meeting of a concave wall with a straight wall.

Let A D E B (Fig. 21) be the plan of the bracketing on the straight wall, and D M G E the plan on the circular wall, C A B the elevation on the straight wall, and G M H on the circular wall. Divide the curves

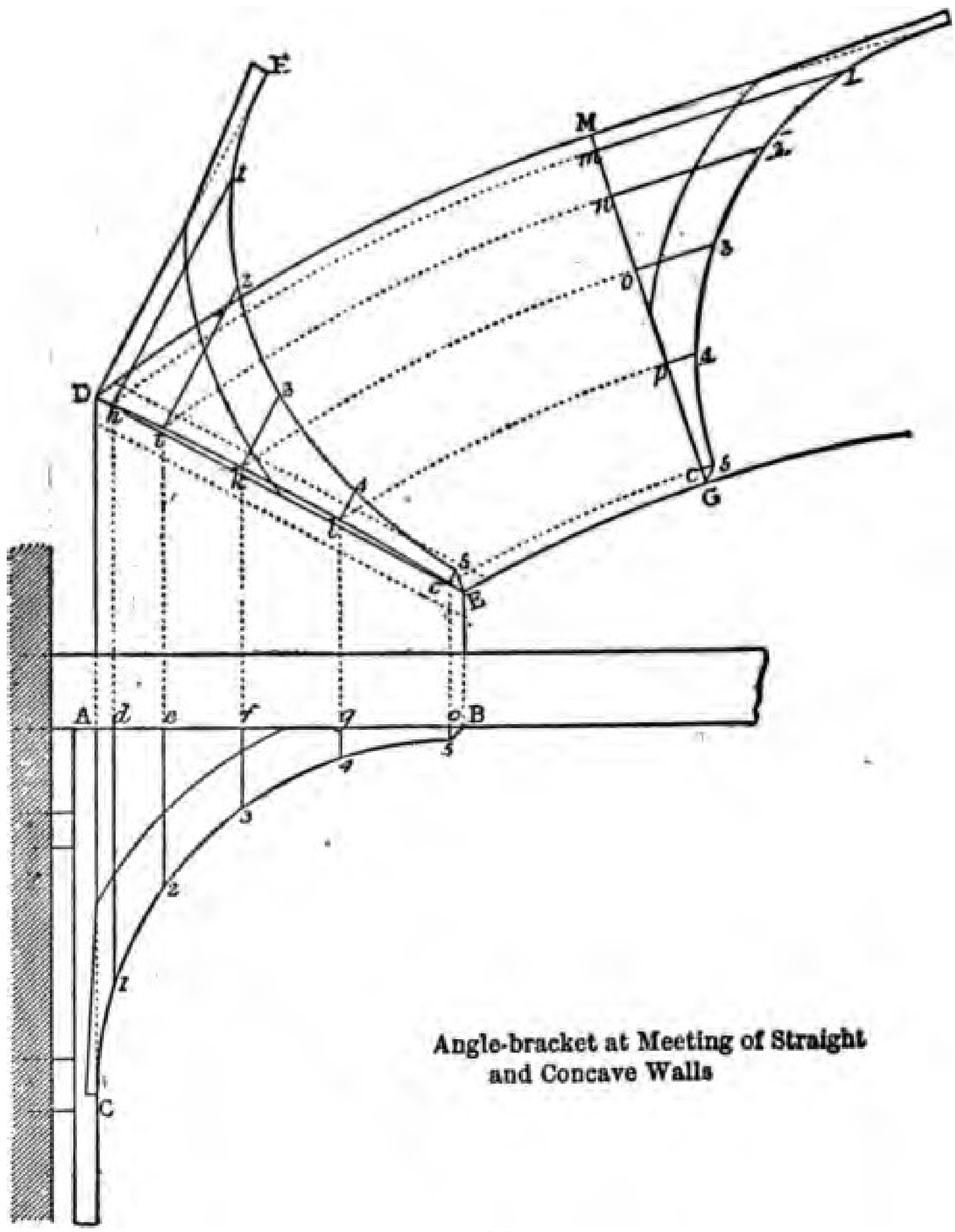
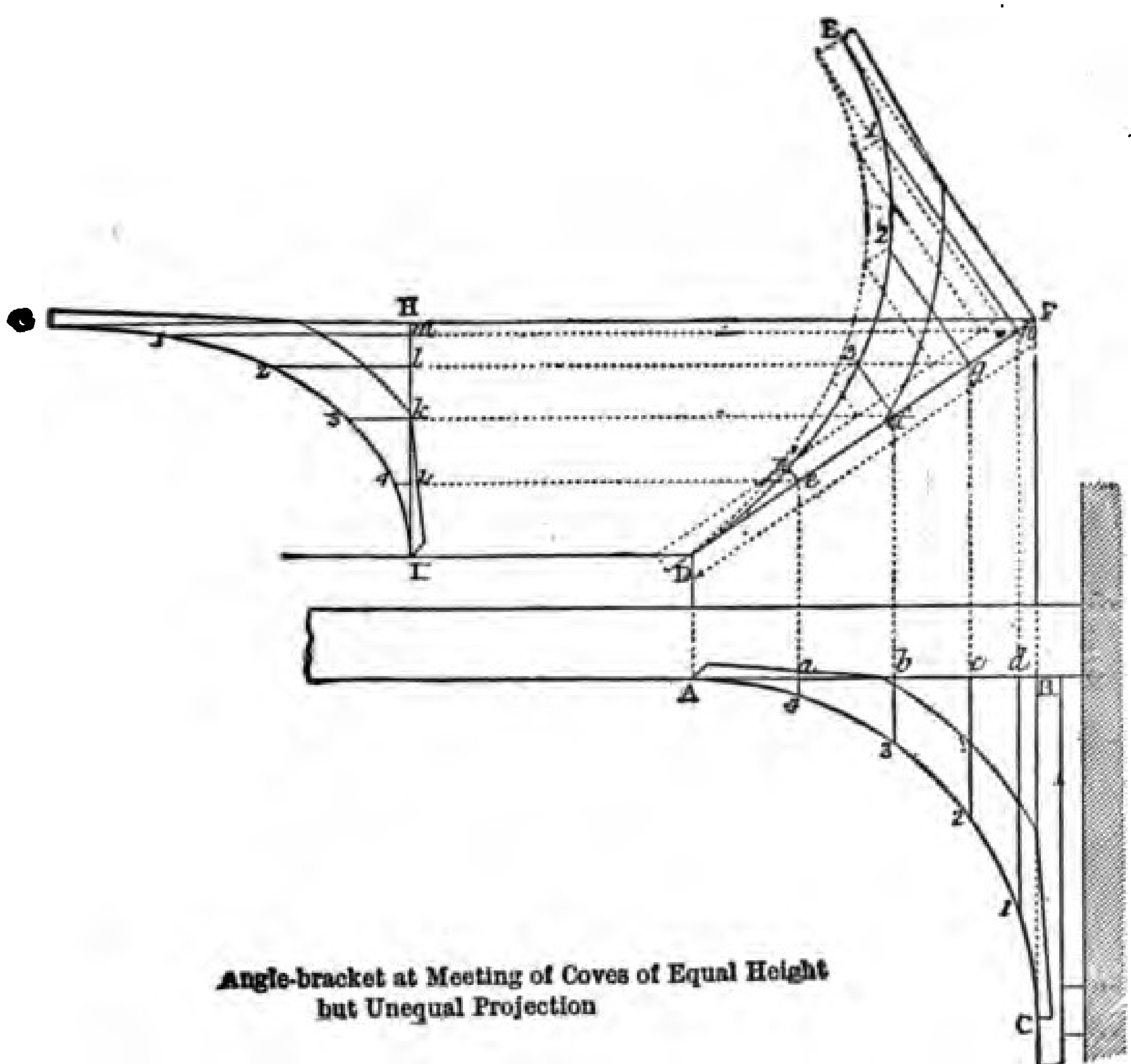


Fig. 21.

C B, G H into the same number of equal parts; through the divisions of C B draw the lines C D, 1 d h, 2 e i, &c., perpendicular to A B and through those of G H draw the parellel lines, part straight and part curved 1 m h, 2 n i, 3 o k, &c. Then through the intersections h i k l of the straight end curved lines draw the curve D E, which will give the line from which to measure the ordinates h 1, i 2, k 3, &c.

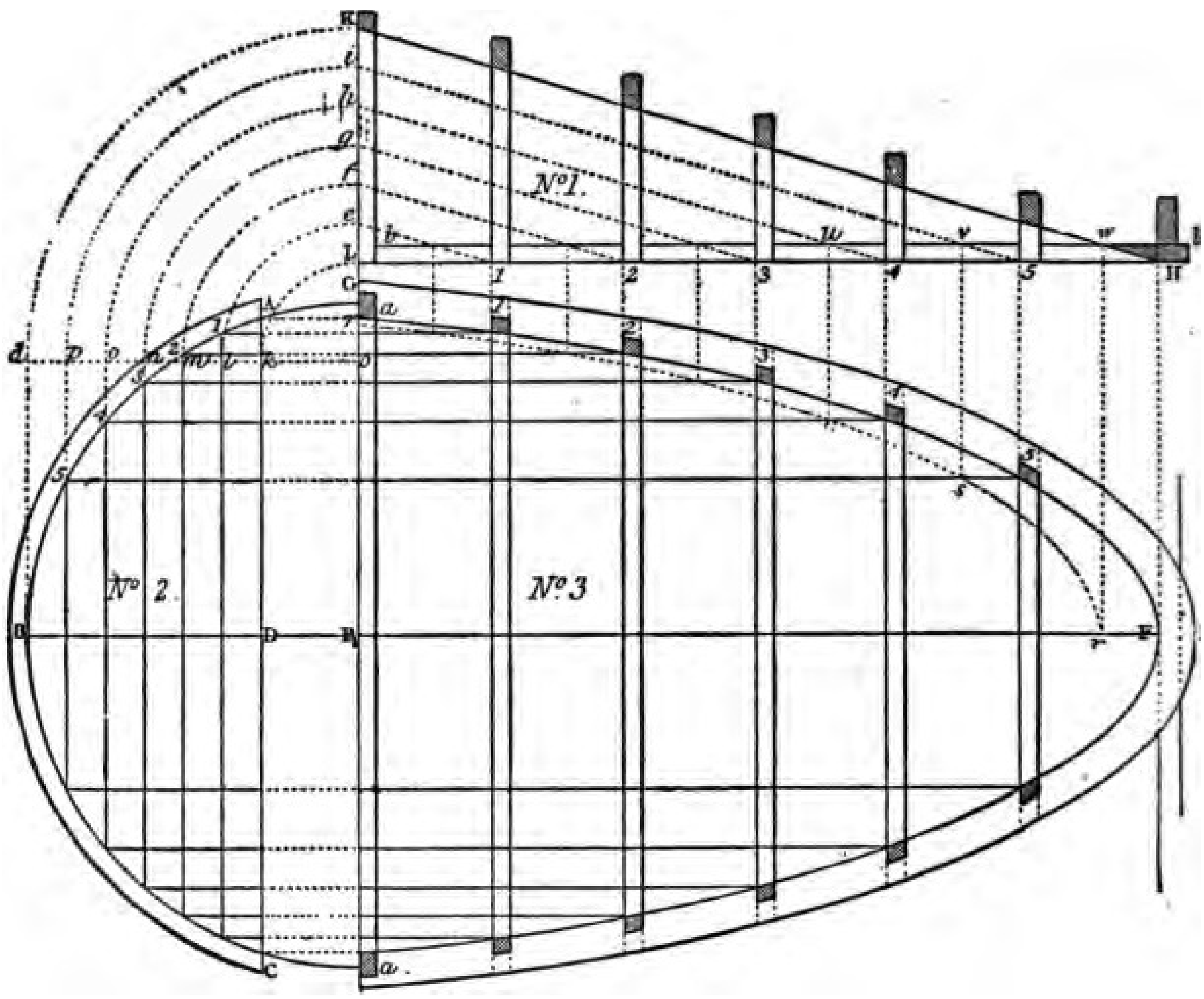


Angle-bracket at Meeting of Coves of Equal Height but Unequal Projection

Fig. 22.

Fig. 22 shows the method of finding the angle-bracket at the meeting of coves of equal height but unequal projection. The height C B is equal to G H, but the projection B A is greater than H I.

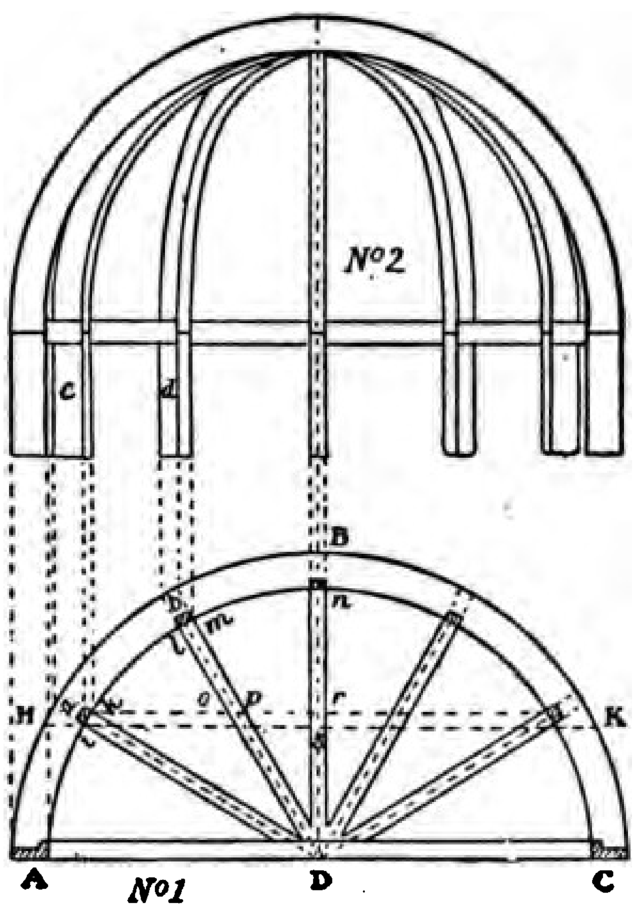
Fig. 23, Nos. 1, 2, 3, shows the curb and ribs of a circular opening (C B A, No. 2), cutting in on a sloping ceiling. No. 1 is a section through the centre B D, No. 2 and E F I, No. 3. The height L K is divided into

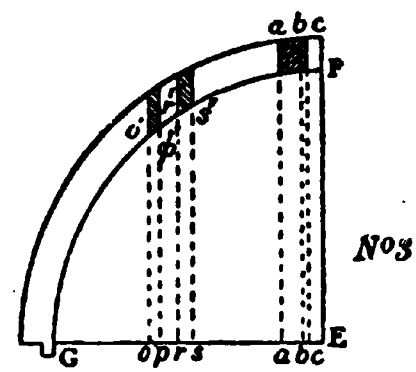


Curb and Ribs of Circular Arch Cutting into Sloping Ceiling

Fig. 23.

equal parts in e, f, g, h, i, and the same heights are transferred to the main rib in No. 2 at A, 1, 2, 3, 4, 5, B. Through the points A, 1, 2, 3, 4, 5, in No. 2 lines are drawn parallel to the axis B E I; and through the points e, f, g, h, i, in No. 1 lines are drawn parallel to the slope K H. The places of the ribs 1, 2, 3, 4, 5, in the latter, and their site on the plan, No. 3, and also the curve of the curb, are found by intersecting lines in the manner with which the student is already acquainted.





Spherical Niches on Semicircular and Segmental Plans

Fig. 24.

ON NICHES.

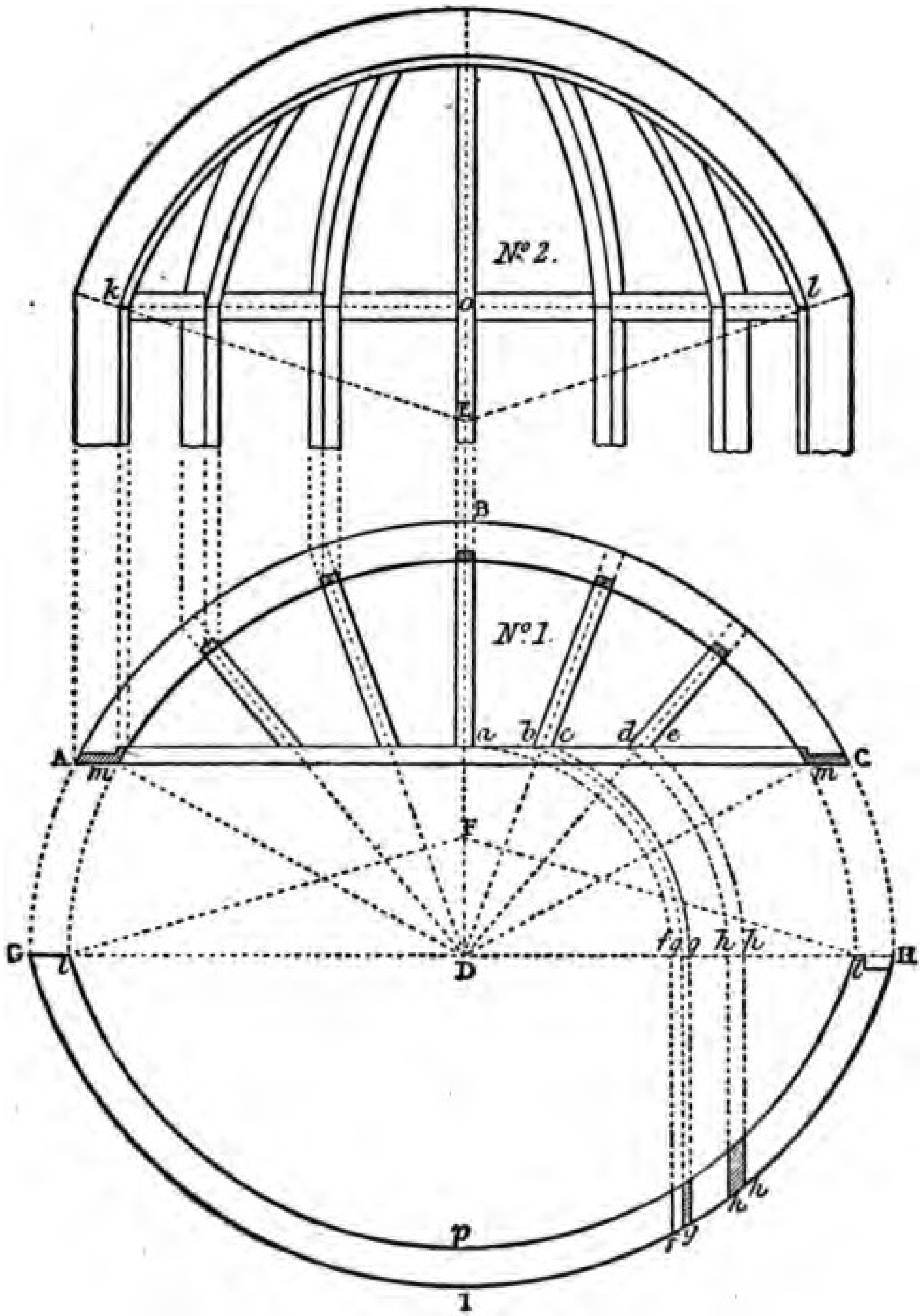
To describe a spherical niche on a semi-circular plan. The construction of this (Fig. 24) is precisely like that of a spherical dome. The ribs stand in planes, which would pass through the axis if produced. They are all of similar curvature. No. 2 shows an elevation of the niche, and No. 3 the bevelling of the ribs a, b, against the front rib at D on the plan; a b is the bevel of a, and b c of b.

Let H B K (Fig. 24) be the plan. It is obvious that the ribs m p, nr, will be parts of the quadrant G F (No. 3). Transfer the lengths lo, m p, n r, and r s to the line G E, as shown at o p r s, and raise perpendiculars from these points to the quadrant; G p is the rib m p, and o p is the bevel; G r is the rib nr, and r s is the section of the front rib at the crown; the vertical projection of the upper arris of this rib will be a semicircle with radius s s or s H.

The niche of which both the plan and elevation are segments of a circle.

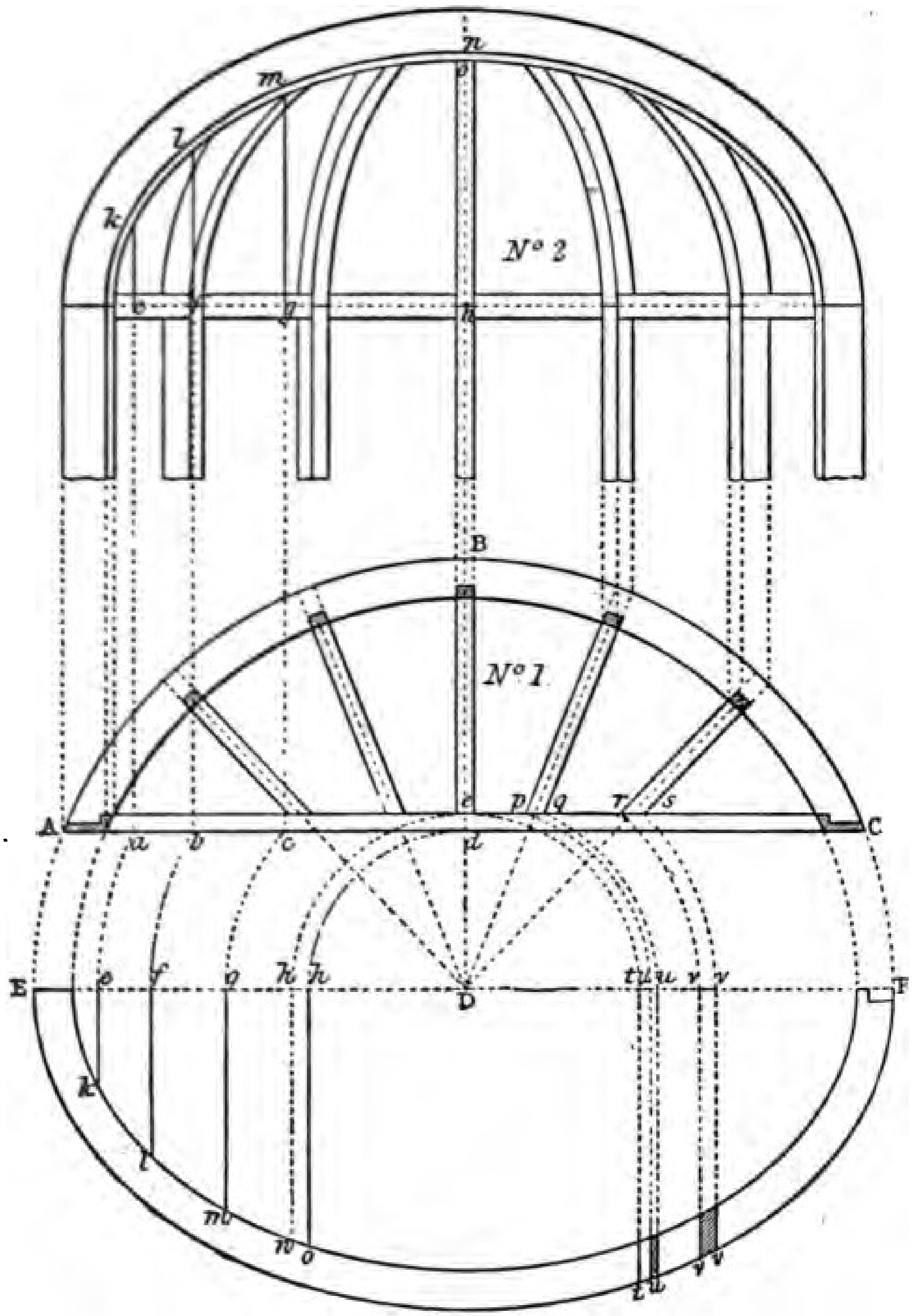
No. 25 is the elevation of the niche, being the segment of a circle whose centre is at E. No. 1, A B C, is the plan, which is a segment of a circle whose centre is D. Having drawn on the plan as many ribs as are required, radiating to centre D, and cutting the plan of the front rib in a, b c, d e; then through the centre D draw the line G H parallel to A C; and from D describe the curves m l, A G, C H, cutting the line G H; and make D F equal to E O, No. 2. From F as a centre describe the curves l p l and G I H for the depth of the ribs; and this is the true curve for all the back ribs.

To find the lengths and bevel of the ribs:—From the centre D describe the quadrant and arcs a f, b g, c g, d h, &c., and draw f f, g g, h h perpendicular to D H, cutting the curve l p l, and the lines of intersection will give the lengths and bevels of the several ribs.



Segmental Niche on Segmental Plan

Fig. 25.



Elliptical Niche on Segmental Plan

Fig. 26.

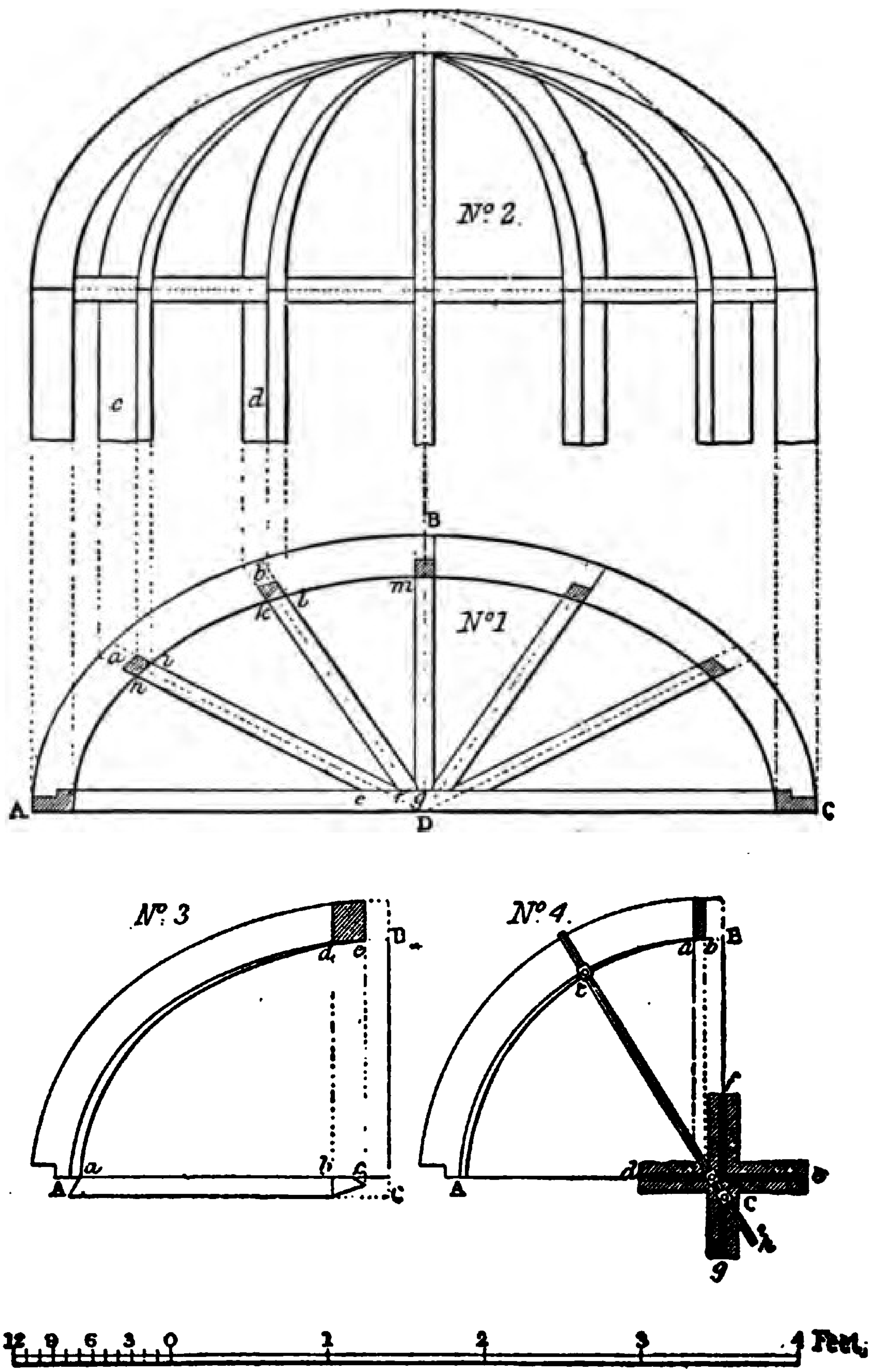
Let D in the plan (No. 1, Fig. 26) be the centre of the segment. Through D draw E F parallel to A C, and continue the curve of the segment to E F. Then to find the curve of the back ribs:—From k l m n, any points in the curve of the front rib (No. 2) let fall perpendicular to the line A B, cutting it in a b c d. Then from D as a centre describe the curves a e, b f, c g, e h, d h, and from the points where they meet the line E F draw the perpendiculars e k, f l, g m, h n, h o, and set up on e k the height ek of the elevation and the corresponding heights on their other ordinates, when k l m n o will be the points through which the curve of the radial ribs may be traced. The manner of finding the lengths and bevels of the ribs is shown at t u u v v.

A niche semi-elliptic in plan and elevation.

Let No. 1, Fig. 27, be the plan, and No. 2 the elevation of the niche. The ribs in this case radiate from the centre D, and with the exception of m g (which will be the quadrant of a circle) they are all portions of ellipses, and may be drawn by the trammel, as shown in No. 4, which gives the true curve of the rib marked d in No. 2 and b D in No. 1. The rib c, in the elevation, is seen at a D in No. 1; the bevel of the end h i is seen at A a in No. 3, and that of the end e f at b c.

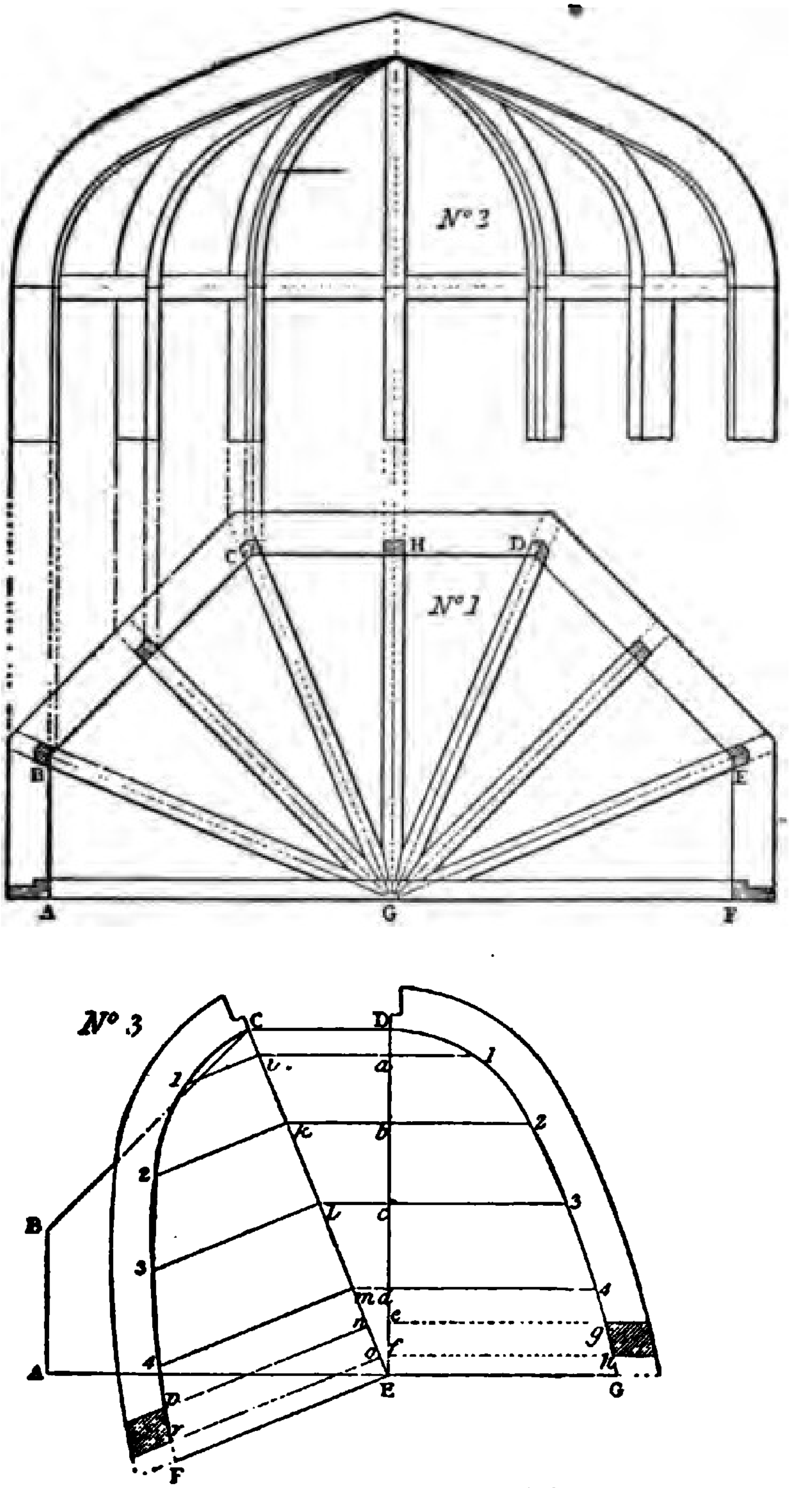
To draw the ribs of a regular octagonal niche.

Fig. 28.—Let No. 1 be the plan, and No. 2 the elevation of the niche. It is obvious that the curve of the centre rib H G will be the same as that of either half of the front rib A G, F G. In No. 3, therefore, draw A B C D E, the half-plan of the niche, equal to A B C H G, No. 1, and make D G E equal to half the



Elliptical Niche on Elliptical Plan

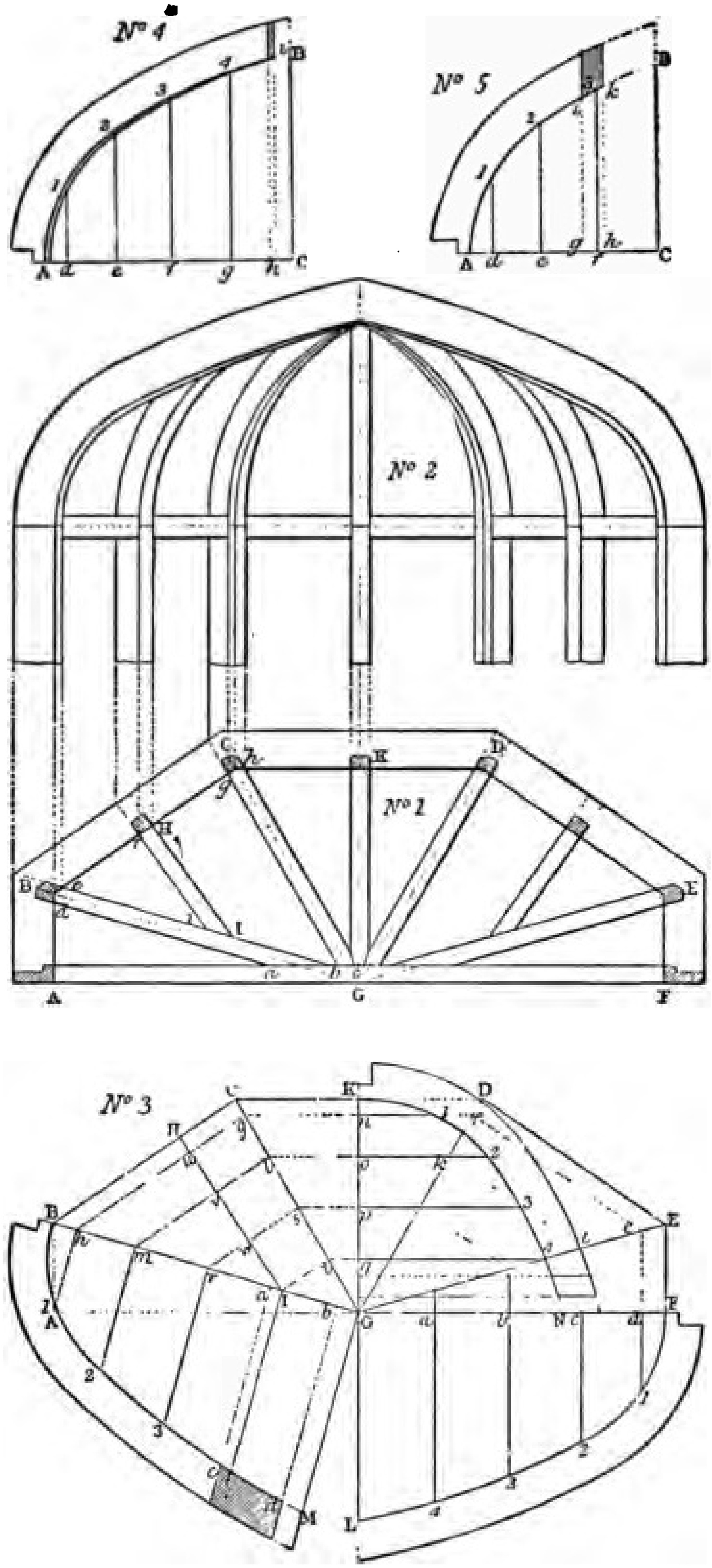
Fig. 27.



Regular Octagonal Niche

Fig. 28.

ont rib. Divide D G into any number of parts 1 2 3 4, &c., and through the points of division draw lines parallel to A G, meeting the seat of the centre of the angle



Irregular Octagonal Niche

Fig. 29.

rib C E in i k l m n o. On these points raise indefinite perpendiculars, and set up on them the heights a 1 in i 1, b 2 in k 2, and so on. The shaded parts show the bevel at the meeting of the ribs at G in No. 1.

To draw the ribs of an irregular octagonal niche.

Fig. 29.—Let No. 1 be the plan, and No. 2 the elevation of the niche. Draw the outline of the plan of the niche at A B C D E F (No. 3), and draw the centre lines of the seats of the ribs B G, H I, &c.; draw also G L F equal to the half of the front rib, as given in the elevation No. 2, and divide it into any number of parts 1 2 3 4. Through the points of division draw d1, c 2, b 3, a 4, perpendicular to G F, and produced to the seat of the first angle rib G E. Through the points of intersection draw lines parallel to the side E D of the niche meeting the second angle rib D G; through the points of intersection again draw parallels to D C, and so on. The curve of the centre rib is found by setting up from n o p q G the heights d 1, c 2, &c., on the parallel lines which are perpendicular to K G. The curve of the rib B G or E G is found by drawing through the points of intersection of the parallels perpendiculars to the seat of the rib, and setting upon them, at h m r I G, the heights d1, c2, &c. No. 4 shows the rib C G, and No. 5 the intermediate rib H I.

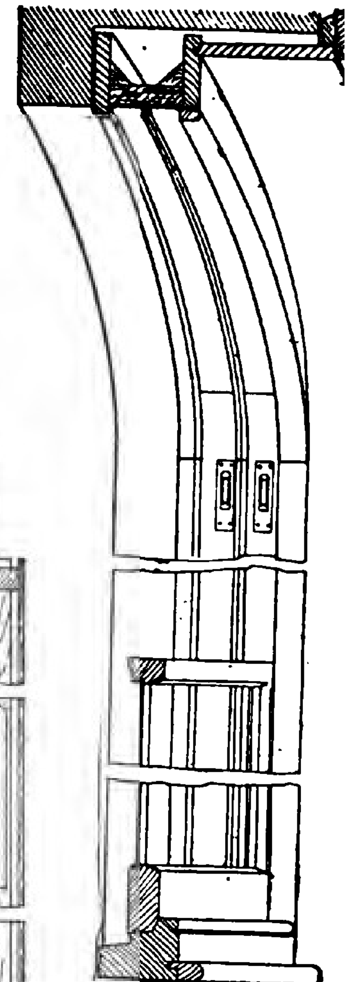
DOUBLE CURVATURE WORK.

To Obtain to Soffit Mould for marking the veneer (see Fig. 35), divide the elevation of lower edge of the head (Fig. 30) into a number of equal parts, as A, B, C, D, E, S, and drop projectors from these points into the plan cutting the chord line A″ S″ in A″, B″, C″, D″, E″, S″. Draw the line s′ s′, Fig. 35, equal in length to the stretch out of the soffit in the elevation (the length of a curved line is transferred to a straight one by taking a series of small steps around it with the compasses, and repeating a like number of the straight line), and transfer the points A B C, &c., as they occur thereon, repeating them on each side of the centre line. Erect perpendiculars at the points, and make each of these lines equal in length to its cordespondingly marked line in the plan, as A′ a′ a′ Fig. 35, equal to A″ a a Fig. 32, these letters referring respectively to the chord line and the inside and outside edges of the head. Draw the curves through the points so found. As will be seen by reference to Fig. 35, the mould is wider at the springing than at the crown; this is in consequence of the pulley stiles being parallel. If they were radial their width would be the same as the width of the head at the crown, and the head would be parallel; the gradual increase in width from the crown to the springing is also apparent in the sash-head and the beads, as indicated by the line O O, Fig. 35, which is the inside of the sash-head, and the outside of the parting head; this variation in width renders it impossible to gauge to a thickness from the face or the groove in the head from its edges.

To Form the Head. Having prepared the cylinder (Fig. 33) to the correct size, prepare a number of staves to the required section, which may be obtained by drawing one or two full size on the elevation, as shown to the right in Fig. 36. The staves should be dry straight-grained yellow deal, free from knots and

Fig. 30.

Fig. 31.



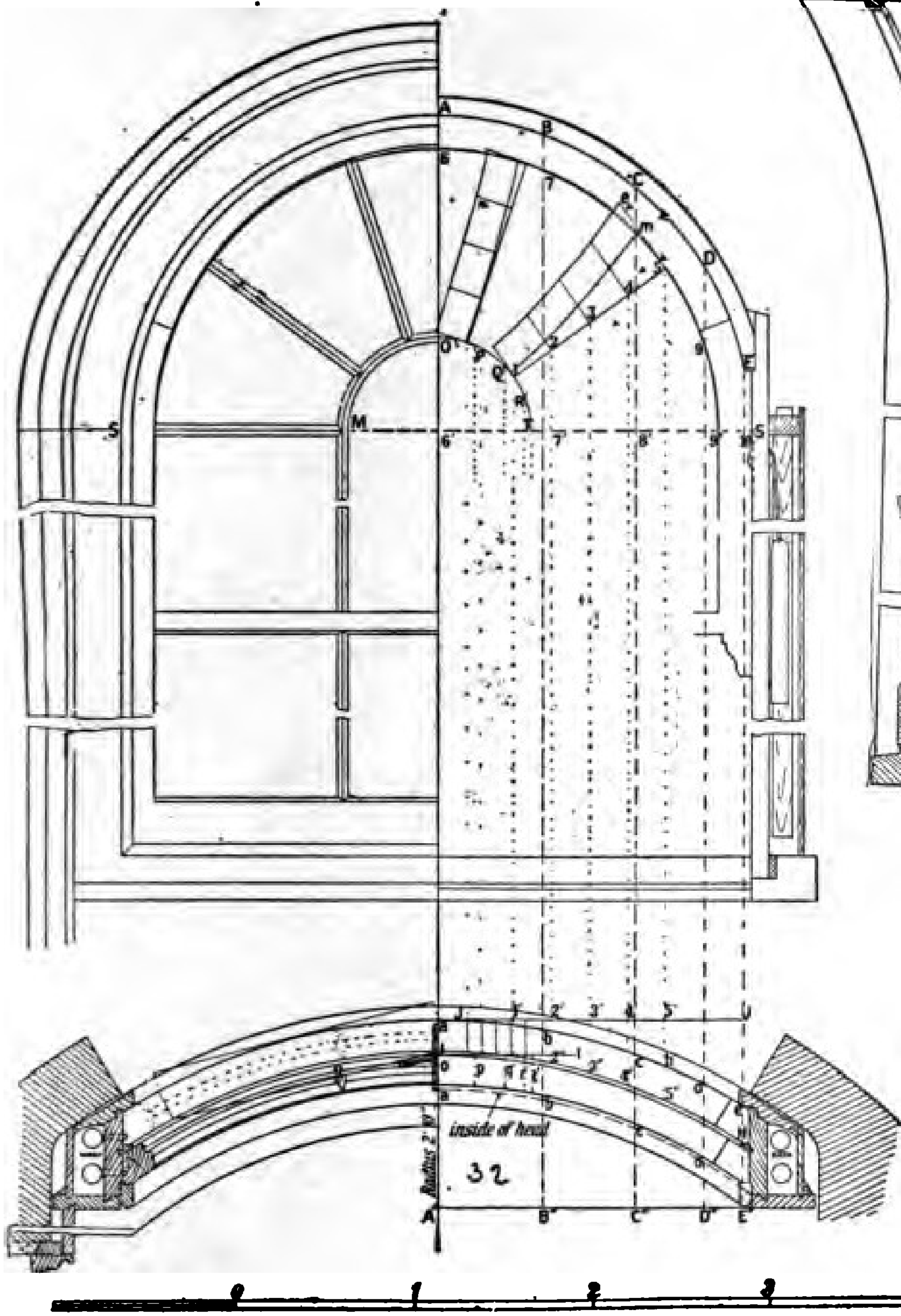


Fig. 32.

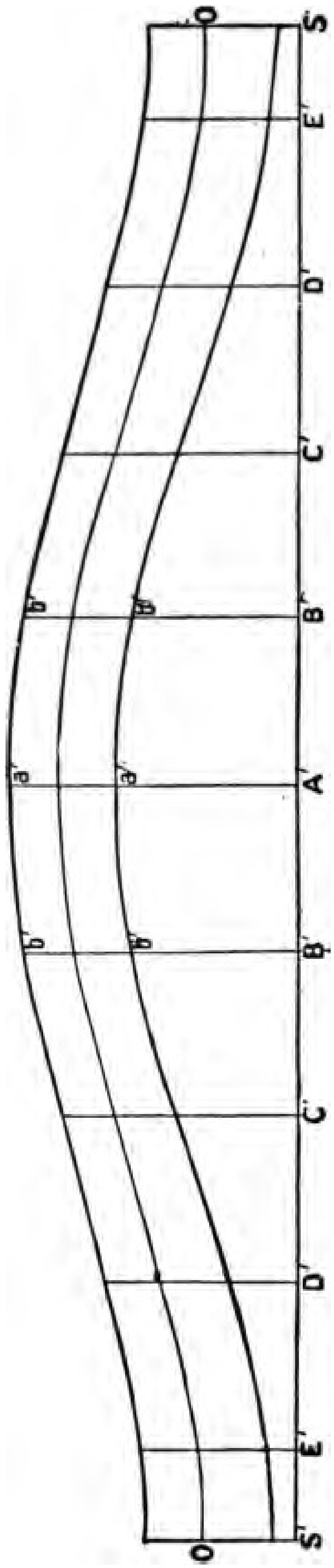


Fig. 35.

sap, and not be so wide that they require hollowing to fit. If the veneer is pine, it will probably bend dry, but hardwood will require softening with hot water. One end

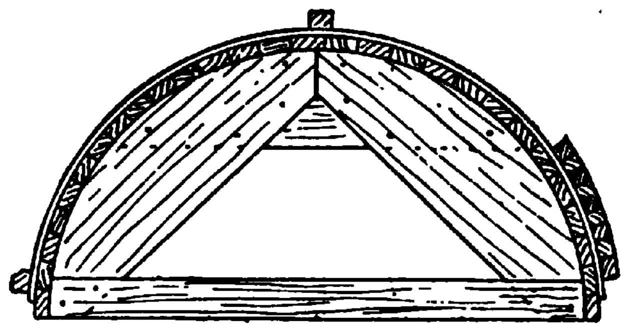


Fig. 33.

should be fixed as shown in Fig. 34, by screwing down a stave across it. Then the other end is bent gently over until the crown is reached,

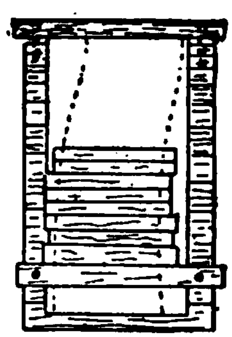


Fig. 34.

when another stave is screwed on, and the bending continued until the veneer is well down all round, and a third stave secures it until it is thoroughly dry, when the remainder may be glued on. It is as well to interpose a sheet of paper between the cylinder and the veneer, in case any glue should run under, which would then adhere to the paper instead of the veneer. The head should not be worked for at least twelve hours after glueing. If a band saw is at hand, the back should be roughly cleaned off and the mould bent round it, the shape marked, and the edges can then be cut vertically with the saw, by sliding it over the cylinder sufficiently

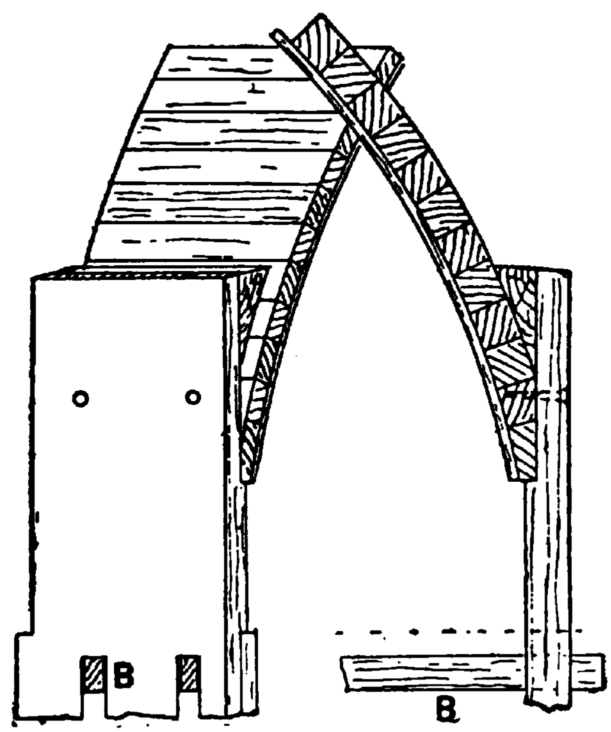


Fig. 36.

for the saw to pass. When cut by hand, the mould is applied inside and the cut is made square to the face, the proper bevel being obtained with the spokeshave, and found by standing the head over its plan and trying a set square against it. When fitting the head to the pulley stiles the correctness of the joints is tested by cutting a board to the same sweep as the sill, with tenons at each end, and inserting it in the mortises for the pulleys, as shown at B, Fig. 36. A straight-edge applied to this and the sill will at once show if the head is in the correct position, and if the edges are vertical as they should be.

To Obtain a Developed Face Mould. Make the line i h, Fig. 37, equal to the stretch out of the plan of the face of sash-heads, viz., I H, Fig. 32. Transfer the divisions as they occur, and erect perpendiculars thereon.

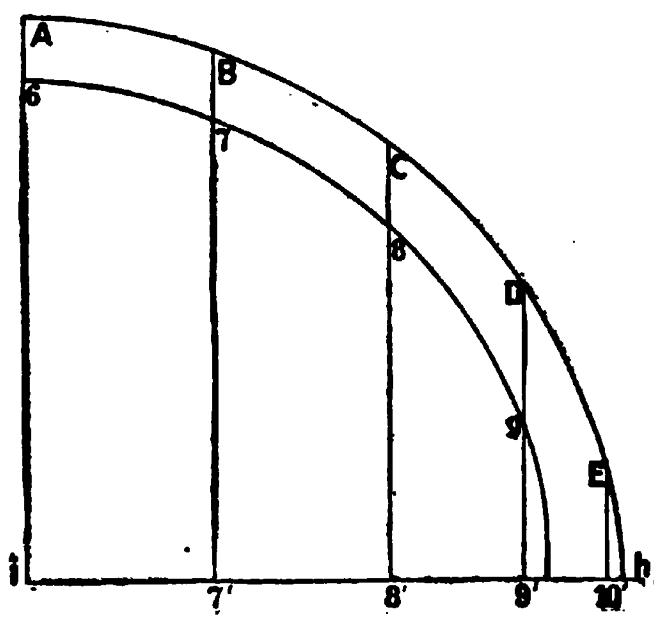


Fig. 37.

Make these equal in height to the corresponding ordinates over the springing line in the elevation, and draw the curves through the points so found. The groove for the parting bead can be marked by running a ⅜ in. piece around the inside of the sash-head, this bead being generally put in parallel. It is sometimes omitted altogether, the side beads being carried up until they die off on the head.

To Find the Mould for the Cot Bar. Divide its centre line in the elevation into equal parts, as O, P, Q, R, T. Drop projectors from these into the plan, cutting the chord line 1 1 in o, p, q, r, t. These lines should be on the plan of the top sash, but are produced across the lower to avoid confusion with the projectors from the other bars. Set out the stretch out of the cot bar

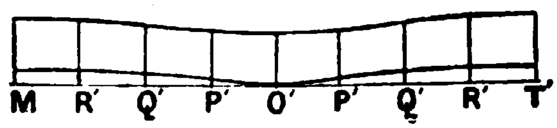


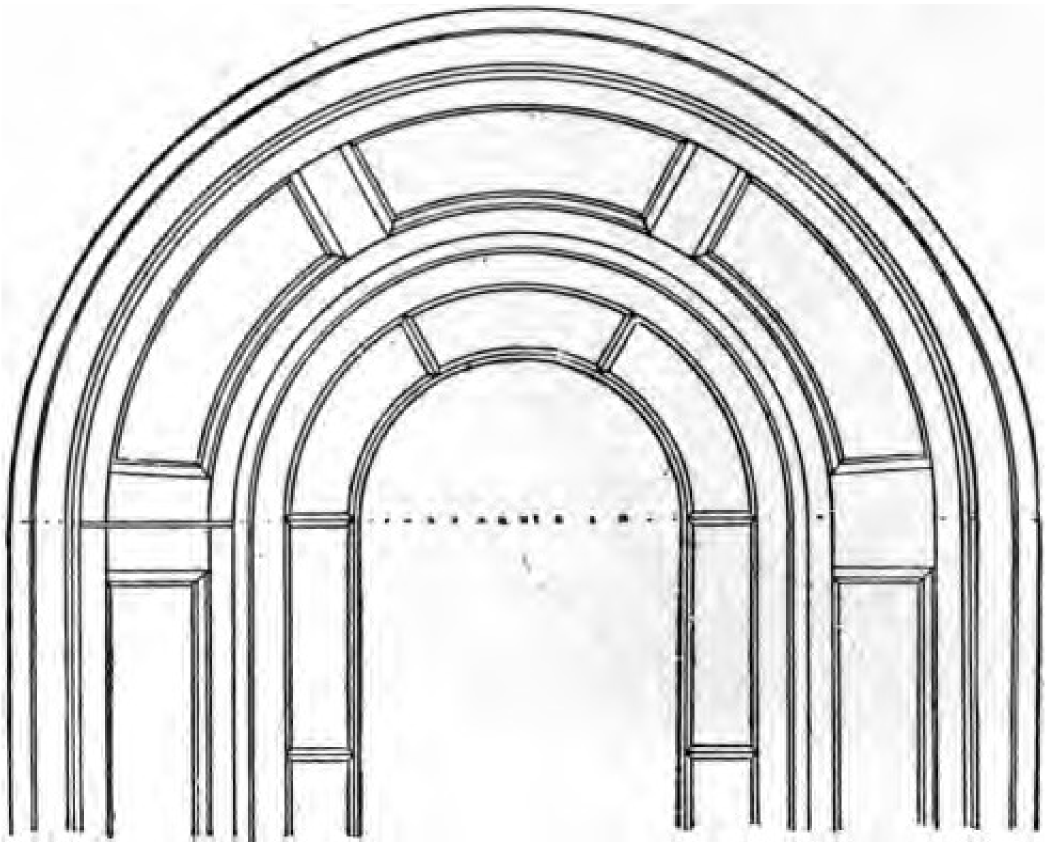
Fig. 38.

on the line M′ O′ T′, Fig. 38, and erect perpendiculars at the points of division, and make them equal in length to the correspondingly marked lines in the plan. The cot bar is cut out in one piece long enough to form the two upright sides as well as the arch. The straight parts are worked nearly to the springing, and the bar which is got out wider in the centre is then steamed and bent around a drum, and afterwards cut to the mould (Fig. 38) and then rebated and moulded. The arched bar should not be mortised for the radial bars, but the latter scribed over it and screwed through from inside.

To Find the Mould for the Radial Bars. Divide the centre line of the bar into equal parts, as 1, 2, 3, 4, 5, Fig. 30, and project the points into the plan, cutting the chord line J J in 1′, 2′, 3′, 4′, 5′. Erect perpendiculars upon the centre line from the points of division, and make them equal in length to the distance of the corresponding points in the plan from the chord line, and draw the curve through the points so obtained.

The other bar is treated in the same manner, the projectors being marked with full lines in the plan.

Fig. 39.



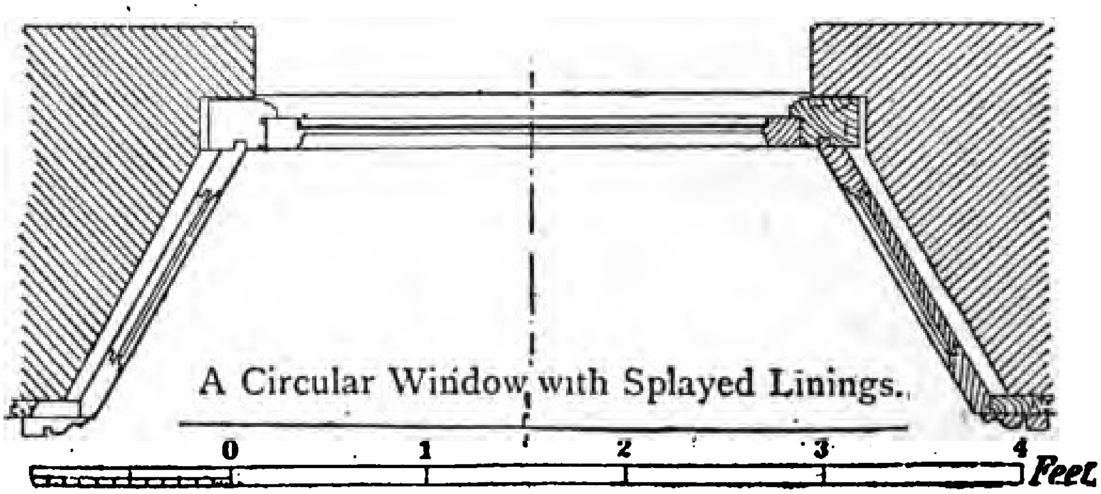


Fig. 40.

The soft moulds for the head linings are obtained in similar manner to those for the head mould, the width being gauged from the head itself.

A Frame Splayed Lining with Circular Soffit as shown in part elevation in Fig. 39, plan Fig. 40, and section Fig. 41. The soffit stiles are worked in the solid

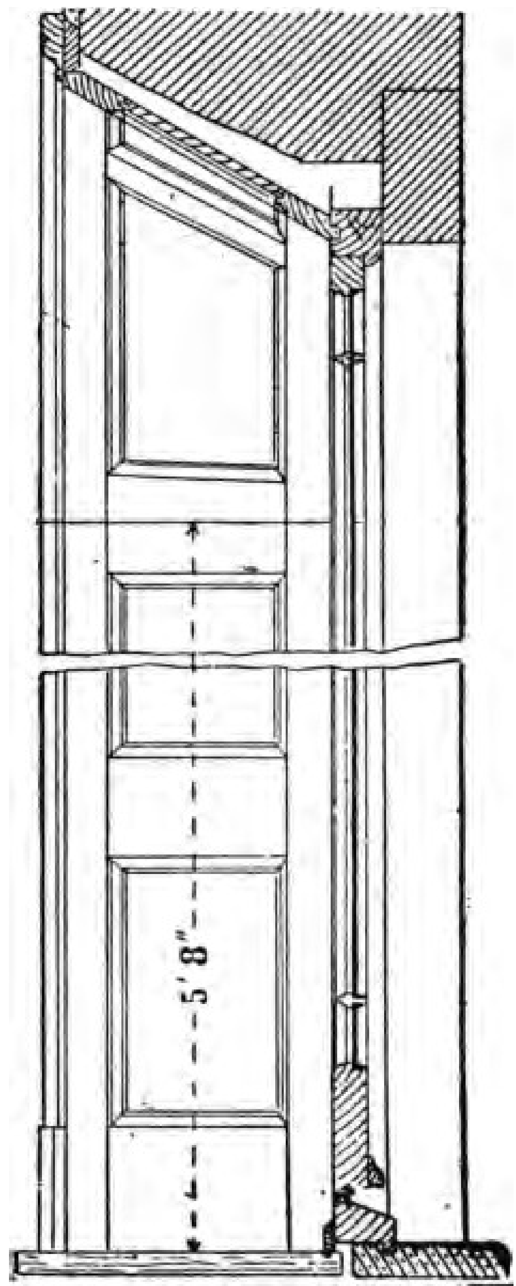
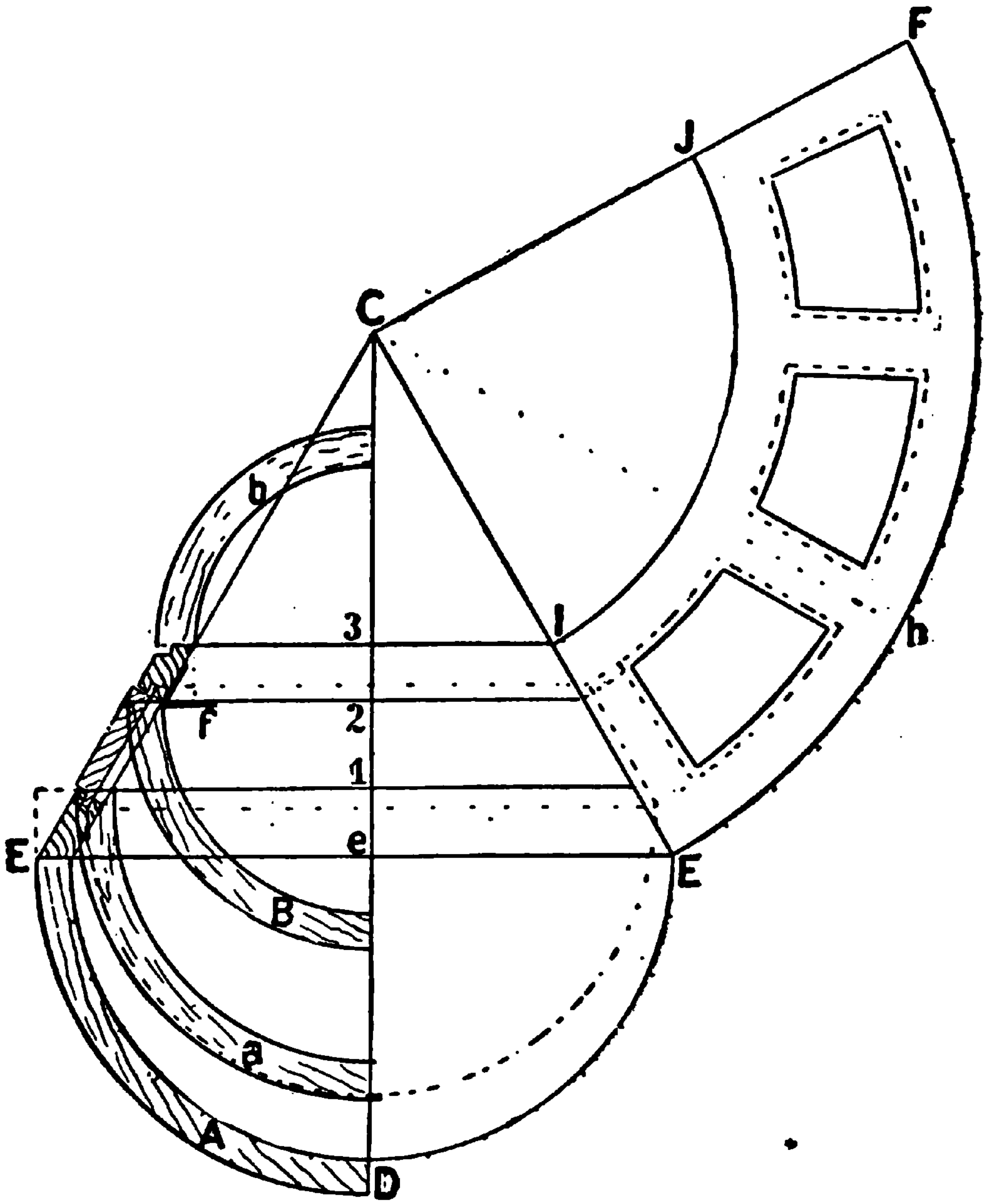


Fig. 41.

in two pieces joined at the crown and springings. The rails are worked with parallel edges, their centre lines radiating from the centre of the elevation. Edge moulds only are required for the stiles, and a developed mould for the panels. The method of obtaining these has been shown on a separate diagram, Fig. 42.



Method of Obtaining Moulds for Circular Splayed Linings.

Fig. 42.

To Obtain the Face Moulds for the Soffit Stiles. Draw the line E E, Fig. 42, and produce the faces of the jambs until they meet in point C. From C draw the line C D perpendicularly to E E. This line will contain the centres of the various moulds, which are located by producing the plans of the edges of the stile across it e, 1, 2, 3 being the respective centres, and the inside and outside faces of the jambs affording the necessary radii for describing the arcs A a and B b.

To Apply the Moulds. Prepare the stuff equal in thickness to the distance between the lines e 1 and 2 3, Fig. 42. Apply the mould A to the face of the pieces intended for the front stile, and cut the ends to the mould, and square from the face. Set a bevel as at F, Fig. 42, and apply it on the squared ends, working from the lines on the face, and apply the mould a at the back, keeping its ends coincident with the joints, and at the points where the bevel lines intersect the face. The piece can then be cut and worked to these lines, and the inside edge squared from the face. The outside edge is at the correct bevel, and only requires squaring slightly on the back to form a seat for the grounds. The inside stile is marked and prepared similarly.

The Development of the Conical Surface of the Soffit is shown on the right hand of the diagram, Fig. 42, and is given to explain the method of obtaining the shape of the veneer, but it is not actually required in the present construction, as the panel being necessarily constructed on a cylinder, its true shape is defined thereon, and its size is readily obtained by marking direct from the soffit framing when the latter is put together. Let the semi-circle E D E, Fig. 42, represent a base of the semi-cone, and the triangle E C E its vertical section. From the apex C, with the length of one of its sides as radius, described the arc E F, which make equal in length to the semi-circle E D E by stepping lengths as previously described. Join F to C, and E C F is the covering of the semi-cone. The shape of the frustum, or portion cut off by the section line of the linings, is found by projecting the inner edge of the lining upon the side C E, and drawing the con-

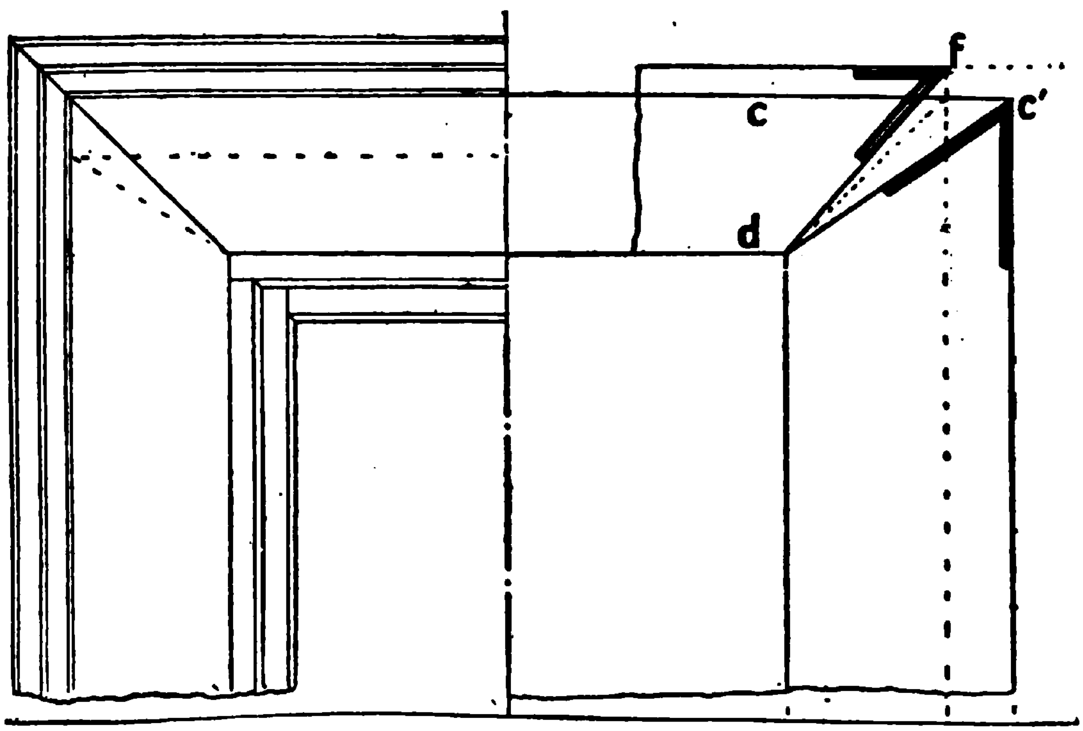


Fig. 43.

centric arc I J; then E I J E represents the covering of the frustum. Any portion of this, the panels for instance, is found in the same way. To draw the rails, divide their centre lines equally on the perimeter, and draw lines from the divisions to the centre as H C; make the edges parallel with these lines.

A Window With a Splayed Soffit and Splayed Jambs is shown in part elevation, Fig. 43, plan Fig. 44; and section Fig. 45. The linings are grooved and tongued together, as shown in the enlarged section, Fig. 46. To obtain the correct bevel required for the shoulder of the jamb and the groove in the soffit, the lining must

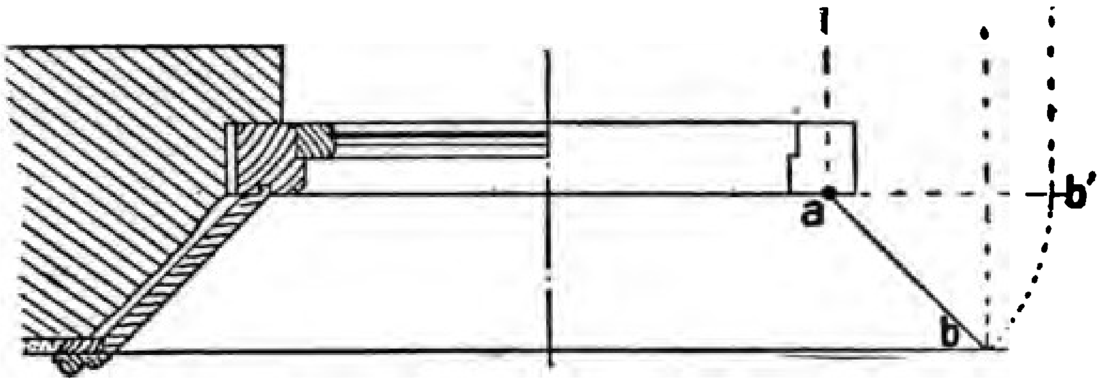


Fig. 44.

be revolved upon one of its edges until it is parallel with the front, when its real shape can be seen. This operation is shown in the diagram on the right-hand

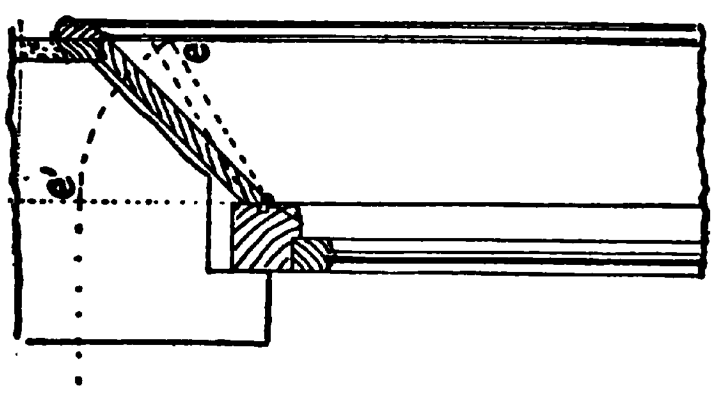


Fig. 45.

half of the plan and elevation. Draw the line a b, representing the face of the jamb in plan, at the desired angle. Project the edges into the elevation, and intersect them by lines c d, projected from the top and bottom edges of the soffit in section. This will give the projection of the linings in elevation. Then from point a as centre, and the width of the lining a b as radius, describe the arc b b′, bringing the edge b into the same plane as the edge a. Project point b′ into the elevation, cutting the top edge of the soffit produced in c′. Draw a line from c′ to d, and the contained angle is the bevel for the top of the jamb. When the soffit is splayed at the same angle as the jambs, the same bevel will answer for both; but when the angle is different, as shown by the dotted lines at e, Fig. 45, then the soffit also must be turned into the vertical plane, as shown at e′ and a line drawn from that point to intersect the projection of the front edge of the jamb in F; join this point to the intersection of the lower edges, and the contained angle is the bevel for the grooves in the soffit. (See Fig. 46.)

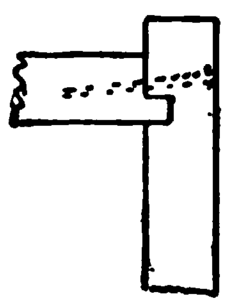


Fig. 46.

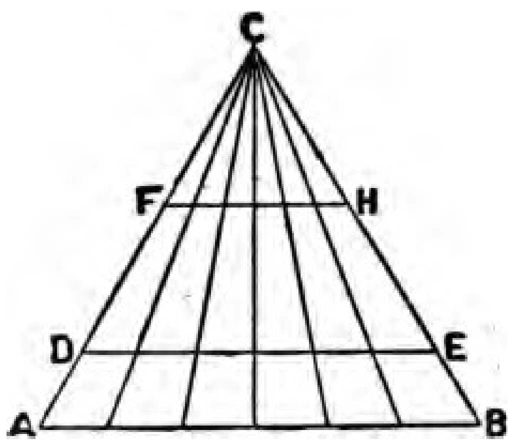


Fig. 47.

The Enlarging and Diminishing of Mouldings. The design of a moulding can be readily enlarged to any desired dimensions by drawing parallel lines from its members, and laying a strip of paper or a straight-edge of the required dimensions in an inclined direction between the boundary lines of the top and bottom edges;

and at the points where the straight-edge crosses the various lines, make marks thereon which will be points in the new projection, each member being increased proportionately to the whole. Projections drawn at right angles to the former from the same points will give data for increasing the width in like manner.

To Diminish a Moulding. The method to be explained, which is equally applicable to enlargement, is based upon one of the properties of a triangle, viz., if one of the sides of a triangle is divided into any number of parts, and lines drawn from the divisions to the opposite side of the triangle, any line parallel with the divided side will be divided in corresponding ratio. See Fig. 47, where A B C is an equilateral triangle, the side A B being divided into six equal parts, and lines drawn from these to the apex C. The two lines D E and F H, parallel to A B, are divided into the same number of parts, and each of these parts bears the same ratio to the whole line that the corresponding part bears to A B, viz., one-sixth; the application of this principle will now be shown. Let it be required to reduce the cornice shown in Fig. 48 to a similar one of smaller proportions. Draw parallel projectors from the various members to the back line A B, and upon this line describe an equilateral triangle. Draw lines from the points on the base to the apex, then set off upon one of the sides of the triangle from C a length equal to the desired height of the new cornice as at G or H, and from this point draw a line parallel to the base line. At the points where this line intersects the inclined division lines, draw horizontal projectors corresponding to the originals. To obtain their length or amount of projection, draw the horizontal line b E, Fig. 48, at the level of the lowest member of the cornice, and upon this line drop projectors at right angles to it from the various members. Describe the equilateral triangle b i E upon this side, and draw lines from the divisions to the apex i. To ascertain the length

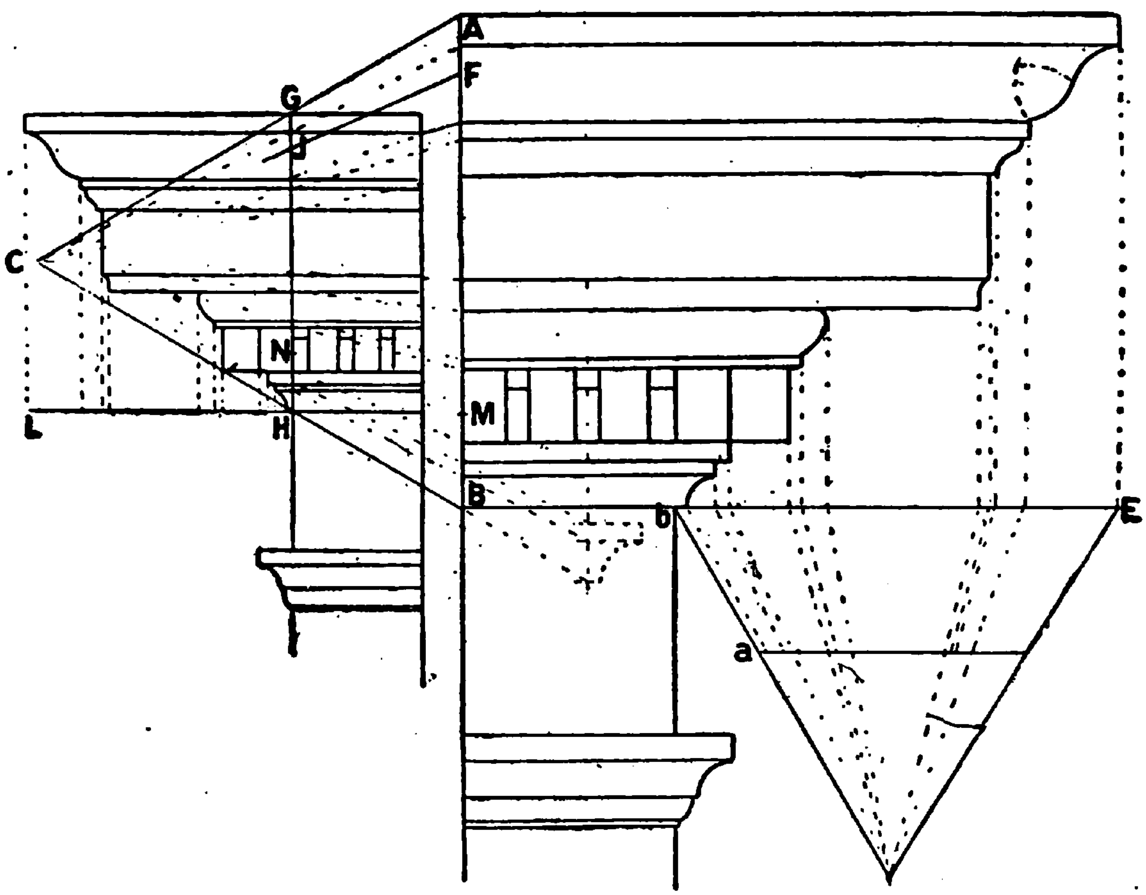


Fig. 49. Fig. 48.

that shall bear the same proportion to b E that the line G H bears to A B, place the length of b E on the line B A from B to F, and draw a line from F to C: the portion of the line G H cut off from J to H is the proportionate length required. Set this length off parallel to b E within the triangle, as before described, and also draw the horizontal line L H, Fig. 49, making it equal in length to a a, Fig. 48. Upon this line set off the divisions as they occur on a a, noting that their direction is reversed in the two figures. Erect perpendiculars from these points to intersect the previously drawn horizontals, and through the intersections trace the new profile. The frieze and architrave are reduced in like manner, M B, Fig. 48, representing the height of the original architrave, and N H the reduction. The cornice can be enlarged similarly by pro-

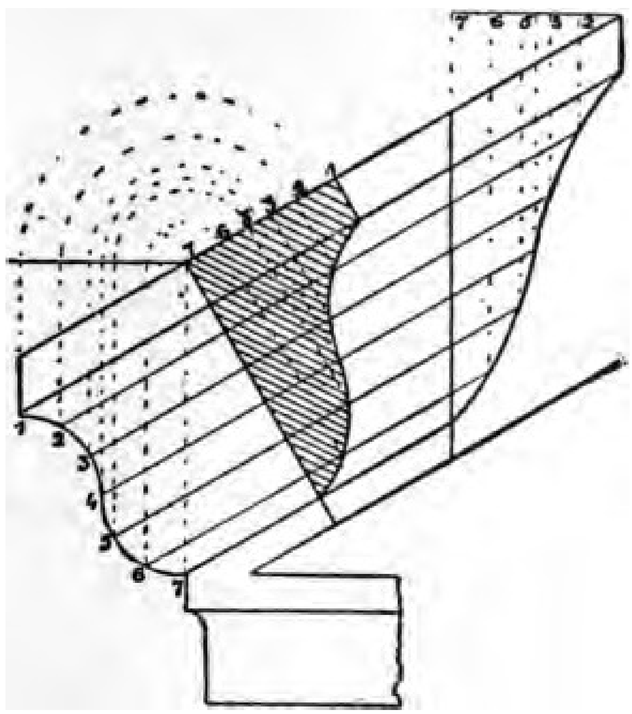


Fig. 50.

ducing the inclined sides of the triangle, as shown by the dotted lines on Fig. 48, sufficiently to enable the required depth to be drawn within it parallel to A B. One member has been enlarged to indicate the method, which should be clear without further explanation.

Raking Mouldings. Fig. 50 shows the method of finding the true section of an inclined moulding that is required to mitre with a similar horizontal moulding at its lower end, as in pediments of doors and windows. The horizontal section being the more readily seen, is usually decided first. Let the profile in Fig. 50 represent this. Divide the outline into any number of parts, and erect perpendiculars therefrom, to cut a horizontal line drawn from the intersection of the back of the moulding with the top edge, as at point 7. With this point as a centre and the vertical projectors from 1 to 7 as radii, describe arcs cutting the top of the inclined mould, as shown. From these points draw perpendicu-

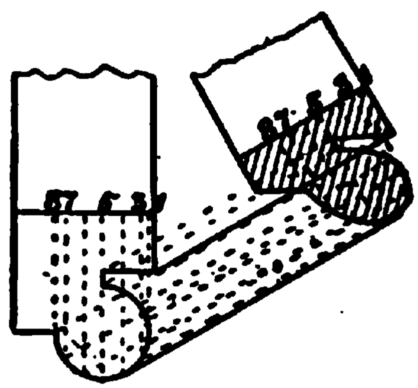


Fig. 51.

lars to the rake, to meet lines parallel to the edges of the inclined moulding drawn from the corresponding points of division in the profile, and their intersections will give points in the curve through which to draw the section of the raking mould. When the pediment is broken and a level moulding returned at the top, its section is found in a similar manner, as will be clear by inspection of the drawing. If the section of the raking moulding is given, that of the horizontal mould can be found by reversing the process described above. Fig. 51 shows the application of the method in finding the section of a return bead when one side is level and the other inclined, as on the edge of the curb of a sky-light with vertical ends.

Sprung Mouldings. Mouldings curved in either elevation or plan are called "sprung," and when these are used in a pediment, require the section to be determined as in a raking moulding. The operation is similar to that described above up to the point where the back of the section is drawn perpendicular to the inclination, but in the present case this line E x, Fig. 52,

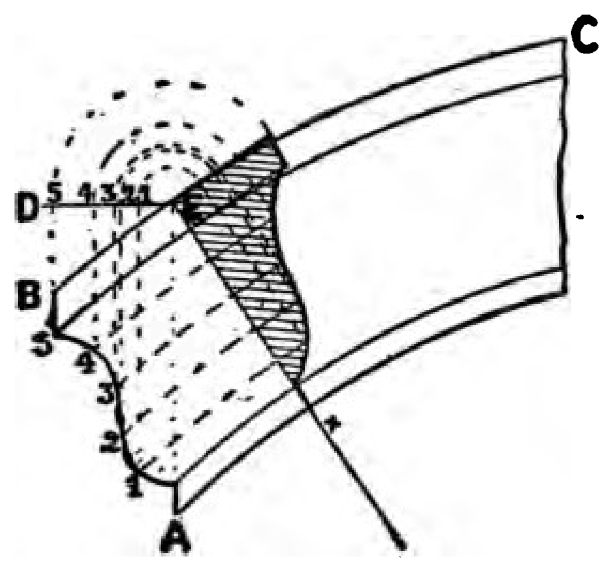


Fig. 52.

is drawn radiating from the centre of the curve, and the projectors are drawn parallel to this line. The parallel projectors, 1, 2, 3, 4, 5, are also described from the centre until they reach the line E x, when perpendiculars to this line are raised from the points of intersection to meet the perpendicular projectors.

Mitreing Straight and Curved Mouldings Together. If a straight mitre is required, draw the plan of the mouldings, as in Fig. 53, and the section of the straight mould at right angles to its plan as at A. Divide its profile into any number of parts, and from them draw parallels to the edges intersecting the mitre line. From these intersections describe arcs concentric with the plan of the curved mould, and at any convenient point thereon draw a line radial from the centre. Erect perpendiculars on this line from the points where the arcs intersect it, and make them equal in height to the corresponding lines on the section of the straight moulding A, and these will be points in the profile of

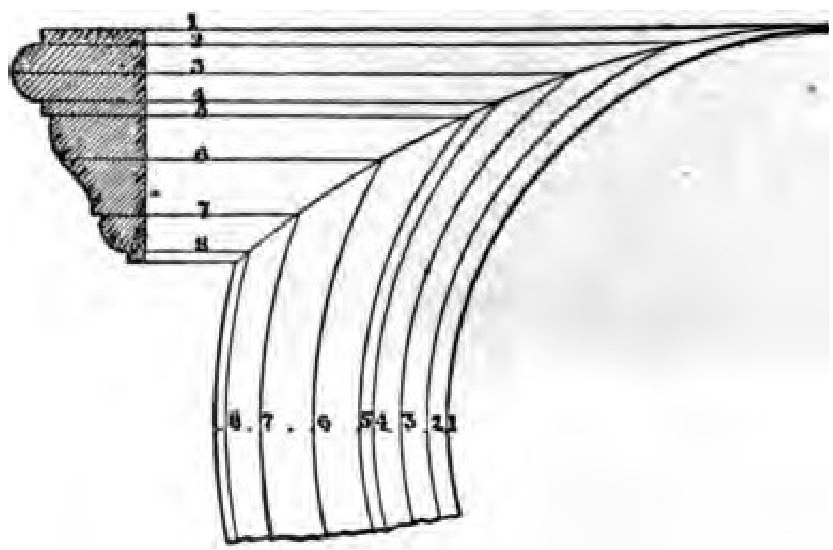


Fig. 53.

the curved moulding B. When it is required that the section of both mouldings shall be alike, a circular mitre is necessary, and its true shape is obtained as shown in Fig. 53. Draw the plan and divide the profile of the straight moulding as before, drawing parallels to the edge towards the seat of the mitre. Upon a line drawn through the centre of the curved moulding set off divisions equal and similar to those on the straight part, as 1 to 8 in the drawing. From the centre of the curve describe arcs passing through these points, and through the points of intersection of these arcs with the parallel projectors, draw a curve which will be the true shape of the mitre. Cut a saddle templet to this shape, and use it to mark the mouldings and guide the chisel in cutting.

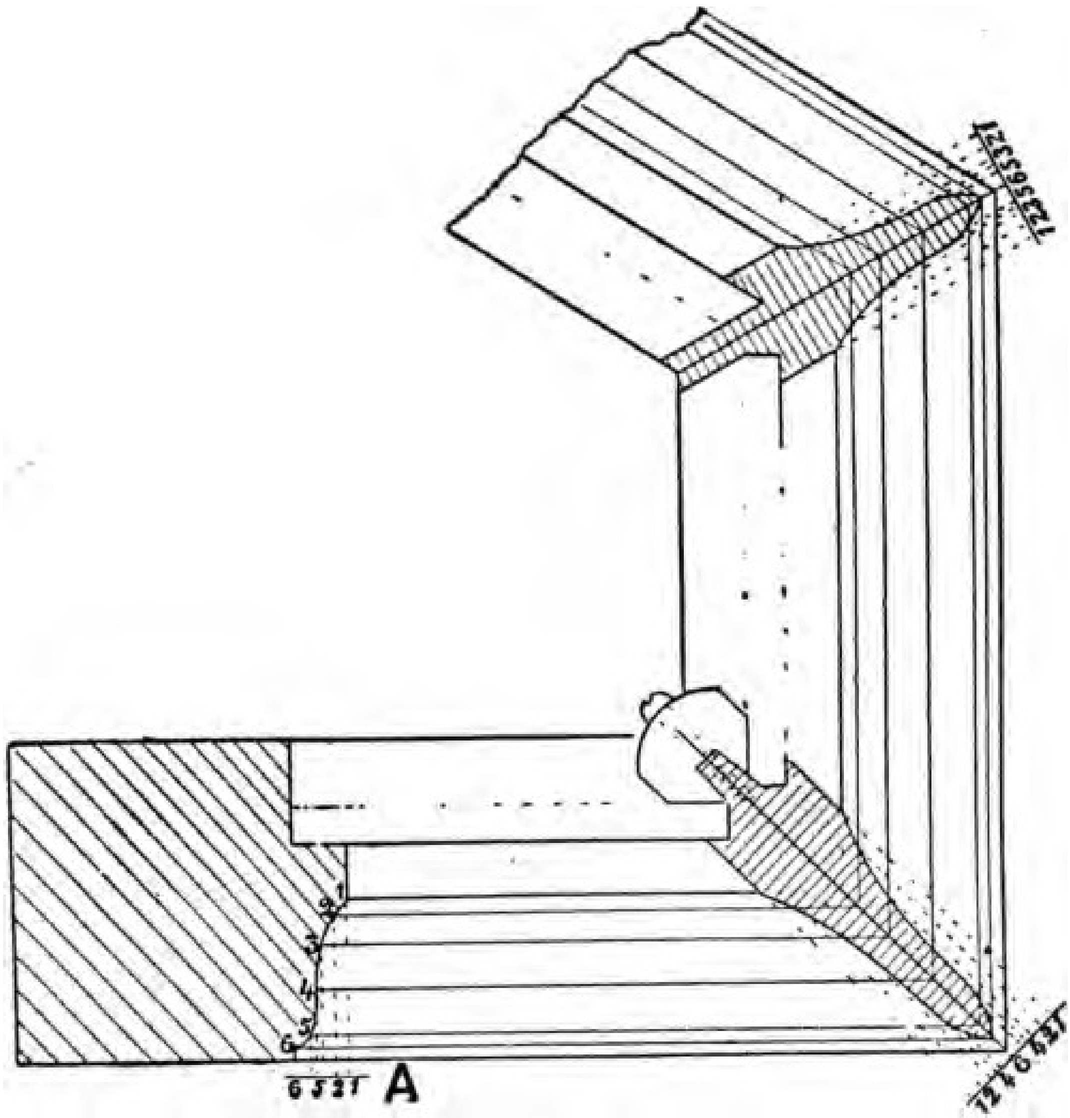


Fig. 54.

To Obtain the Section of a Sash Bar Raking in Plan. Fig. 54 represents the plan of a shop front sash with bars in the angles. On the left hand is shown the section of the stile or rail into which the bars have to mitre. Divide the profile of moulding into a number of parts, as shown from 1 to 6, and from these points draw parallels to the sides of the rails intersecting the centre line of the bar. Also draw perpendiculars from the same ponts to any line at right angles to them, as at A. Draw a line at right angles to the centre line of the bar, and on it set off the divisions from 1 to 6 as at A. Draw projectors from these points parallel to the centre line of the bar, and where they intersect the correspondingly numbered lines drawn parallel with the sides of the sash will be points in the curve of the section of the bar. It will be noticed that there is no fillet or square shown on the bar, and that in transferring the points from the line A they must be reversed on each side of the centre. Should a fillet be required on the bar, additional thickness must be given for the purpose. Three methods of forming the rebates in the bar are shown, the screwed saddle beads being the best for securing the glass.

Interior Shutters Include Folding, Sliding, Balanced, and Rolling. Exterior Consist of Hanging, Lifting, Spring, and Venetian Shutters.

FOLDING, or as they are frequently called, BOXING SHUTTERS, because they fold into a boxing or recess formed between the window frames and the walls, are composed of a number of narrow leaves, framed or plain, as their size may determine, rebated and hinged to each other and to the window frames. They should be of such size that when opened out they will cover the entire light space of the sash frame and a margin of a ¼ in. in addition. Care must be taken to make them parallel, or they will not swing clear at the ends; and as a further precaution, they should not be carried right from soffit to window board, but have clearance pieces interposed at their ends about $3/8$ in. thick. The outer leaf, which is always framed, is termed a shutter; the others are termed flaps. It is not advisable to make the shutters less than $1\frac{1}{4}$ in. thick, and flaps over 8 in. wide should be framed; those less than 8 in. may be solid, but should be mitre clamped to prevent warping. In a superior class of work the boxings are provided with cover flaps which conceal the shutters when folded, and fill the void when they are opened out. The sizes and arrangements of the framing are determined by the general finishings of the apartment, but it is usual to make the stiles of the front shutters range with those of the soffit and elbow linings. When venetian or other blinds are used inside, provision is made for them by constructing a block frame from $2\frac{1}{2}$ to 3 in. thick inside the window frame and the shutters are hung to this.

The leaves are hung to each other with wide hinges called back flaps, that screw on the face of the leaves, there not being sufficient surface on the edges for butt hinges. In setting out the depth of boxings, at least $1/8$ in. should be allowed between each shutter to provide room for the fittings; the shutters are fastened by a flat iron bar hung on a pivot plate fixed on the inner left-hand leaf, and having a projecting stud at its other end which fits into a slotted plate, and it is kept in this position by a cam or button. Long shutters are made in two lengths, the joint coming opposite the meeting rails of the sashes; these are sometimes rebated together at the ends.

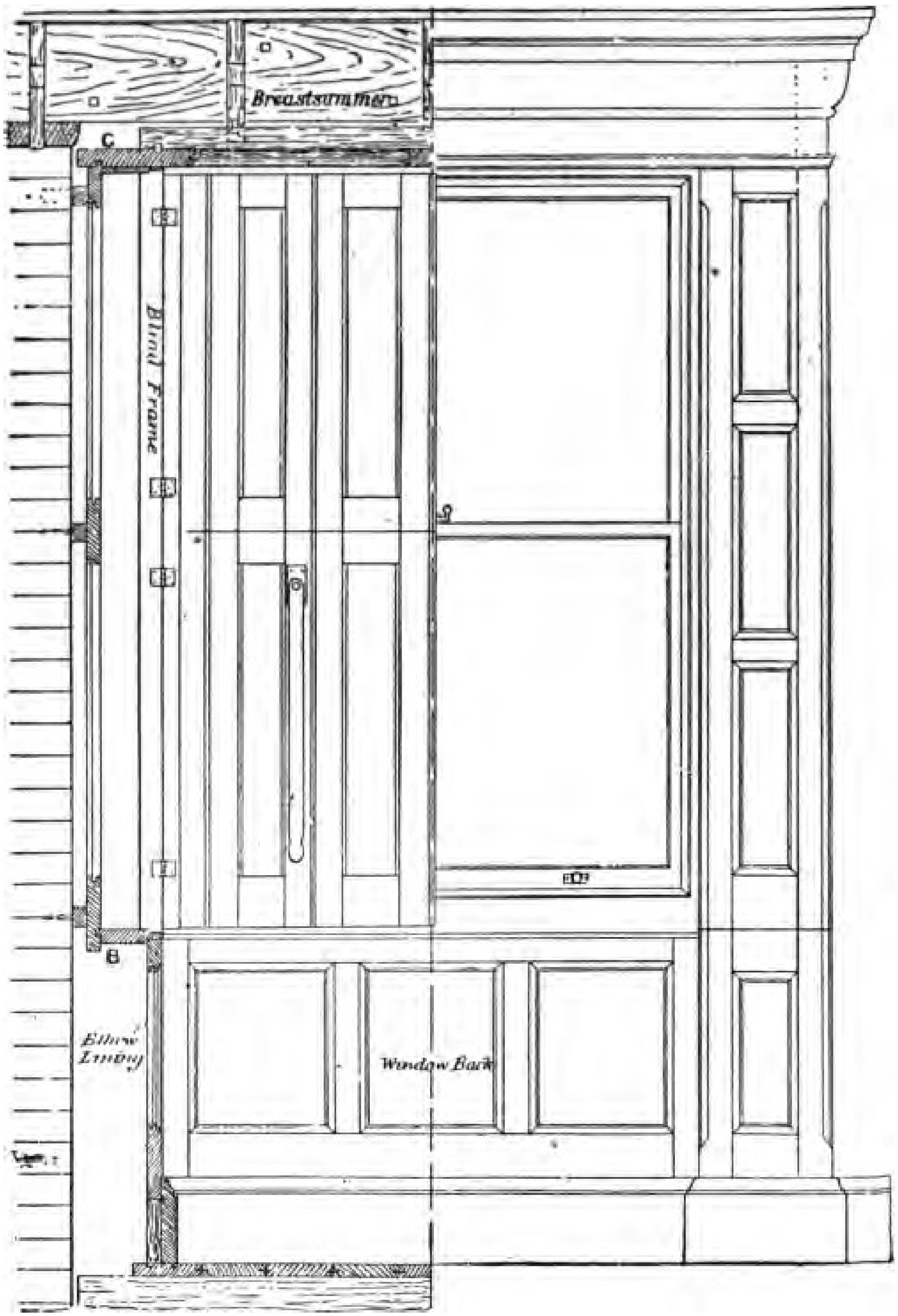


Fig. 55.

Fig. 55 is a sectional elevation of a **Window Fitted with Boxing Shutters** having a cover flap and spaces for a blind and a curtain. One-half of the elevation shows the shutters opened out and the front of the finishings removed, showing the construction of the boxings, &c. The plan, Fig. 56, is divided similarly, one-half showing the shutters folded back, with por-

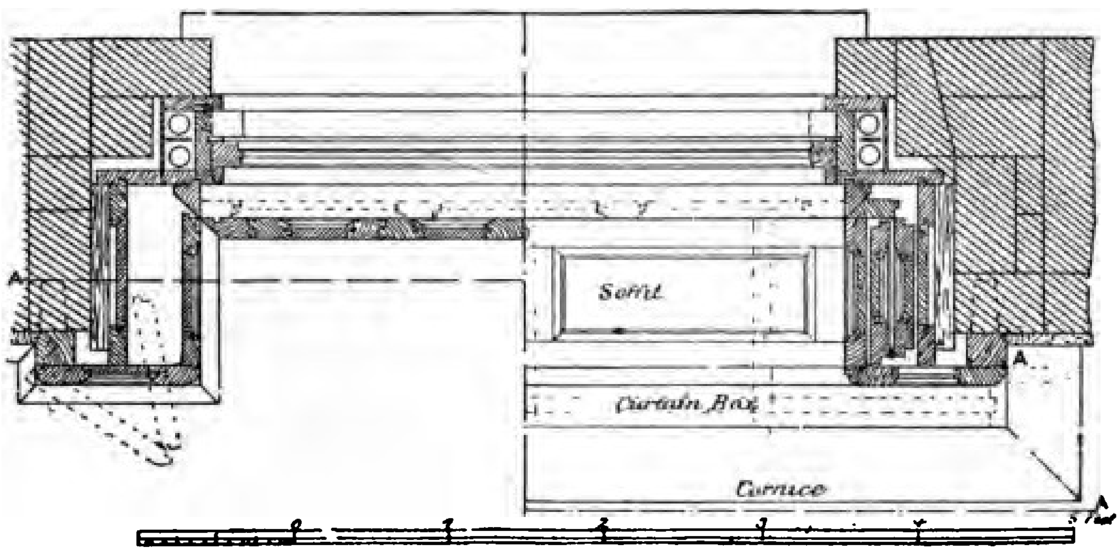


Fig. 56.

tions of soffit cornice, &c.; the other half gives the plan and sections of the lower parts, the dotted lines showing the window back.

Fig. 57 is a vertical section and Fig. 58 an enlarged section through the boxings. The framed pilaster covering the boxing is cut at the level of the window board, and hung to the stud A′, this being necessary for the cover to clear the shutters when open (see dotted lines on opposite half). The cover flap closes into rebates at the top and bottom, as shown in section in Fig. 55. The window back is carried behind the elbows, and grooved to receive the latter. The rails of the soffit must be wide enough to cover the boxings, and should have the boxing back tongued

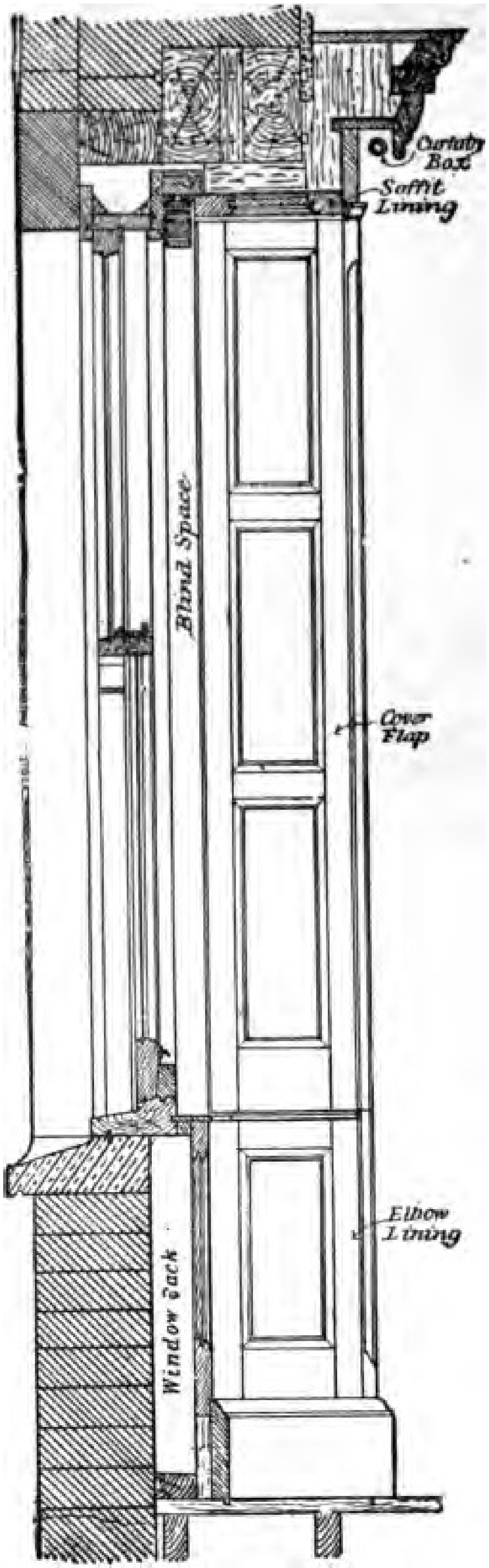


Fig. 57.

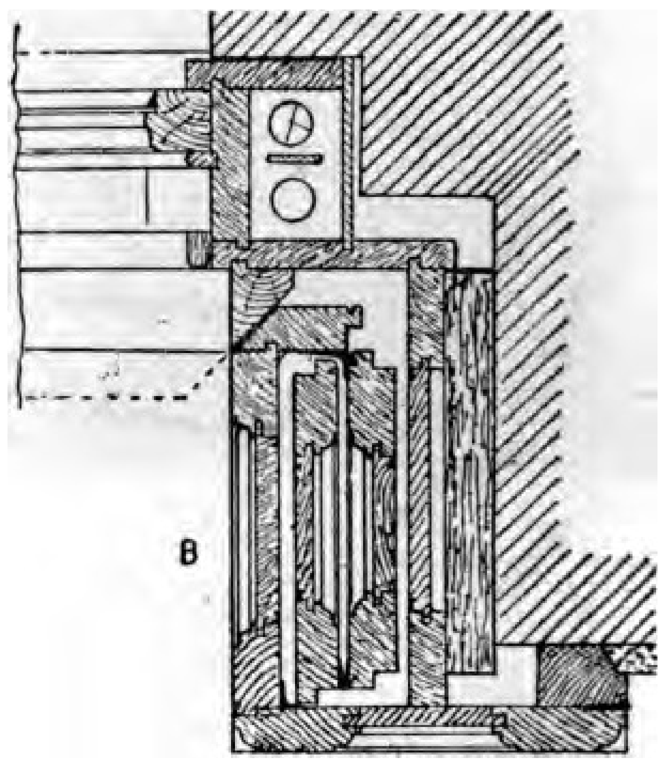


Fig. 58.

into it, as shown in the section, Fig. 55. When the linings of an opening run from soffit to the floor uninterruptedly, they are called jamb linings; but when they commence, as in the present instance, under the window board, they are termed elbow linings, the corresponding framing under the window being the window back.

Sliding Shutters are used instead of folding shutters in thin walls, and consist of thin panelled frames running between guides or rails fixed on the soffit and window board, which are made wider than usual for

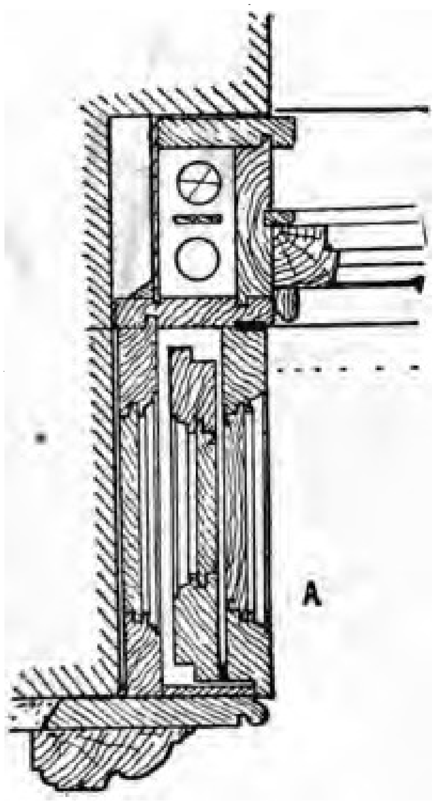


Fig. 59.

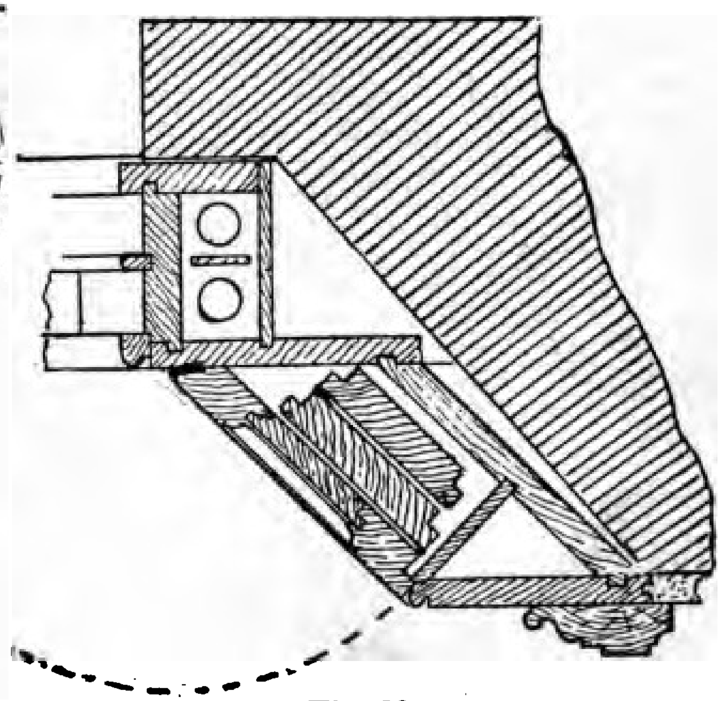


Fig. 60.

that purpose. When open, they lie upon the face of the wall adjacent to the opening. They are so seldom used that they do not call for illustration.

Balanced or Lifting Shutters are shown in section in Fig. 61, and plan in Fig. 62. They consist of thin panelled frames. the full width of and each half the height

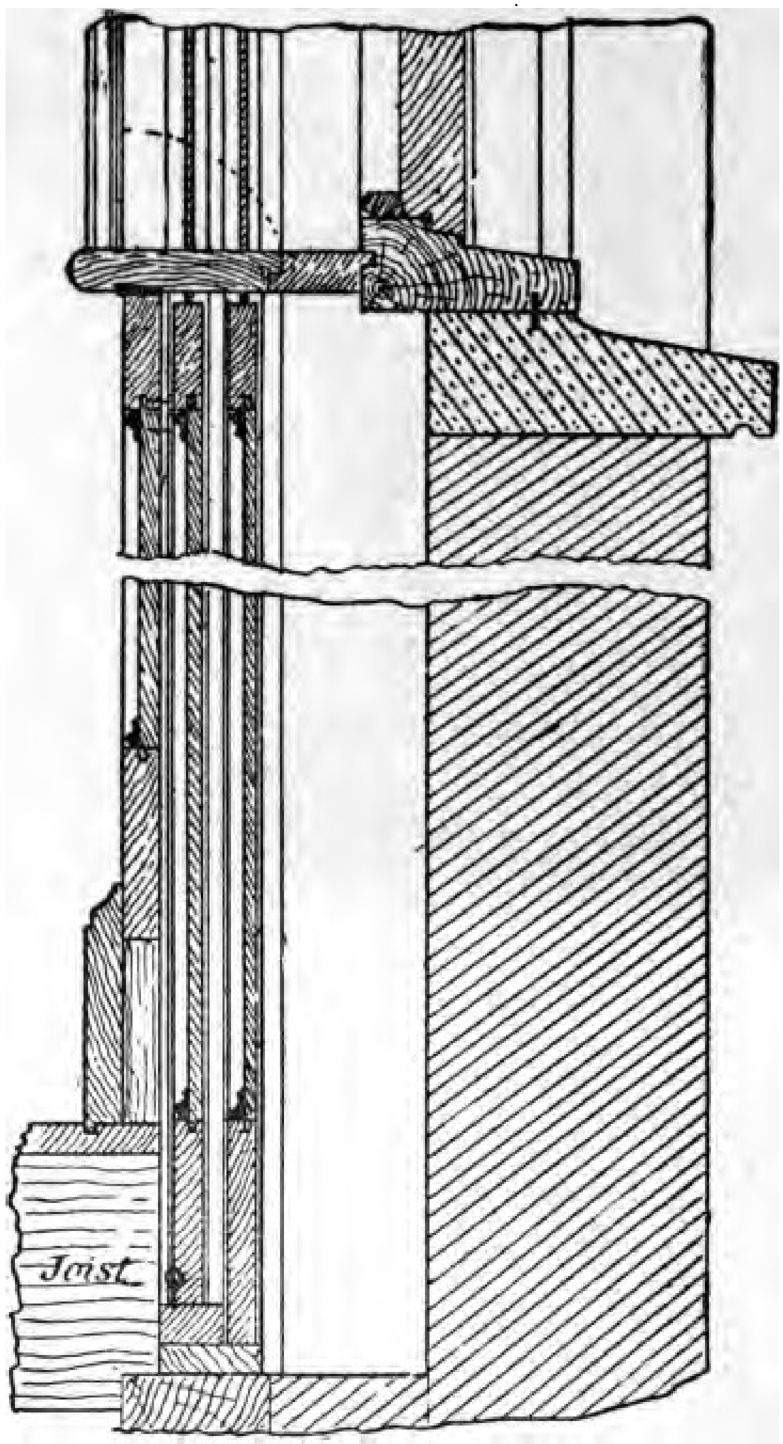
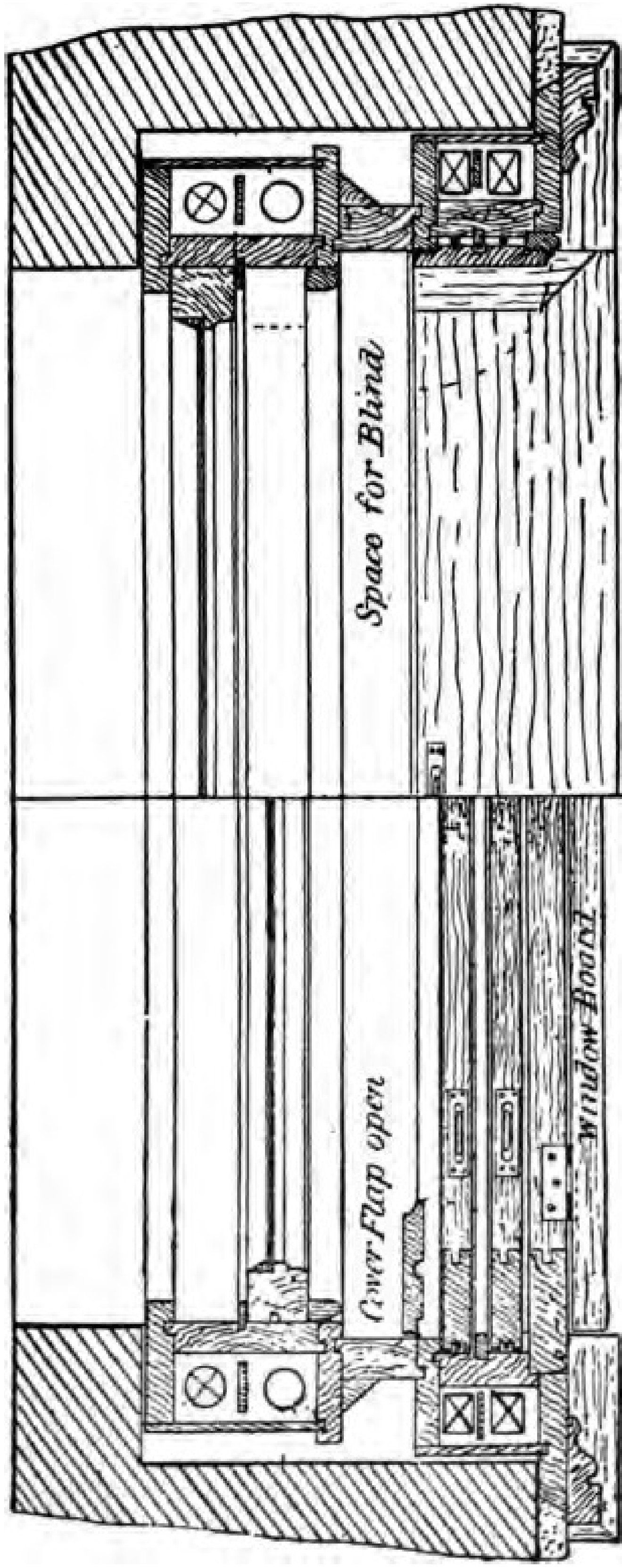


Fig. 61.

of the window, hung with weights in a cased frame in a similar manner to a pair of sashes. The frame extends the whole height of the window, and is carried down behind the floor joists to the set off. The win-



Half Plan Window Board closed

Half Plan with Window Board open

Fig. 62.

dow board is hinged to the front panelling or shutter back, so that when the former is lifted, the shutters can pass down behind the back out of sight. Cover flaps hung to the outer linings on each side close over the face of the pulley stiles and hide the cords. When it is desired to close the shutters, the cover flaps are opened out flat, as shown to the left of the plan. The

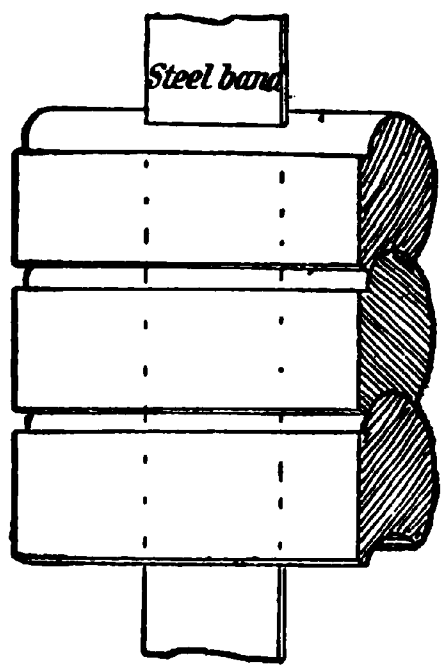


Fig. 63.

window board can then be lifted, and the shutters run up, a pair of flush rings being inserted in the top edge of each for that purpose. The window board is next shut down, the inside shutter brought down upon it, the outside one pushed tight up to the head, and the meeting rails, which overlap an inch, fastened with a thumb-screw. The bottom rail of the upper shutter is made an inch wider than the other, for the purpose of showing an equal margin when overlapped. Square lead weights have generally to be used for these shutters in consequence of their comparative heaviness.

Rolling or Spring Shutters are made in iron, steel and painted or polished woods, and though somewhat monotonous in appearance, are, in consequence of their convenience in opening and closing, and in the case of the metal kinds, the additional security against fire and burglary, fast superseding all other kinds, both for internal and external use, especially in shops and public buildings. They consist, in the case of the wood varieties, of a series of laths of plano-convex, double convex, oval, or ogee section, fastened together by thin steel or copper bands passing through mortises in the centre of the thickness, as shown in Fig. 63, and secured at their upper ends to the spring container. These metal bands are supplemented by several waterproof bands of flax webbing glued to the back sides of the laths. The upper edge of each lath is rounded, and fits into a corresponding hollow in the lower edge of the one above it; this peculiar overlapping joint, whilst preventing the passage of light, etc., between the laths, readily yields when the shutter is coiled around the barrel. The spring barrel is usually made of tinned iron plate, and encases a stout spiral spring wound round an iron mandrel with squared ends to which a key is fitted for winding the spring up. The ends of the mandrel project from the case and are fixed in bracket plates at each end of the shutter—in the case of shop fronts, in a box or recess behind the fascia; one end of the spring is secured to the barrel, the other end to the mandrel; the shutter is secured to the barrel by the

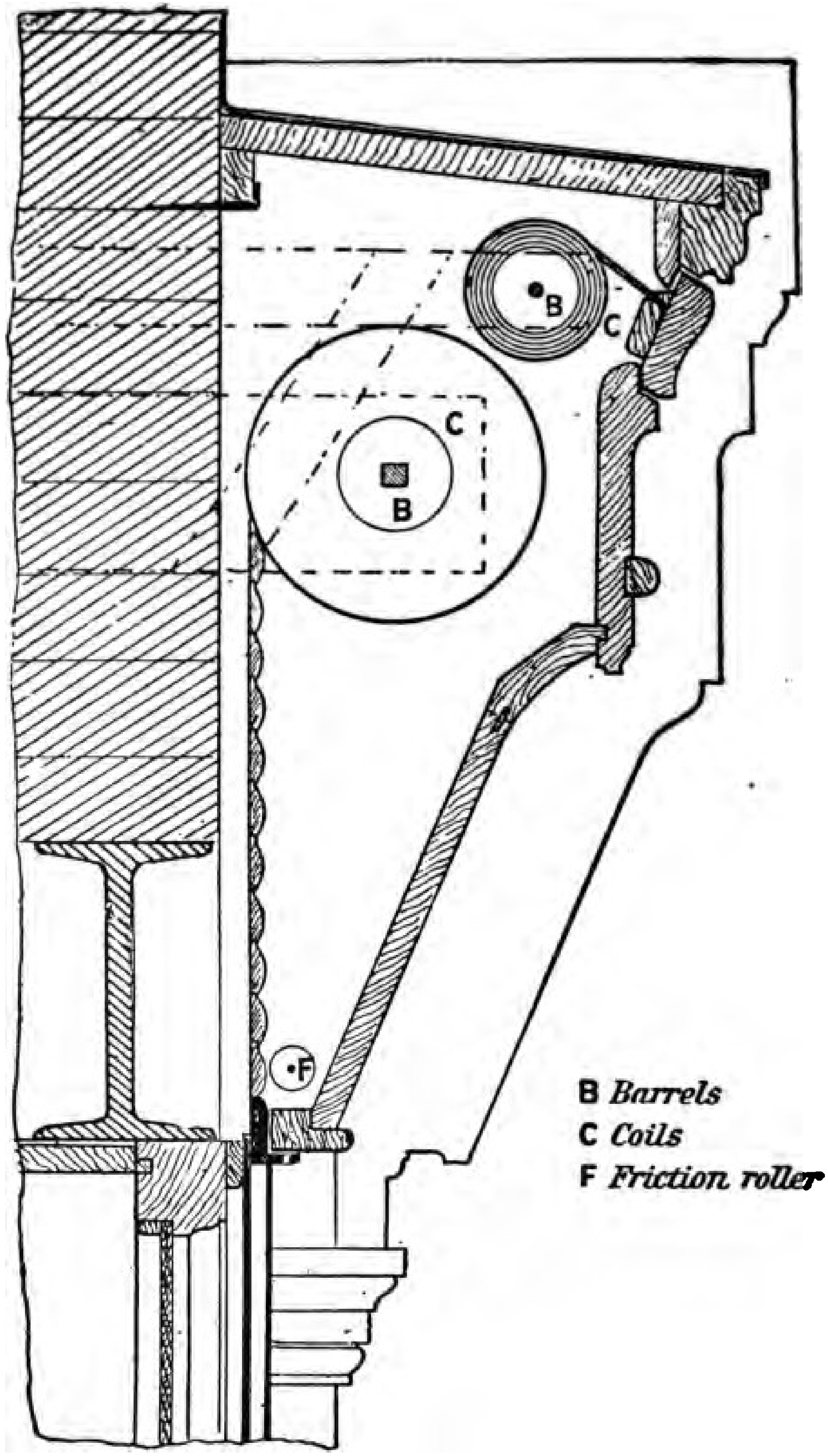


Fig. 64.

metal bands mentioned above, and the normal condition of things is that when the shutter is coiled up the spring is unwound. The pulling down of the shutter winds up the spring, and the tension is so arranged that it does not quite overcome the weight of the shutter and the friction when the shutter is down, so that the latter must be assisted up with a long arm. Great care should be taken in fixing these shutters to arrange the barrel perfectly level and parallel with the front, and to securely fix the brackets. These may be bolted to the girder or breastsummer; or screwed to the fixings plugged in the wall, as shown by dotted lines in Fig. 64. Where possible a wood groove should be formed on each side of the openings for the shutters to work in, but iron channels (Fig. 65) are frequently used. These are cemented into a chase in the wall or pilaster. Fig. 64 is a section through a shop fascia, showing shutter and blind barrels fixed to the face of the wall, where no provision has been made beneath the girder. Fig. 66 shows the adaption of one of these shutters to the inside of a window; the barrel is fixed in a seat formed beneath the window, the top of which is hinged, to gain access to the coil. The minimum space required for a coil, for a shutter about 6 ft. high is 10½ in. A friction roller F should be fixed close to the back rail to prevent chafing as the coil unwinds; the shutter is lifted by means of a flush ring in the L iron bar at the top, and this ring engages with a tilting hook in the soffit to keep the shutter up. The metal varieties of these shutters are wound up by the aid of balance weights or bevel wheel gearing.

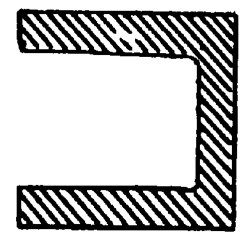


Fig. 65.

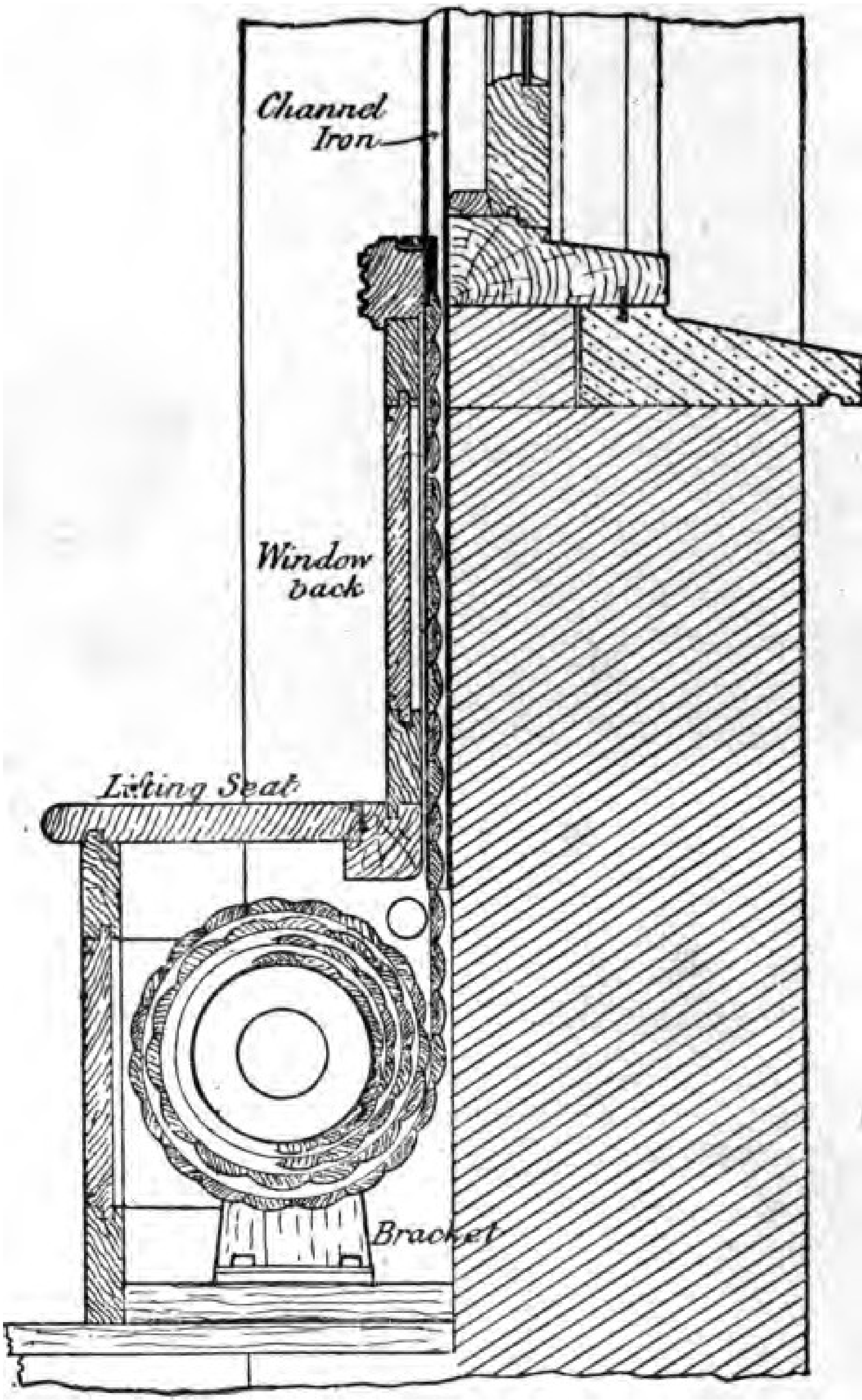


Fig. 66.

WINDOWS GENERALLY.

Size and Position. The sizes and positions of window openings are influenced by the size of the rooms, and the purposes for which the building is used. For the sake of ventilation, and also to secure good lighting, the windows should be placed at as great a height as the construction of the room will allow. In dwelling-houses the height of the sill is usually about 2′ 6″ above the inside floor level.

Construction. The framework holding the glass of the window may be fixed or movable. It must be so prepared that the glass can be replaced easily when necessary. In warehouses, workshops and similar buildings, the frames holding the glass are often fixed as **Fast Sheets** (Fig. 66½). As, however, this arrangement affords no means of ventilation, it is more usual to have the glass fixed in lighter frames called **Sashes.** If the sashes are hung to solid rebated frames, and open as doors do, the windows are called **Casement Sashes.** If they slide vertically and are balanced by weights or by each other, the window is a **Sash and Frame Windows.** Other methods of arranging sashes, either hinged, pivoted or made to slide past each other, are described in detail later.

Sashes. The terms used for the various parts of sashes and fast sheets are somewhat similar to those employed in describing doors. Thus, the **Styles** are the outer uprights, and the **Rails** are the main horizontal cross-pieces: top rails, meeting rails, and bottom rails being distinguished. Any intermediate members, whether vertical or horizontal, are named **Bars.**

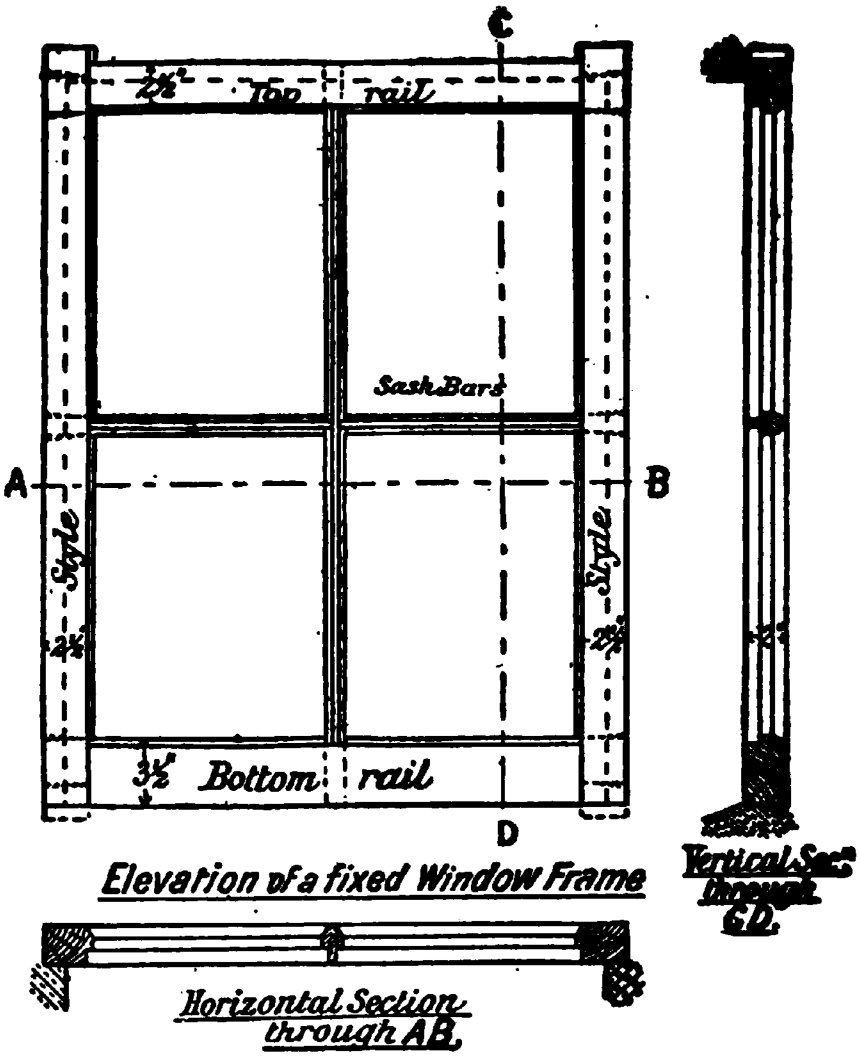


Fig. 66½

Sashes are from 1½ to 3 inches thick. The inner edge of the outer face is **Rebated** to receive the glass. The inner face is left either square, chamfered, or moulded;

Fig. 67.

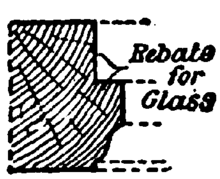


Fig. 68.

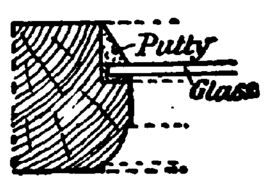


Fig. 69.

Fig. 70.

two common forms of moulding are lamb's-tongue (Fig. 68) and ovolo (Fig. 69). The size of the rebate is indicated in Fig. 70; it varies with the thickness of the sash, its depth being always a little more than one-third this thickness. The width of the rebate varies from a quarter of an inch to half an inch, and the mould is usually sunk the same depth as the rebate. This last fact is of some importance, as it affects the shoulder lines; and with hand work it influences the amount of labor in the making of the sashes.

As little material as possible is used in the sashes, in order that the light shall not be interfered with. In general, the styles and top rail are square in section before being rebated and moulded. In casement sashes, however, it is often advisable to have the outer styles a little wider than the thickness, especially when they are tongued into the frame. The width of the bottom rail is from one and a half to twice the thickness of the sash. Sash bars, which require rebating and moulding on both sides, should be as narrow as possible, in order not to interrupt the light. They are usually from five-eighths of an inch to one and a quarter inches wide.

Joints of Sashes. The sashes are framed together by means of the **Mortise and Tenon Joint** (Fig. 71). The proportions of the thickness and width of tenons, haunched tenons, &c., are to a large extent applicable here. Hardwood cross-tongues are sometimes inserted to strengthen the joints while thick sashes should have **Double Tenons.** An alternative to halving in sash bars is to arrange that the bar which is subjected to the greater stress—as for example, the vertical bars in sliding sashes, and the horizontal bars in hinged casement sashes—shall be continuous; this continuous bar is mortised to receive the other, which is scribed i. e. cut to fit the first, and on which the short tenons are left. This method is called **Franking the Sash Bars,** and is illustrated in Fig. 72.

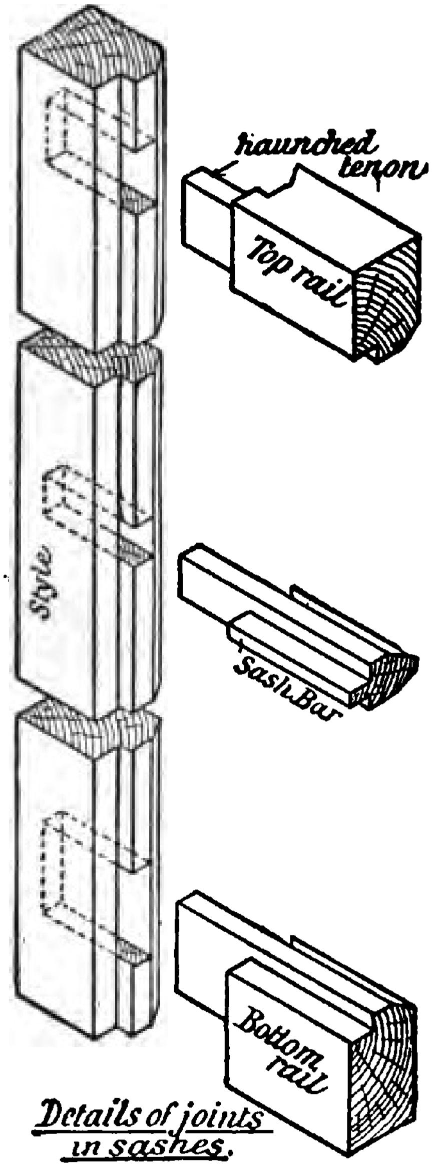


Fig. 71.

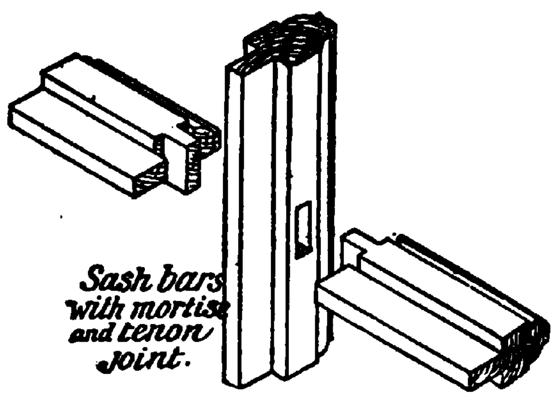


Fig. 72.

Casement Windows. Casement windows may be hinged in such a manner that they open either inwards or outwards. They may consist either of one sash, or of folding sashes, and are hung with butt hinges to solid rebated frames. These **Frames** consist of jambs, head and sill. The head and sill "run through" and are mortised near the ends to receive tenons formed on the ends of the jambs. The upper surface of the sill is weathered to throw off rain water. Casement windows which reach to the floor are usually called **French Casements.** Their sashes require an extra depth of bottom rail.

Casement Sashes Opening Inwards. Figs. 73, 74 show the elevation and vertical and horizontal sections, of a window opening in a 14″ brick wall fitted with a casement window having folding sashes to open inwards. In this class of window the frame is rebated for the sashes on the inner side. Each sash has, on the outer edge of the outer style, a semi-circular tongue, which fits into a corresponding groove in the jamb of the frame. This tongue renders the vertical joint between the sash and frame more likely to be weather proof; it is to provide for the tongue that the extra width of style already referred to is necessary. The tongue, however, is often omitted, as in Fig. 77. It will be seen readily that, if the sash were in one width, it would be impossible to have a tongue on more than one edge of it. With casement sashes opening inwards the greatest difficulty is found, however, in making a water-tight joint between the bottom rail of the sash and the sill of the frame. Figs. 77 and 78 show two methods by which this may be accomplished. An essential feature of all these sashes is a small groove or **Throating** on the under edge of the bottom rail; this prevents the water from getting through. The groove in the rebate of the sill (Fig. 78) is provided to collect any water that may drive through the joint. This

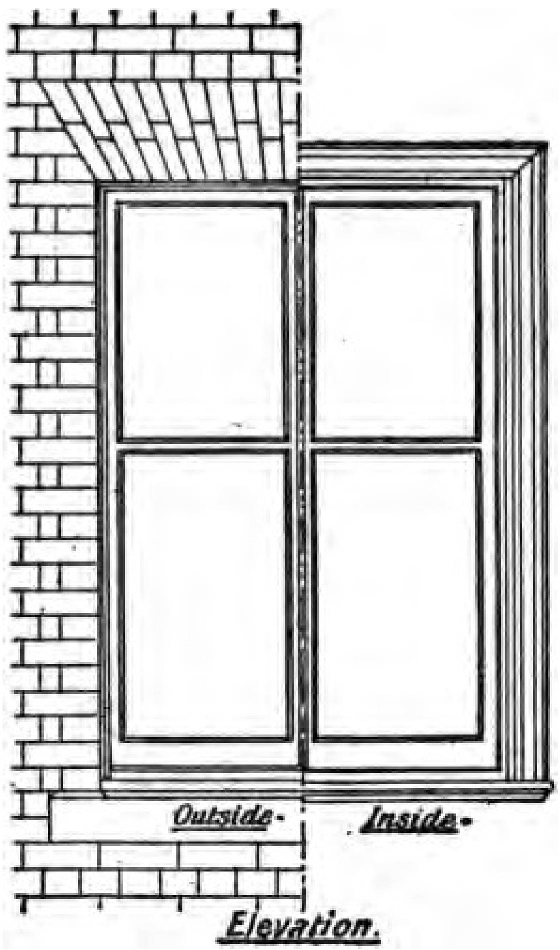


Fig. 73.

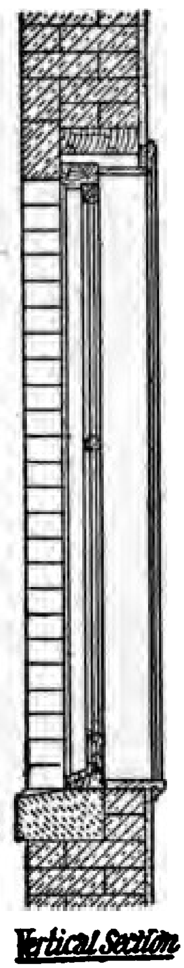


Fig. 74.

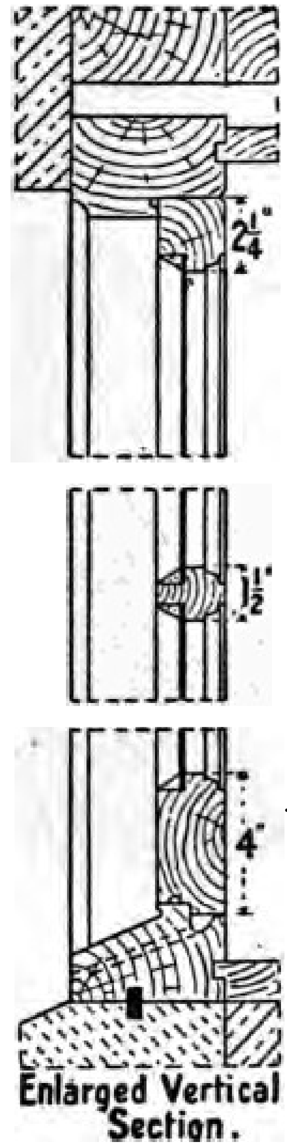


Fig. 75.

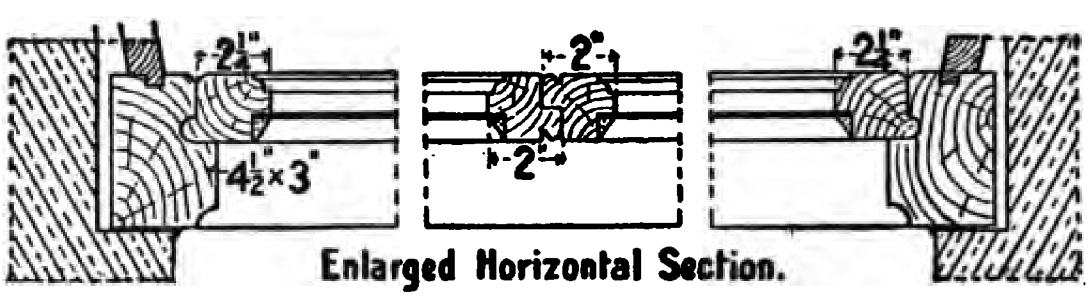


Fig. 76.

water escapes through the hole bored in the centre of the sill.

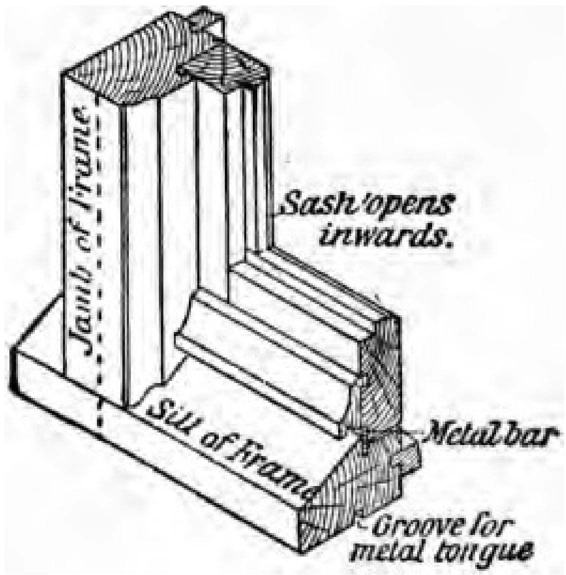


Fig. 77.

When casement sashes are hung after the manner of folding doors, the vertical joint between the meeting styles is rebated. Alternative methods of rebating

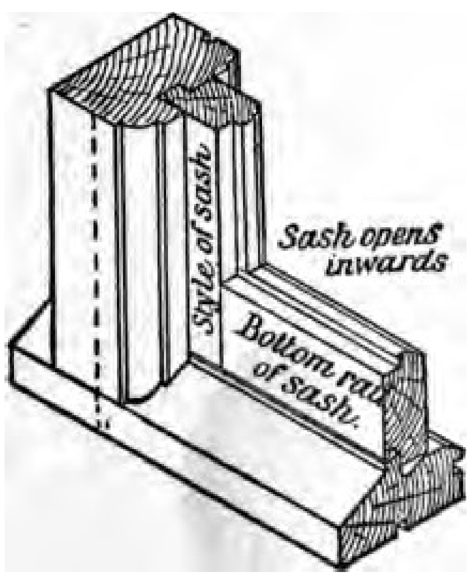


Fig. 78.

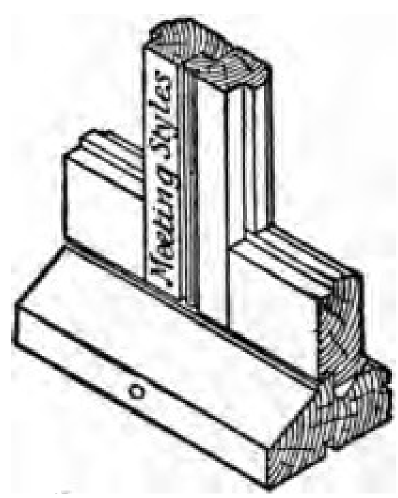


Fig. 79.

are shown in Fig. 79 and 80. Fig. 80 is known as a **Hook Joint** and is the better one.

Casement Sashes Opening Outwards. These are more easily made weatherproof than inward-opening sashes. The chief objections to their adoption are that they are not easily accessible for cleaning the outside, especially in upper rooms and that they are also liable, when left open, to be damaged by high winds and to let in the rain during a storm. Fig. 81 is a sketch of one corner of such a window. It will be noticed that these

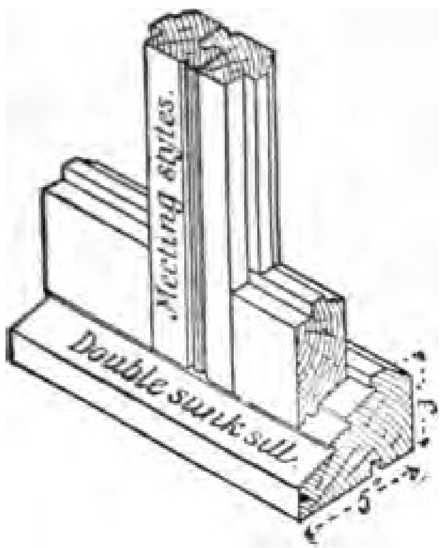


Fig. 80.

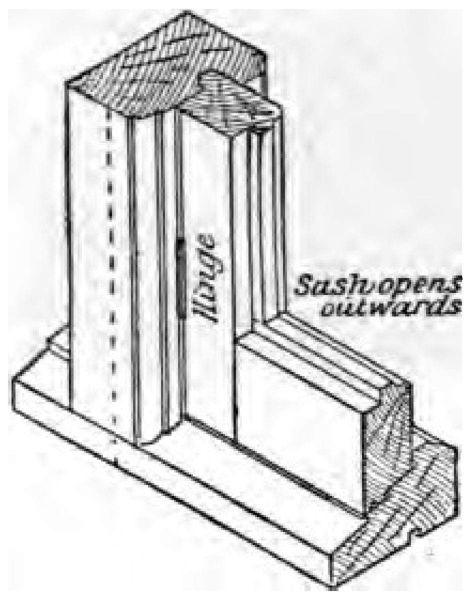


Fig. 81.

frames, like door frames, have the exposes arrises moulded in various ways, and that the sashes may either be hung flush with one face of the frame (as in Figs. 76 and 77), or fit in the thickness of the frame (Figs. 78 and 81). The sill shows to be **Double Sunk,** i. e., to have the upper surface—upon which the bottom rail of the sash fits—rebated with two slopes (weatherings).

Sash and Frame Window. In this class of window, which is by far the most common, because it is easily made weather proof, there are **Two Sashes,** which slide past each other in vertical grooves, and are usually

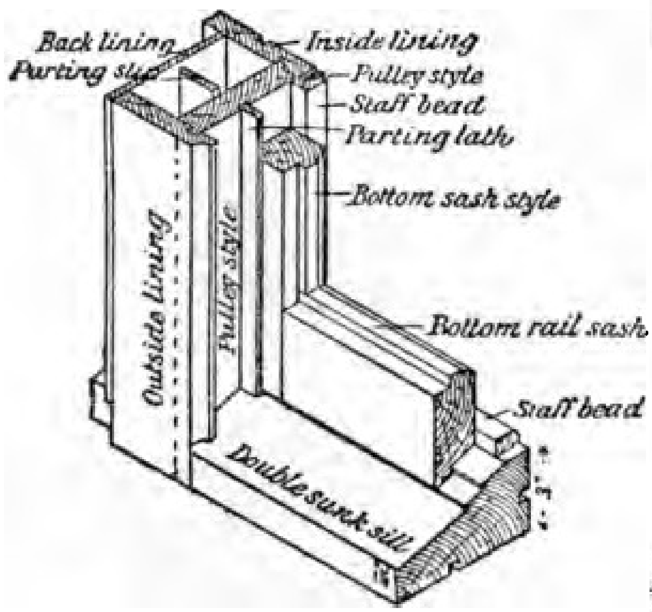


Fig. 82.

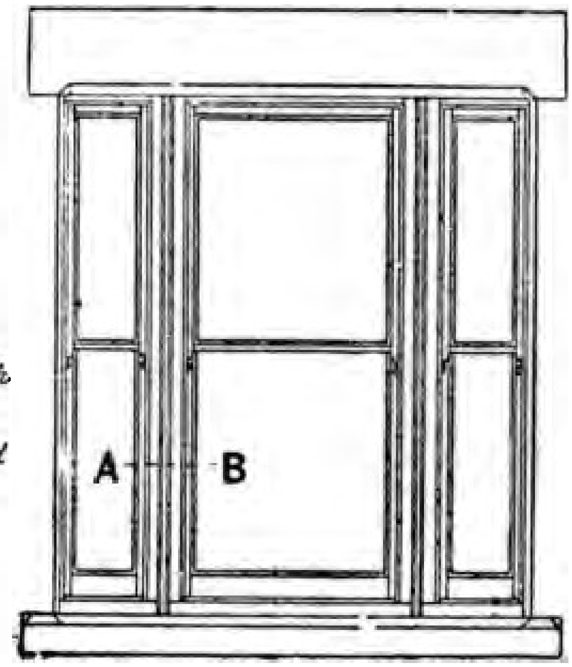


Fig. 82½.

balanced by iron or leaden weights. As will be seen from Fig. 82 the frames form cases or boxes in which the weights are suspended. They are hence called **Cased Frames. Pulley Styles** (Fig. 86) take the place of the solid rebated jambs of casement windows. The pulley styles, **Outside** and **Inside Linings,** and **Back Lining** (Fig. 82) together form a box which is subdivided by a vertical **Parting Slip** suspended as shown in Fig. 82. In superior window frames of this kind, the pulley styles and linings are tongued and grooved together as shown in Fig. 83. In commoner work the tongues and grooves are often omitted. The frame must be so constructed that the sashes can be removed easily for the purpose of replacing broken sash-lines. To enable this to be done, the edge of the inside lining is either made flush with the face of the pulley style (Fig. 83), or it is rebated slightly as shown in Fig. 83. The edge of the outside lining projects for a distance of about three-quarters of an inch beyond the face of the pulley style, to form a rebate against which the outer (upper) sash slides. The outer sash is kept in position by the **Parting Lath** (Fig. 82) which fits into a groove in the pulley style. The groove for the inner (lower) sash is formed by the parting lath and a **Staff Bead** or **Stop Bead** which is secured by screws. The staff bead on the sill is often made from two to three inches deep to allow the lower sash to be raised sufficiently for ventilation at the meeting rails without causing a draught at the bottom (Fig. 83).

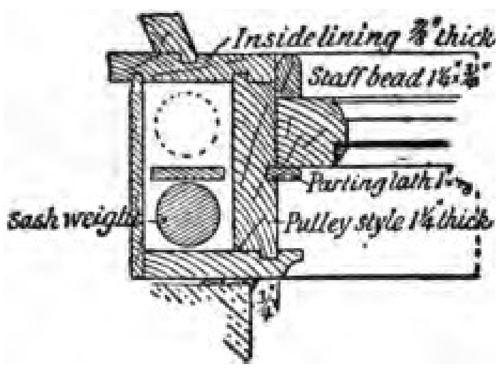


Fig. 83.

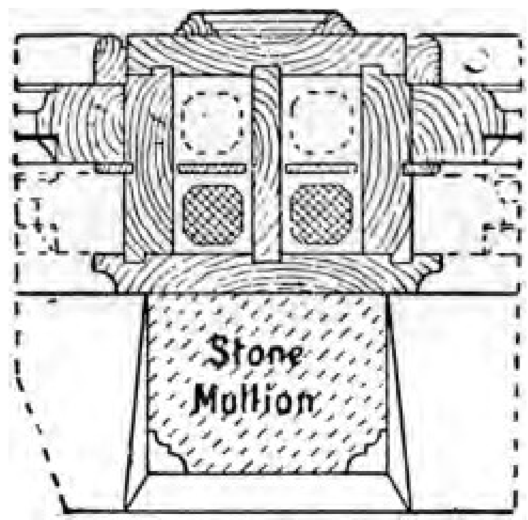


Fig. 83½

A vertical section through the head of the frame is similar to a horizontal section across the pulley style, except that the back lining and parting slip are of course absent.

The sill of the frame is solid and weathered, and should always be of hardwood, preferably oak or teak. The sill has a width equal to the full thickness of the frame. When the weathering has two steppings, it is known as a **Double Sunk Sill.** An alternative to the plan of having the width of the sill of the full thickness of the frame, is to arrange it so that the outside edge is flush with the outside face of the bottom sash, as shown in Fig. 89. With a sill arranged in this manner, and double sunk, there is less danger of water driving through the joint between the sash and the sill than with a sill the full thickness of the frame. In order

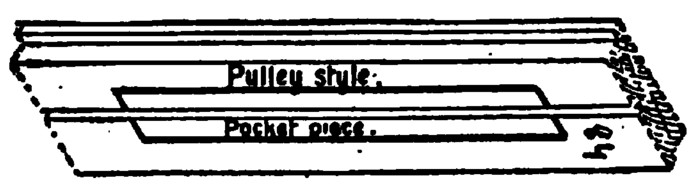


Fig. 84.

to render watertight the joint between the wooden and stone sills of window frames, a metal tongue is often fixed into corresponding grooves cut into the under side of the wooden sill and the upper surface of the stone sill. A rebated joint between the two sills serves the same purpose as the metal tongue.

Fig. 84 shows the methods of fixing the pulley style into the head and sill respectively, when the width of the sill is equal to the full thickness of the frame. The **Pulleys** on which the sash lines run—sash or axle pulleys are fixed in mortises near the upper ends of the pulley styles. It is also necessary to have a removable piece in the lower part of each pulley style, to allow of access to the weights. This piece is named the **Pocket Piece.** It may be cut as shown in Fig. 85; its position is then behind the lower sash, and it is hidden

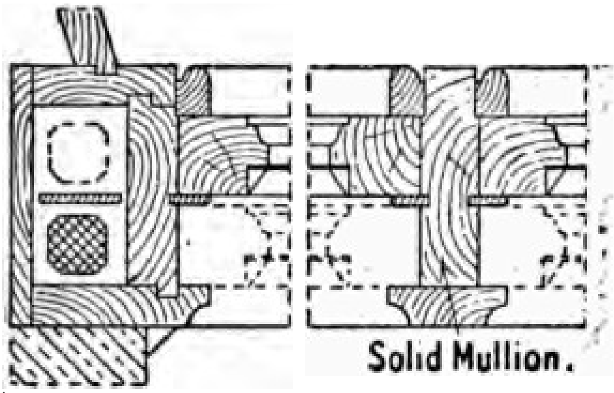


Fig. 84½.

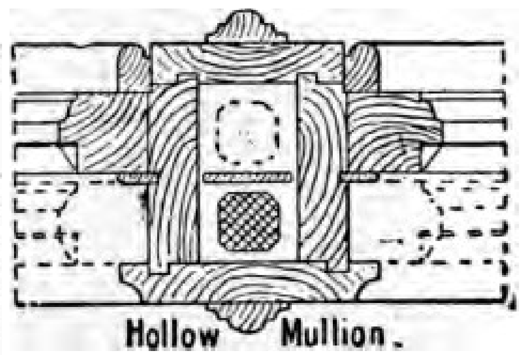


Fig. 85½.

from view when the window is closed. Or, the pocket piece may be in the middle of the pulley style as shown in Fig. 84; the vertical joints between the pocket piece and the pulley style are then V shaped to prevent damage to the paint in case of removal.

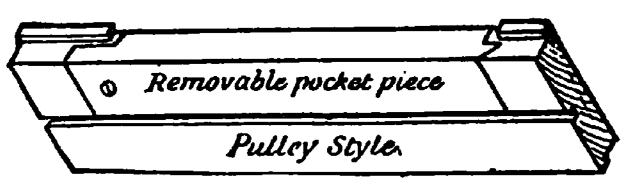
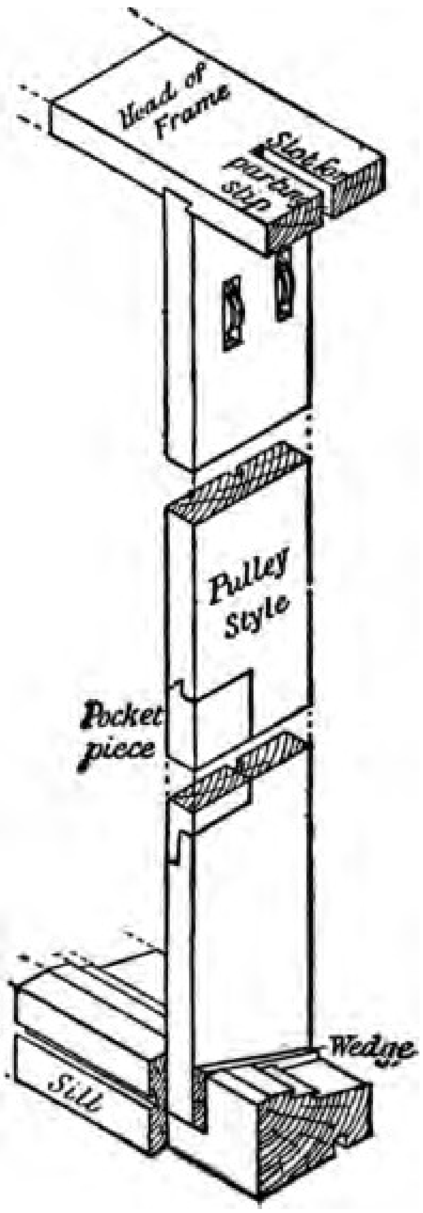


Fig. 85.

Sashes. The only difference between the joints of sliding sashes and those of the casement sashes already described is in the construction of the meeting rails. Each of the meeting rails is made thicker than the sash to the extent of the thickness of the parting lath; otherwise there would be a space between them equal to the thickness of the parting lath. The joint between them may be rebated or splayed. The angle joints between the ends of the sash styles and the meeting rails are



Joints at ends of Pulley Style.
Fig. 86.

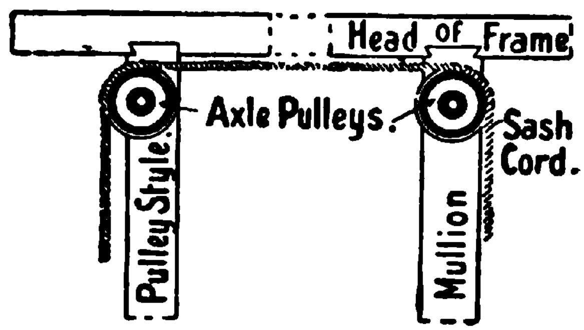
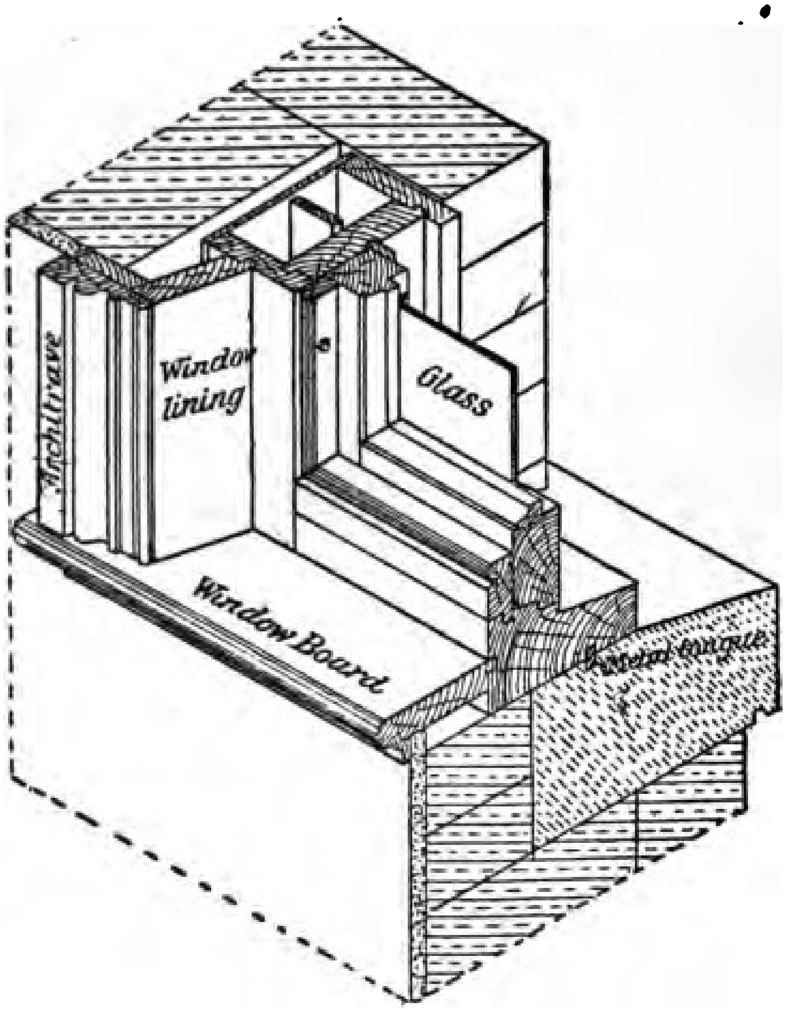


Fig. 86½.

often dovetailed as shown in Fig. 86. They are, however, stronger if the styles are made a little longer, the projecting part being moulded, and mortise and tenon joints used as shown in Figs. 86 and 87. The projecting ends of the styles are called **Joggles**; they assist in enabling the sashes, especially in wide windows, to slide more freely. When as is usually the case, both sashes slide and are balanced by weights, the



Sketch of one corner of a Sash and Frame Window.
Fig. 87.

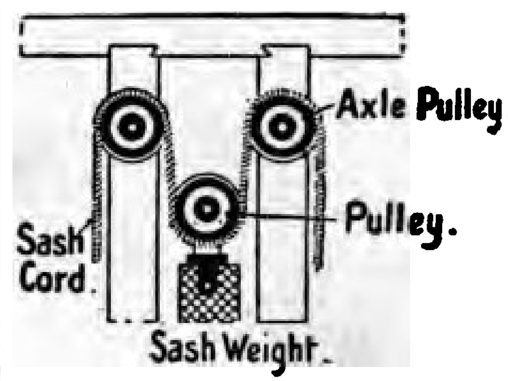


Fig. 87½.

window is known as a **Double-Hung** sash and frame window. If one sash only slides, and the other is fixed in the frame, the window is **Single-Hung**. Figs. 78 to 80 show the details of a sash and frame window fixed in a one-and-a-half-brick-thick wall and having a stone head and sill.

For the sake of appearance, or when it is required to have wider windows than can be arranged with one pair of sashes, two or three pairs of sashes are often constructed side by side in the same frame. When three pairs of sashes are used, it is usual to have the middle pair wider than the others; such a combination (Fig. 73) is named a **Venetian Window.** The vertical divisions between adjacent pairs of sashes are called **Mullions.** These mullions may be constructed in several different ways. If the middle pair of sashes only is required to slide, the mullions may be solid from 1¼″ to 2″ thick, and the sash-cord conducted by means of additional pulleys to the boxes, which are at the outer edges of the frame. Figs. 84 and 86 show this arrangement. If it is desirable to have all the sashes to slide, the mullions must be hollow to provide room for the weights. Figs. 85 and 87 show details of a mullion with provision made for one weight to balance the two sashes adjacent to it. With this arrangement the sash-cord passes round a pulley fixed into the upper end of the weight. If stone mullions are used in the window opening, separate boxings may be made so that each pair of sashes is hung independently as shown in Fig. 83, and the window becomes, as it were, two or three—as the case may be—separate window frames, with the sill and head each in one length for the sake of strength.

The Hanging of Vertical Sliding Sashes. As shown in illustrations already given, the sashes of sash and frame windows are balanced by cast-iron or leaden weights. The best cord is employed for hanging sashes of ordinary size, while for very heavy sashes the sash lines are often of steel or copper. The staff bead and parting bead having been removed, the cords are passed over the axle pulleys (which are best of brass to prevent corrosion) and are tied to the upper ends of the weights. The weights are passed through the pocket holes and suspended in the boxes. The pocket pieces having been replaced, the upper sash, which slides in the outer groove, is hung first, the free ends of the cords being either nailed into grooves in the outer edges of the sash or secured by knotting the ends after passing them through holes bored into the styles of the sash. The upper sash having been hung, the parting laths are fixed into the grooves in the pulley styles, and the lower (inner) sash is hung in a similar manner, after which the staff beads are screwed in position. Care should be taken to have the cords of the right lengths; if the cords for the upper sash are too long the weights will touch the bottom of the frame, and cease to balance the weight of the sash before the latter is closed. If the cords for the lower sash are too short, the weights will come in contact with the axle pulleys, and thus prevent it from closing. Several different devices for hanging sashes—the objects of which are either to render unnecessary the use of weights or to facilitate the cleaning of the outside of the window—have been patented, and are in more or less general use. A detailed description of these is, however, beyond the scope of this book.

Bay Windows. A bay window is one that projects beyond the face of the wall. The side lights may be either splayed or at right angles to the front. The window openings may be formed by having stone or brick mullions or piers at the angles, against which the window frames are fixed, or the wooden framework of the window may be complete in itself. When the latter is the case, it is usual to have stone or brick work to the sill level as shown in Fig. 88. Bay win-

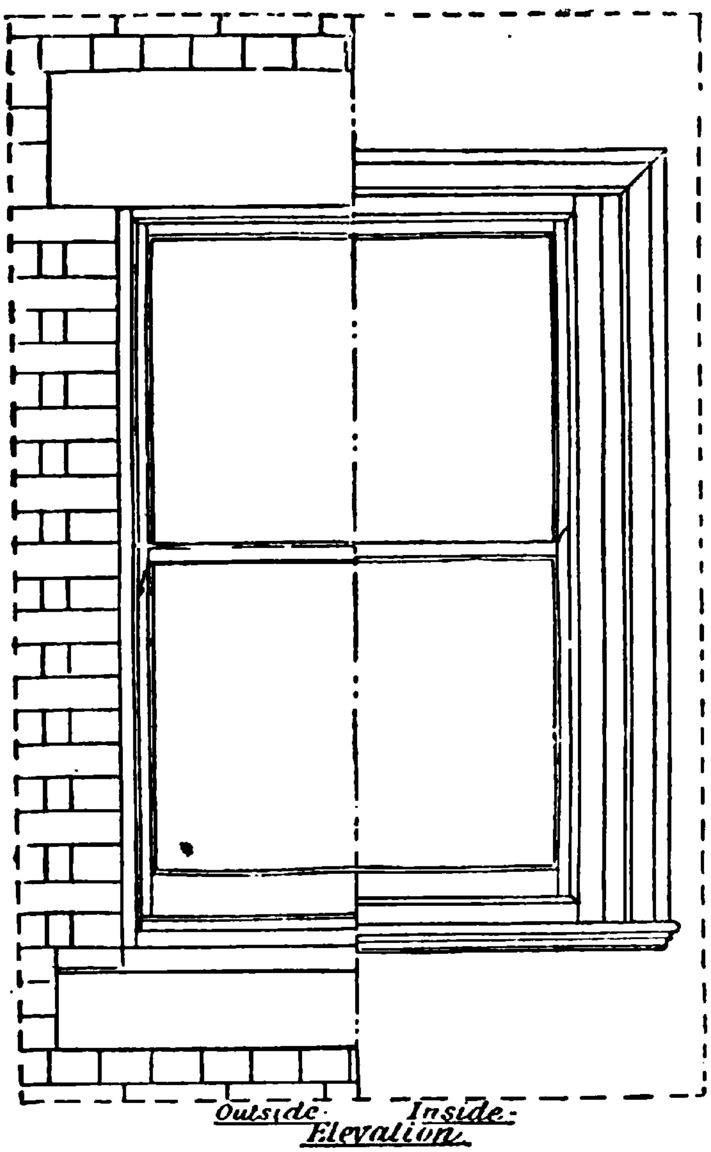


Fig. 88.

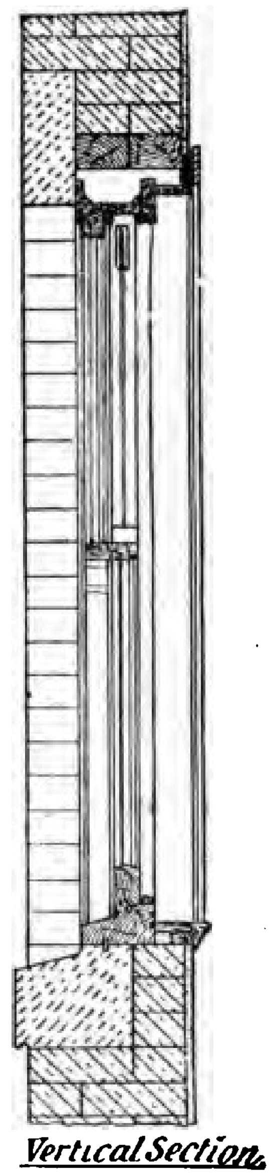
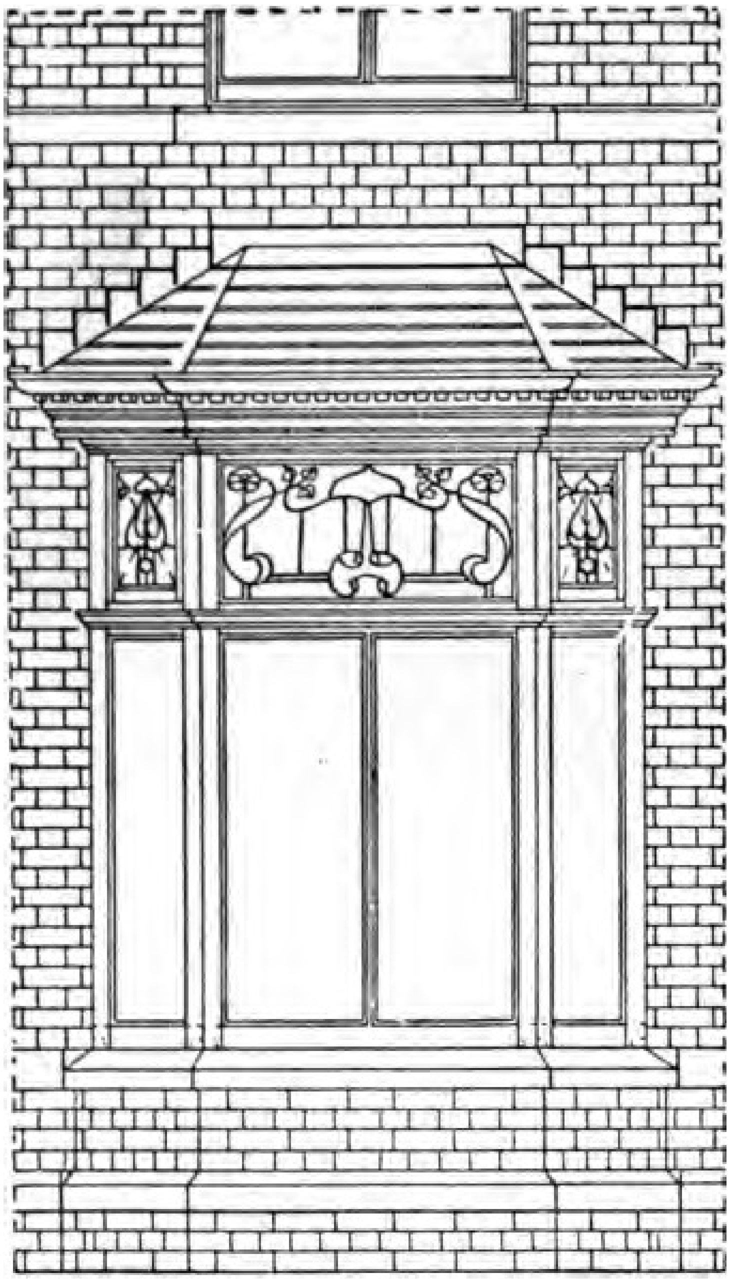


Fig. 89.

dows naturally lend themselves to decorative treatment. With the addition of masonry or brickwork they often assume a massive and bold appearance. When usually by a wooden cornice, and the wooden roof is covered with lead, slates or tiles. The window frames may be arranged as fixed lights, sash and frame, or



Front Elevation.
Fig. 88½.

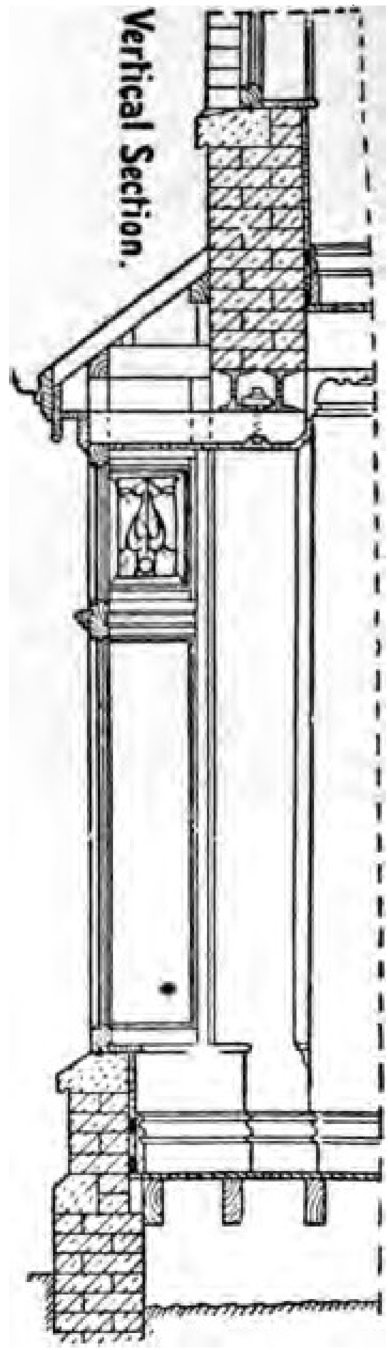
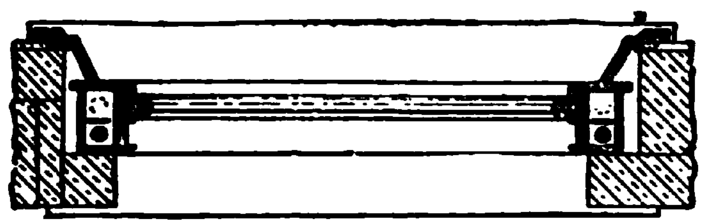


Fig. 89½.

casements. The most usual arrangement is to have the lower lights fixed, and the upper ones as sashes hinged to open for ventilating purposes. Figs. 88 to 90 show the details of a bay window with splayed side lights, the upper side lights being hinged on the tr som to open inwards.

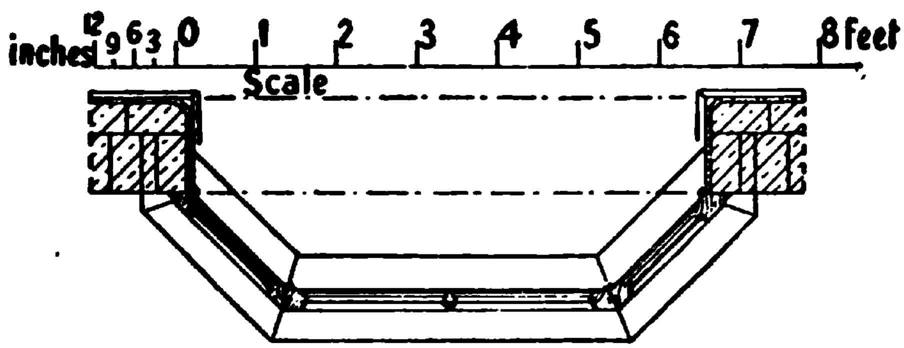
Windows With Curved Heads. When a wind opening is surmounted by an arch, the top of the w dow frame requires to be off the same curvature as under side (soffit) of the arch. In the case of fi



Horizontal Section

Fig. 90.

sashes, or of solid frames with casement sashes, head of the frame is "cut out of the solid." A he which, owing to the size of the curve, cannot easily obtained in one piece, is built up of segments,

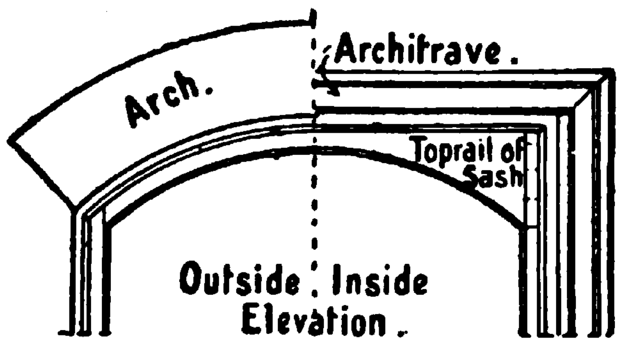


Horizontal Section.

Fig. 90½.

joints being radial to the curve, and secured by ha wood keys. As an alternative method, the head n be built up of two thicknesses—with overlappi joints—and secured together by screws.

A sash and frame window in such an opening may have only the outside lining cut to the curve of the arch, the inner side of the frame being left square. The upper sash will then require a top rail with a straight upper edge and a curved lower edge, as shown in Fig. 91.



Elevation of upper part of a Window having Curved Head.

Fig. 91.

When the head of the frame has to be curved, it may

(1) be built up of two thicknesses with overlapping joints, and secured by screws; it may

(2) be formed of three thicknesses of thin material, bent upon a block of the correct radius, and well glued and screwed together; or

(3) the head may be of the same thickness as the pulley styles, with trenches cut out of the back (upper) side, leaving only a veneer on the face-side under the trenches. Wooden keys are glued and driven into the trenches after the head has been bent upon a block to the required shape.

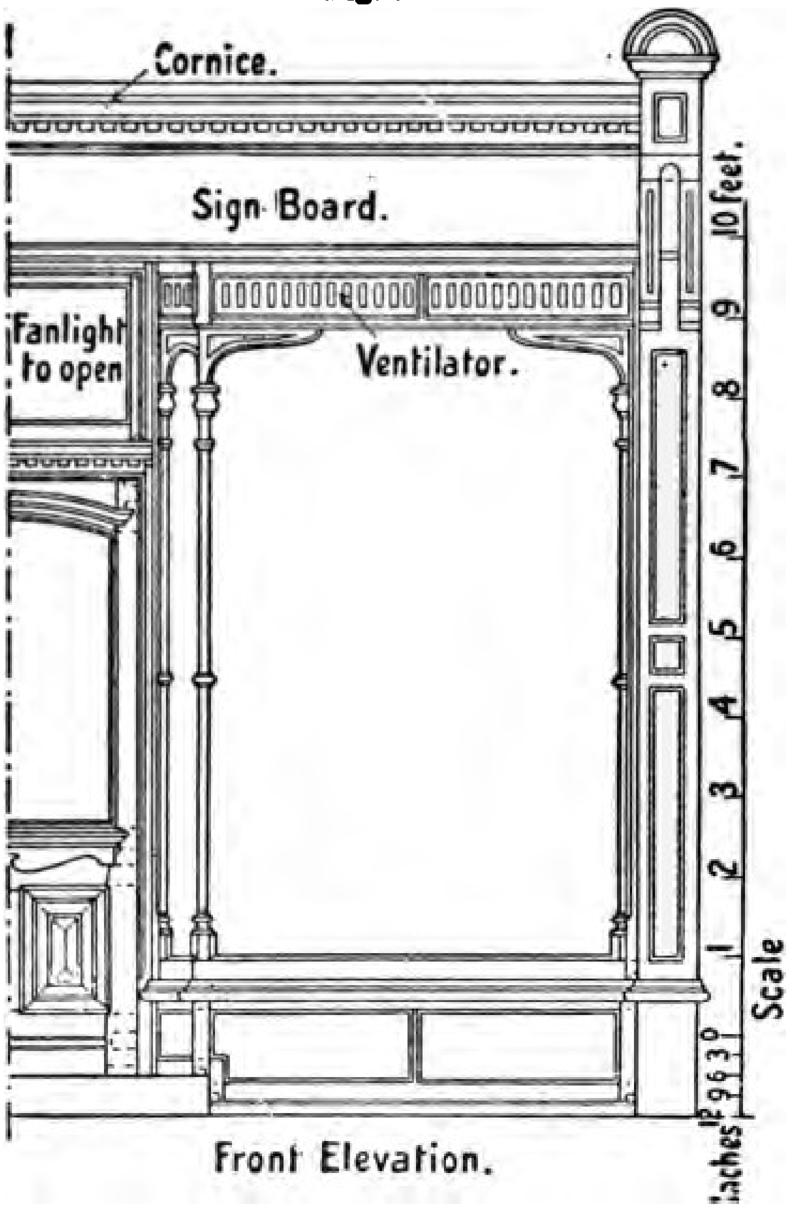
A strip of stout canvas glued over the upper side will strengthen the whole materially. The outside and inside linings are in such a case cut to the required curvature, and when nailed in position hold the head in shape. The end joints of the linings may have hardwood cross-tongues.

Shop Windows. The main object in view in the construction of shop windows is to admit the maximum of light, and to give opportunity for an effective display of the goods. The glass is in large sheets, and therefore is specially thick to secure the necessary strength. Shop windows are usually arranged as fast sheets, with provision for ventilation at the top. The glass is held in position by wooden fillets, and is fixed from the inner side. The chief constructional variations are found in the pilasters, cornice, provision for sign-board, sunblind, and the arrangement of the side windows. Figs. 92 to 95 show the details of a typical example.

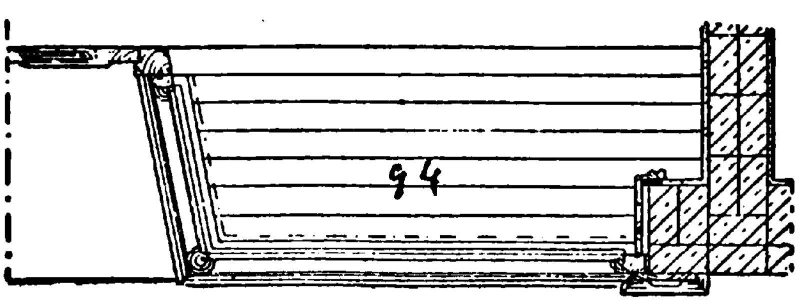
The Fixing of Window Frames. Window frames may be built into the wall—which has usually a recessed opening to receive them—as the brickwork proceeds, or they may be fixed later. In the former case, the ends of the sill and head project and form **Horns,** which are built into the brickwork and help to secure the frame. Wooden bricks or slips may also be built into the wall, the frames being nailed to them.

In the latter case, the frames are secured by wooden **Wedges,** which are driven tightly between the frame and the wall. These wedges should be inserted only at the ends of the head and sill and directly above the jambs; otherwise the frame might be so strained as to interfere with the sliding of the sashes. Window frames as well as door frames should be bedded against a layer of hair-mortar placed in the recess.

Fig. 92.

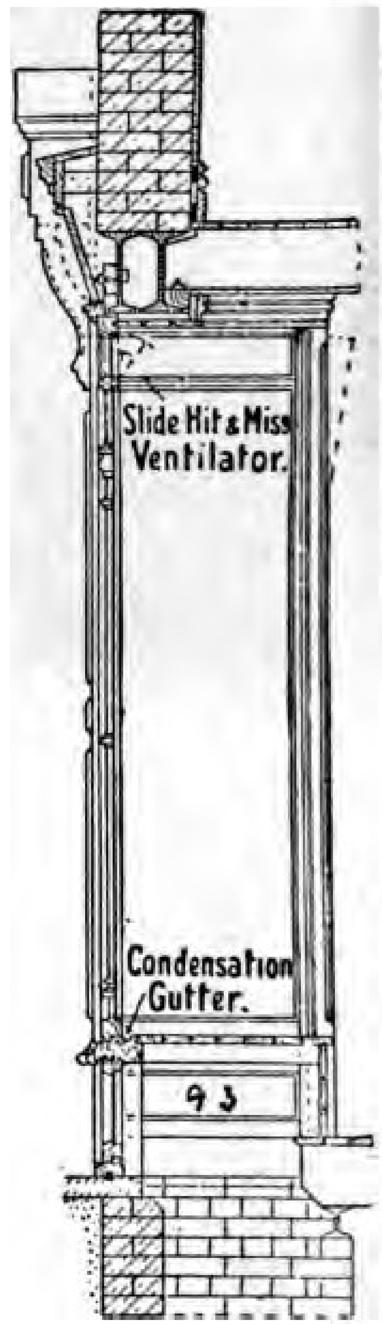


Front Elevation.



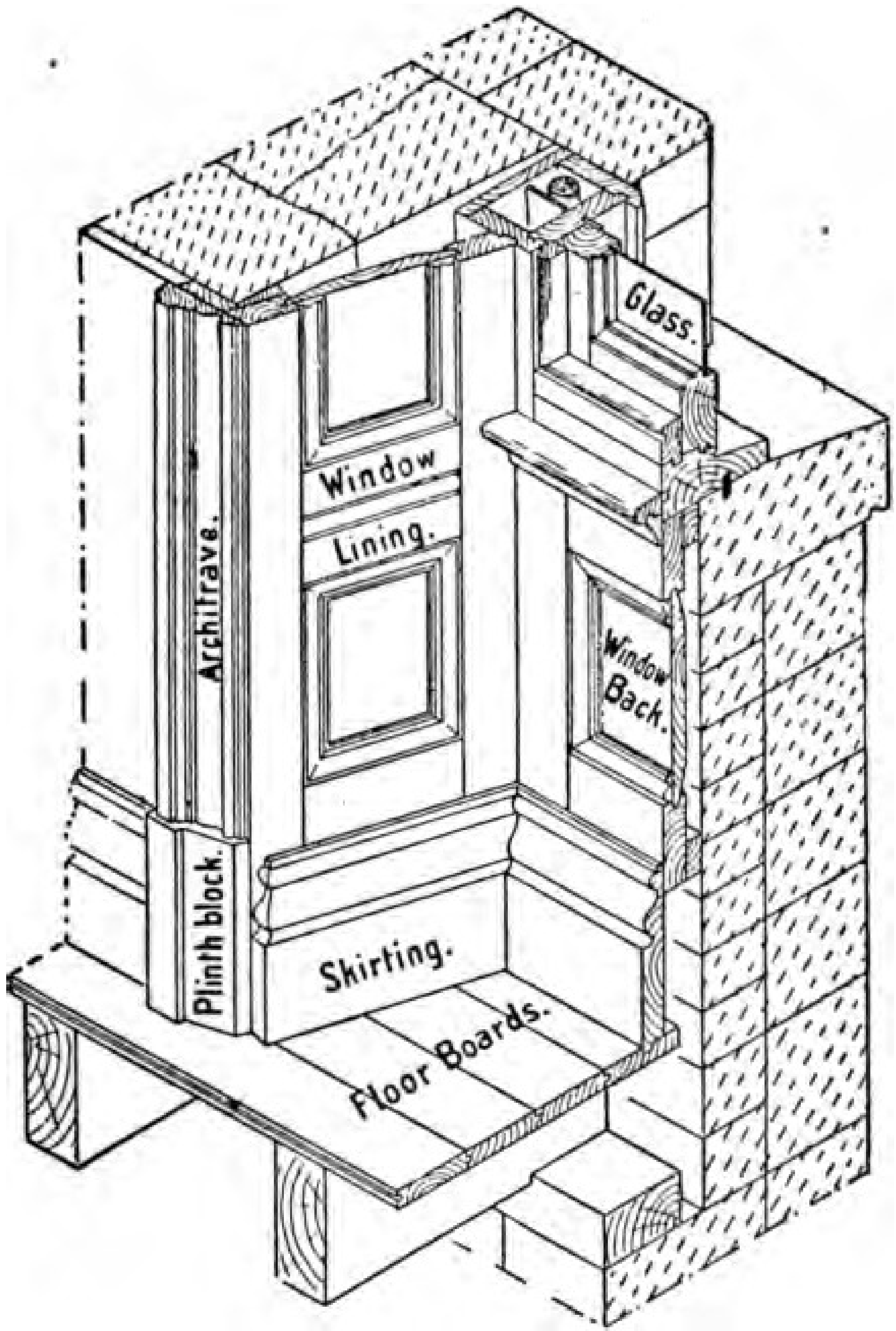
Horizontal Section.

Fig. 94.



Vertical Section.

Fig. 93.



Sketch of part of a Sash and Frame Window, showing Panelled and Moulded Jamb-lining, etc.

Fig. 95.

Linings. When window frames are not of sufficient thickness to come flush with the inner face of the wall, the plaster may be returned round the brickwork and finished against the frame, or a narrow fillet of wood may be scribed to the wall and nailed to the frame. In dwelling-houses, however, the more usual way is to fix linings similar to those used for outer door frames. The width of the linings depends upon the thickness of the wall; they should project beyond the inner face of the wall for a distance equal to the thickness of the

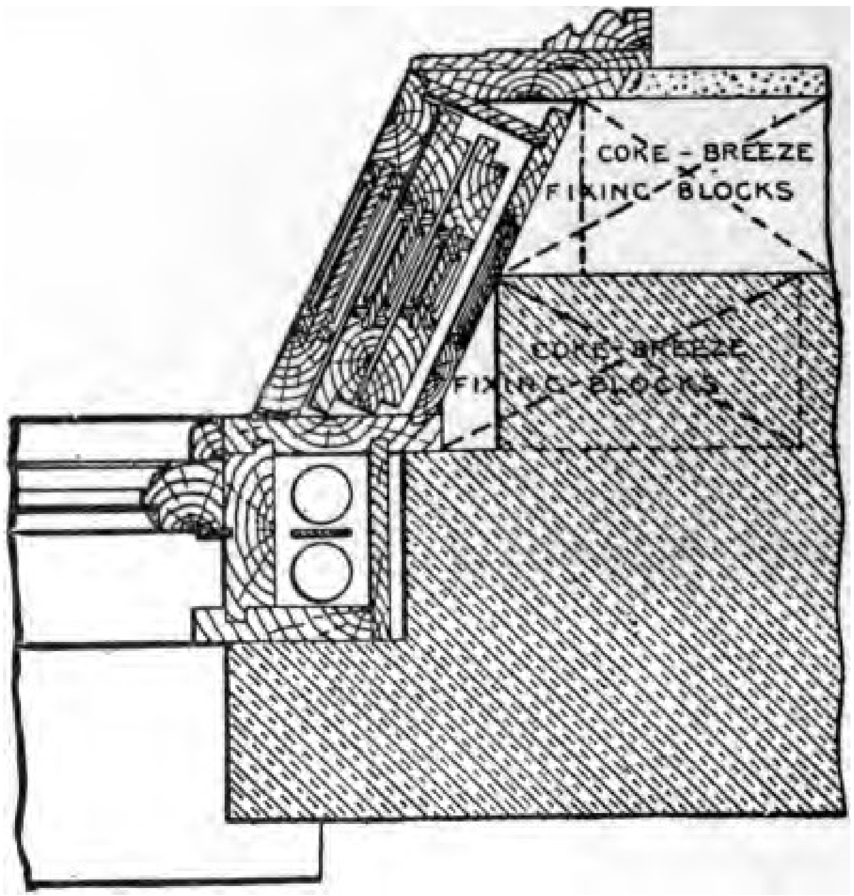


Fig. 96.

plaster, and are usually splayed so that they will not interfere with the admission of light. The inside of window and door openings usually are finished similarly; thus, the **Architrave, or Band Moulding,** which is secured to the edge of the linings and to rough wooden **Grounds** is fixed along the sides and top in both cases.

The bottom of the window opening is finished with a **Window Board** which is tongued into the sill of the frame. The board is about $1\frac{1}{4}$ inches thick, and is made wide enough to project beyond the surface of the plaster for a distance of about 18 inches. The projecting edge is nosed (rounded) or moulded. It is longer than the opening, to allow the lower ends of the architrave to rest upon it.

When the walls are thick, the linings are often framed and panelled. Such linings may terminate on a window board at the sill level, or the inner side of the wall may be recessed below the sill level and the linings carried to the floor as shown in Fig. 96.

Window Shutters. Although not used to the same extent as formerly, wooden window shutters are fitted occasionally to close up the window opening. Window shutters, which are arranged generally on the inner side of the window, may be hinged as box shutters, or may be vertically sliding shutters.

Box Shutters consist of a number of leaves or narrow frames which are rebated and hinged together, an equal number being on each side of the window opening, the outer ones on each side being hung to the window frame. When closed they together fill the width of the window-space, and when open they fold behind each other so that the front one forms the jamb lining of the window frame. If the walls are thick, the shutters can be arranged to fold in the thickness of the wall; if the wall is a thin one it is necessary to construct projecting boxes into which the shutters fold. The nature of the framing of the shutters depends upon the surrounding work; it is usual to have the outer surface framed and moulded, and the inside finished bead-flush. The arrangement of box shutters requires that the shutters on the same side shall vary in width so that they will fold into the boxes on each side of the window, the outermost shutter (which is the widest) then acting as the window lining. Fig. 96 shows a horizontal section through one side of a window, showing hinged shutters folding so that a splayed lining is obtained. Fig. 97 shows hinged shutters consisting of one

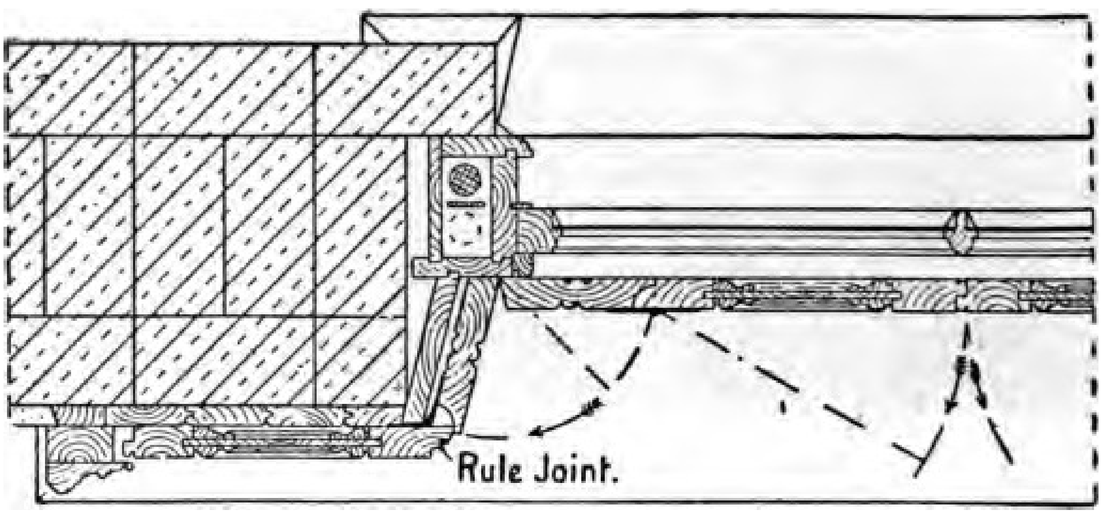


Fig. 97.

narrow and one wide shutter on each side of the opening. This arrangement is suitable for a thin wall, where it is undesirable to have boxes for the shutters projecting beyond the face of the wall. For hanging window shutters it is usual to use back-flap hinges; the joint at the corner of the shutters in Fig. 97 is named a rule joint.

Sliding Shutters, working in vertical grooves and balanced by weights, are sometimes used. They require that the wall under the window sill shall be recessed; the floor also often needs trimming to allow space for them to slide sufficiently low. To hide the grooves in which the shutters slide, thin vertical flaps are hung to the window frame, and the window board is also hinged at the front edge to allow the shutters to slide be-

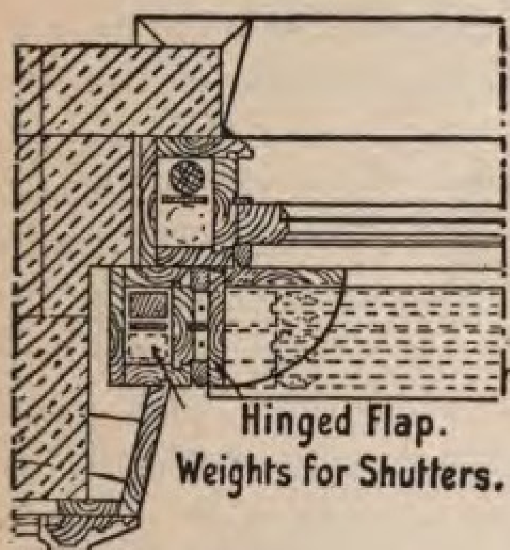


Fig. 99.

low the sill. Figs. 98 and 99 are sections of vertical sliding shutters.

Hinged Skylights.

Skylights which are hinged to open are fitted upon the upper edge of a **Curb** or frame fixed in the plane of the roof, the common rafters

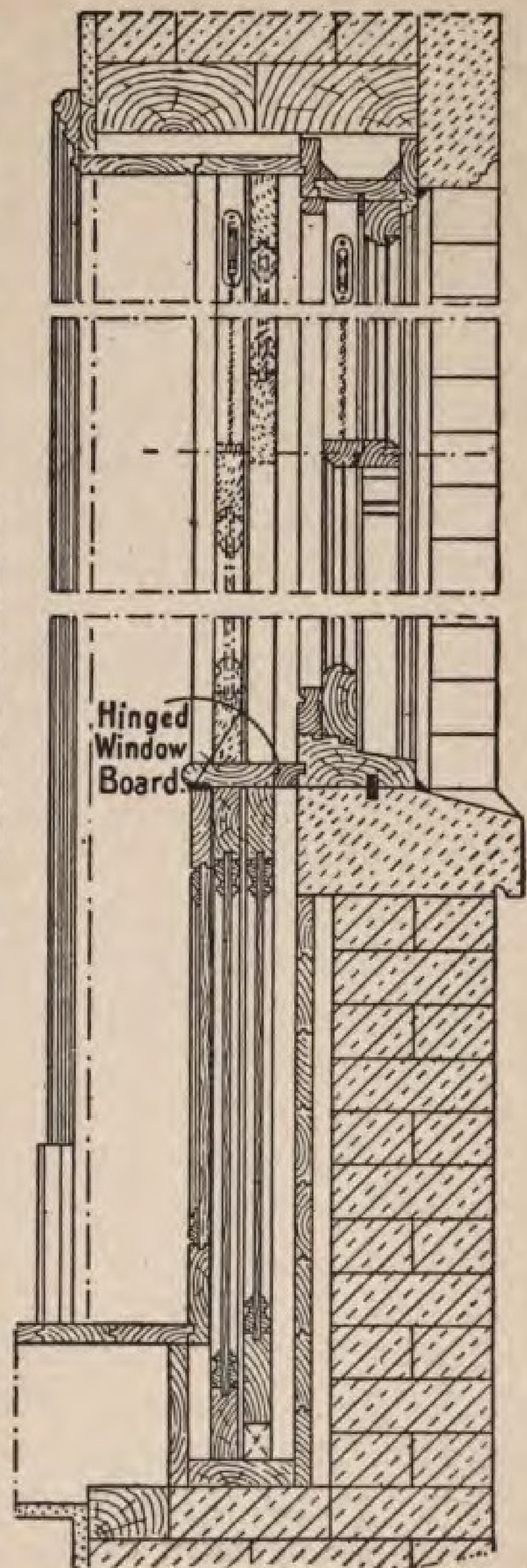


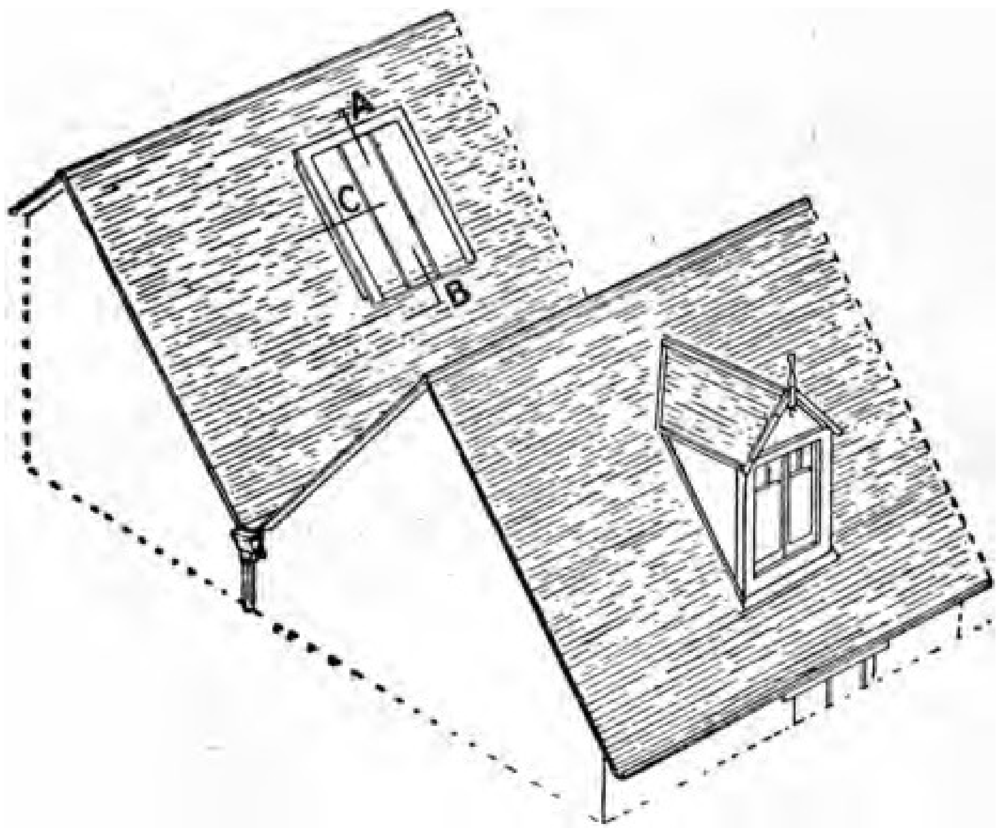
Fig. 98.

being "trimmed" to the required size to receive the curb. The curb is made from material 1¼ to 2 inches thick, and of width such that its upper edge stands from 4 to 6 inches above the plane of the roof. The **Angle Joints** of the curb may be dovetailed or tongued and nailed. The **Sash Frame** rests upon the upper edge of the curb; it is from 2 to 2½ inches thick, and consists of stiles and top rail of the same thickness, and a bottom rail which is thinner than the stiles by the depth of the rebate. Bars are inserted in the direction of the slope of the roof, and the butt hinges used for hanging the sash are invariably fixed as the under side of the top rail.

The joints between the curb and the roofing slates or tiles are made weatherproof with sheet lead. At the upper end—the back of the curb—a small **Lead Gutter** is formed, with the lead going underneath the slates and overlapping the upper edge of the curb. The sides of the curb may be flashed with soakers—short lengths of sheet lead which are worked in between the slates—or the joint may be made with one strip of lead forming a small gutter down the side of the curb. In either case the lead overlaps the upper edge of the curb. At the lower end of the curb, the lead overlaps the slates. To prevent water from rising between the glass and the upper side of the bottom rail, sinkings are cut into the rail. (See Fig. 100).

Dormer Windows. Instead of having the light in or parallel to the plane of the roof, it affords a more artistic treatment of the roof, and often gives a better result in lighting, if the window is fixed vertically. The general arrangement of the framing, as well as of the sashes, depends upon the kind of roof, the width of the window required, and the general style of architecture of the building.

The construction of a dormer window necessitates trimming of the rafters, and the arrangement of projecting framework, the front of which consists of **Cor-**



Sketch of part of the Roof of a Building, showing a Hinged Skylight and a Dormer Window.

Fig. 100.

ner Posts and **Crossrails**—rebated to receive hinged sashes—which are connected to the main roof by other crossrails and by **Braces.** This framework is surmounted by a **Roof** which may be either ridged, or curved outline, or flat. By arranging a ridged roof to overhang, and adding suitable **Barge Boards** and **Finial** (Fig. 101) a dormer window may be made to improve the general appearance of the roof of a building. The sides of the dormer may be either boarded and covered with the same kind of material as the roof, or they may be framed for sidelights.

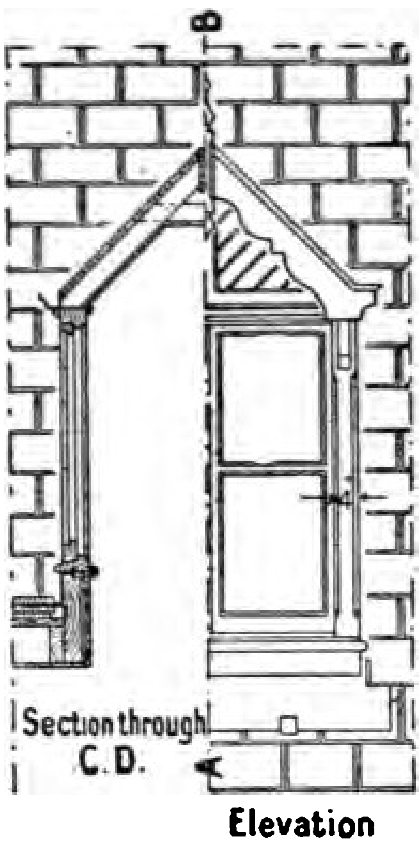


Fig. 101.

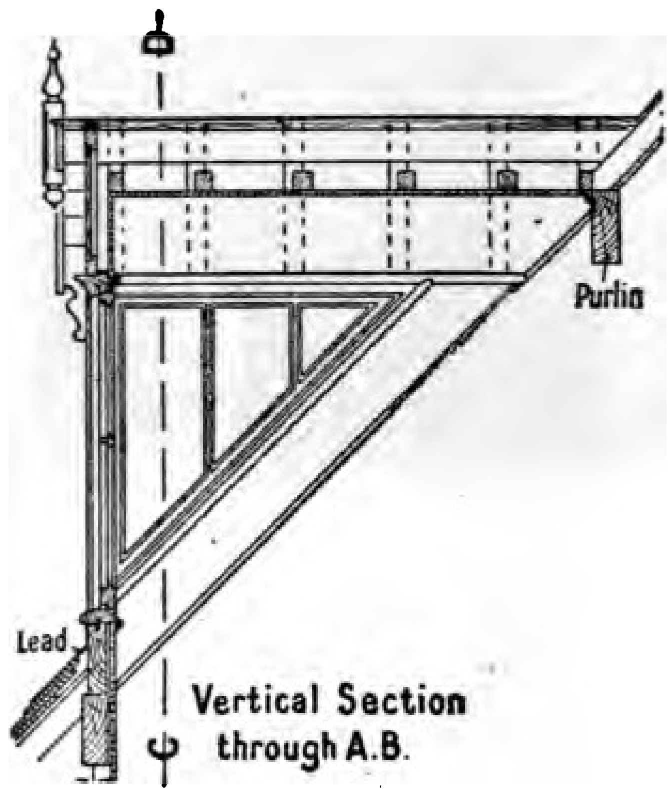


Fig. 102.

As dormer windows are generally in exposed positions, and the sashes are arranged as casements to open, their efficiency depends largely upon the perfection of the joints between the sashes and the frame. It ought to be mentioned however, that with sashes hung folding, semicircular tongues on their hanging stiles are by far the best. Figs. 101 and 102 give the details of a dormer window, with sidelights, fixed in a roof of ordinary pitch. The sashes, which are hung folding, open inwards. The roof may be boarded and covered with lead, or it may be covered with slates or tiles. The joints between the roofing slates of the main roof, and the roof and sides of the dormer, are made weather-proof with sheet lead flashings. Figs. 103 and 104 show a dormer window fixed in a Mansard roof; in this example there are no side lights.

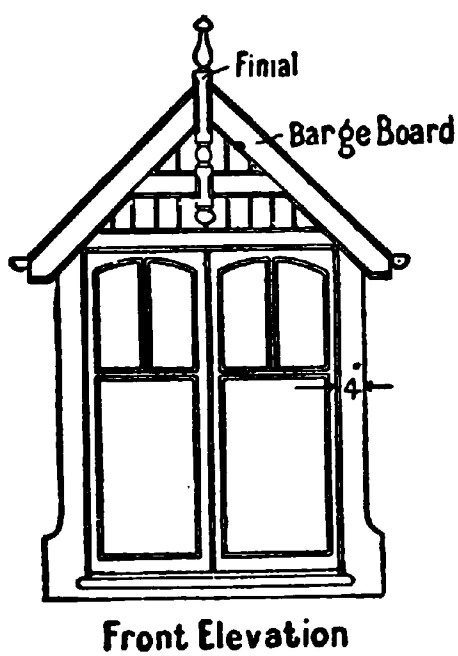


Fig. 103.

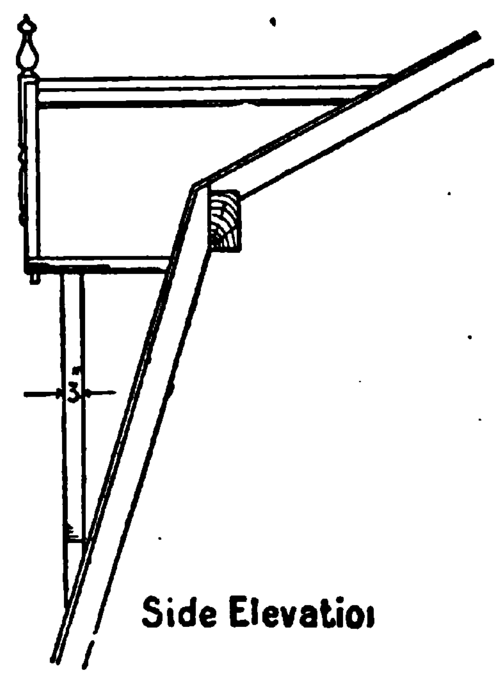


Fig. 104.

Large Skylights and Lantern Lights. For lighting the well of a large staircase, or a room which, for some reason, cannot be lighted with side windows, specially large skylights are often necessary. These are of a more elaborate construction than the skylights already described; they vary considerably in size, shape and design; the plan may be rectangular, polygonal, circular, or elliptical, and the outline may be pyramidal, conical, or spherical. The framework may be of either wood or iron. To support such a skylight, a strong wooden curb is framed into the roof, and projects from 6 to 9 inches above the roof surface. The joints between the curb and the roof are made watertight with sheet lead. The framework of the skylight may consist of rebated quartering; with separate lights which fit into the rebates of the framing: or the sashes them-

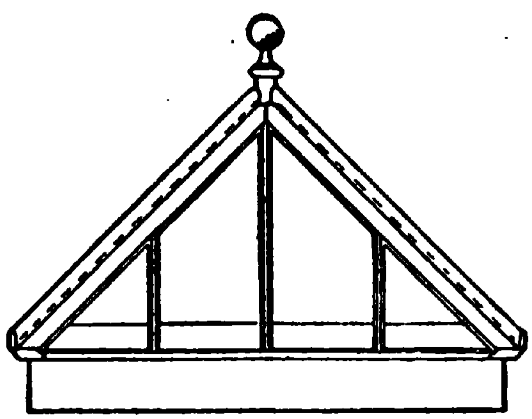


Fig. 105.

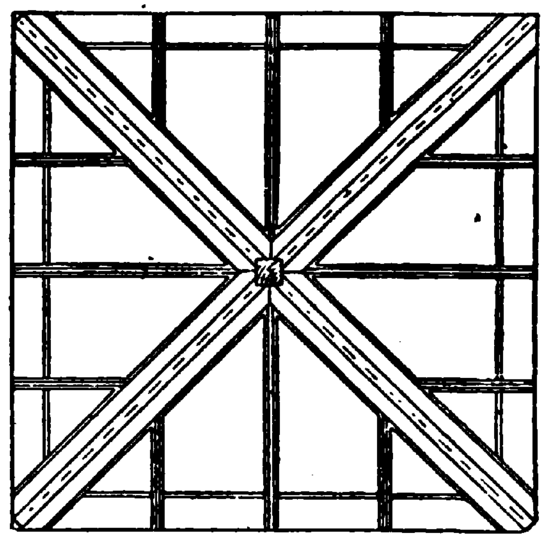


Fig. 106.

selves may be constructed with strong angle stiles, which are mitred together, and provided with either a hardwood tongue inserted in the joint, or with a wooden roll on the top to keep out the water.

With skylights of this description, channels for condensed water should always be provided. These are placed at the upper inner edge of the curb, the remainder of the inside face of the curb being covered by either panelled framing or match boarding.

Figs. 105 and 106 give details of a skylight having the form of a square pyramid. In this example the four triangular lights are mitred at the angles, and have wooden rolls over the joints. Figs. 107 and 108 show elevation and part plan of a skylight with a curved roof surface.

A Lantern Light differs from the skylights just described in having, in addition, vertical **Sidelights.** The sidelights consist of sashes, which, by being hinged or

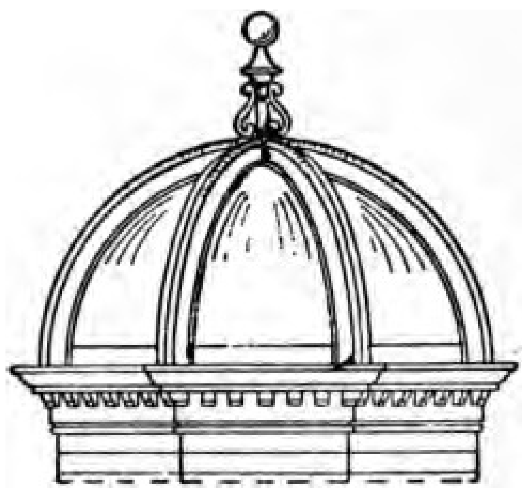


Fig. 107.

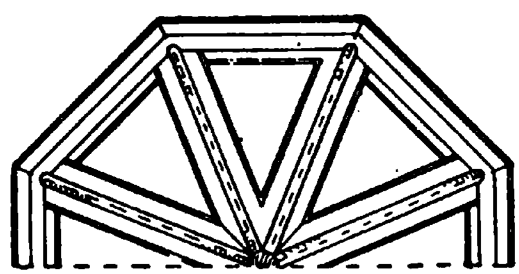


Fig. 108.

pivoted, are often available for ventilation. As they are in exposed positions, the greatest care is required in order to obtain watertight joints. When the sidelights are hinged on the bottom rail, they open inwards; when on the top rail they open outwards. When they are hung on pivots, the pivots are fixed slightly above the middle of the sash. Figs. 109 and 112 show details of a rectangular opening surmounted by a lantern light which is hipped at both ends, and has sidelights arranged to open inwards.

The construction of skylights and lantern lights affords good examples of the application of geometry to practical work as described in previous pages. When

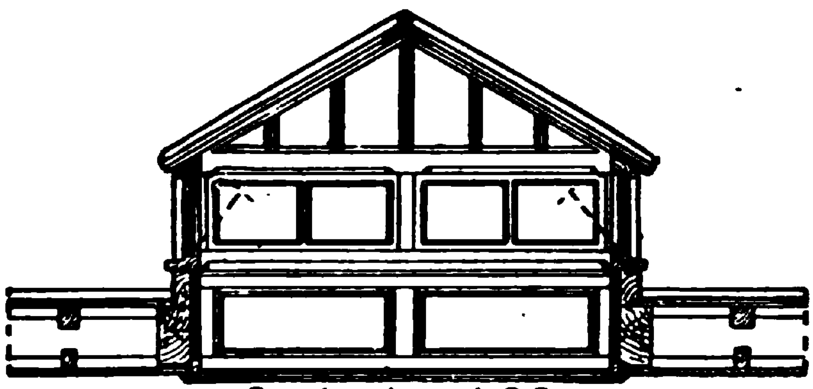
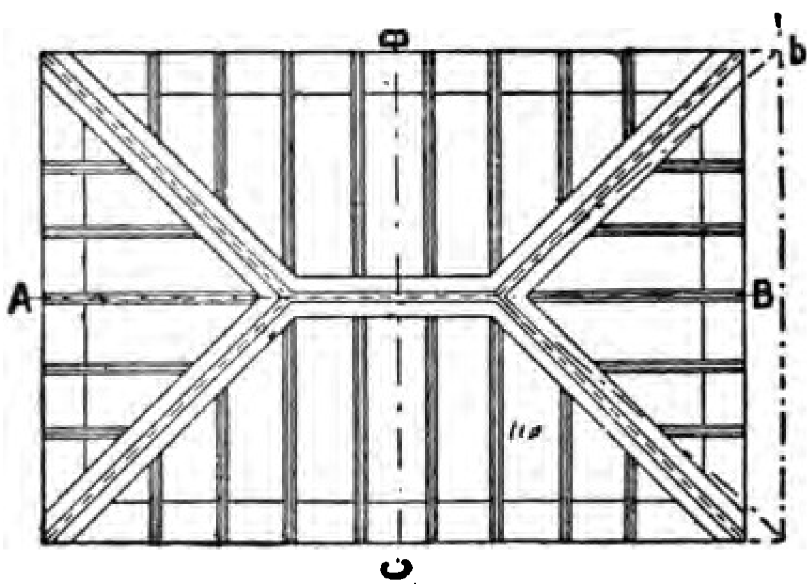


Fig. 109.

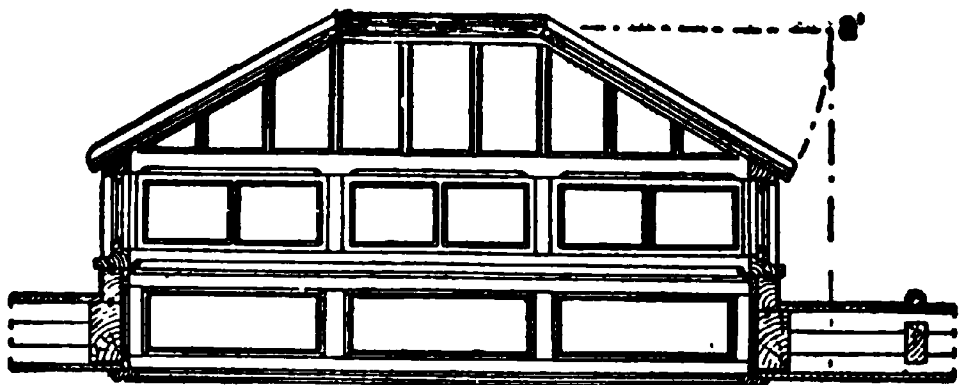
the roof-lights are pyramidal as shown in Figs. 106 and 110, and a separate frame is constructed as shown in



Plan

Fig. 110.

Fig. 110, the methods of obtaining the lengths and bevels of the hip rafters are similar to those described for getting hip rafters. When the roof-lights mitre against one another, the sizes of the lights and the bevels of the angle stiles which mitre together are obtained as shown at X in Fig. 112. With lights of curved



Section through A.B

Fig. 111.

outline, the shapes of the hip rafters or angle-stiles, as well as the developed surfaces, are obtained as explained before.

Lay-Lights. At the ceiling level of roof-lights used for staircase wells, or in similar positions, it is often

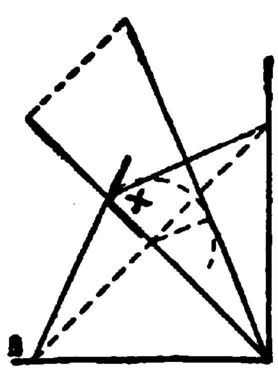


Fig. 112.

Fig. 113.

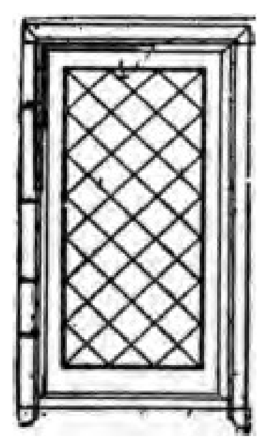


Fig. 114.

considered advisable, for the sake of appearance, to have a horizontal second light called a lay-light. This consists of a sash—or if the space is large, a number of sashes—fixed into frames in the ceiling. The chief feature of lay-lights is in the attempt at decoration by arranging the bars in some ornamental design (Figs. 113 and 114). The lay-lights are often glazed with ornamental glass, which, although it improves the appearance, diminishes the amount of light transmitted.

Greenhouses and Conservatories. In this type of building which is largely constructed of wood and glass, the framework is usually of moulded and rebated quartering, with side sashes fixed in the rebates. As in the case of skylights, the roof-lights, which in this case reach from the ridge to the eaves, have no crossbars, since these would impede the flow of water running down the slope of the roof. Care should be taken to have the bars strong enough to carry the glass without sagging; and it is well to remember that when a roof is of flat pitch a heavy snowstorm will throw a large additional weight upon it, while with a steep roof the wind has much power. The distance apart of the bars which carry the glass ranges from 12 to 18 inches, and the lengths of the sheets of glass should be as great as possible, so as to diminish the number of cross-joints, since these allow of accumulations of dirt which cannot be removed easily. These roof-lights are constructed in exactly the same manner as skylights; they are, however, often much larger, and require to be thicker, unless purlins are placed to support them. When, as is often the case, part of the roof-light is made to open, this part—often a narrow strip at the highest part of the roof (Fig. 115)—is made as a separate light, which overlaps the upper edge of the fixed lower light. Additional ventilation is secured by arranging the side sashes to open.

The above description is intended merely to outline the broad principles of the construction of conservatories, but it should be remembered that the details, while conforming to casement and roof-light construction generally, lend themselves to considerable variation in design and arrangement.

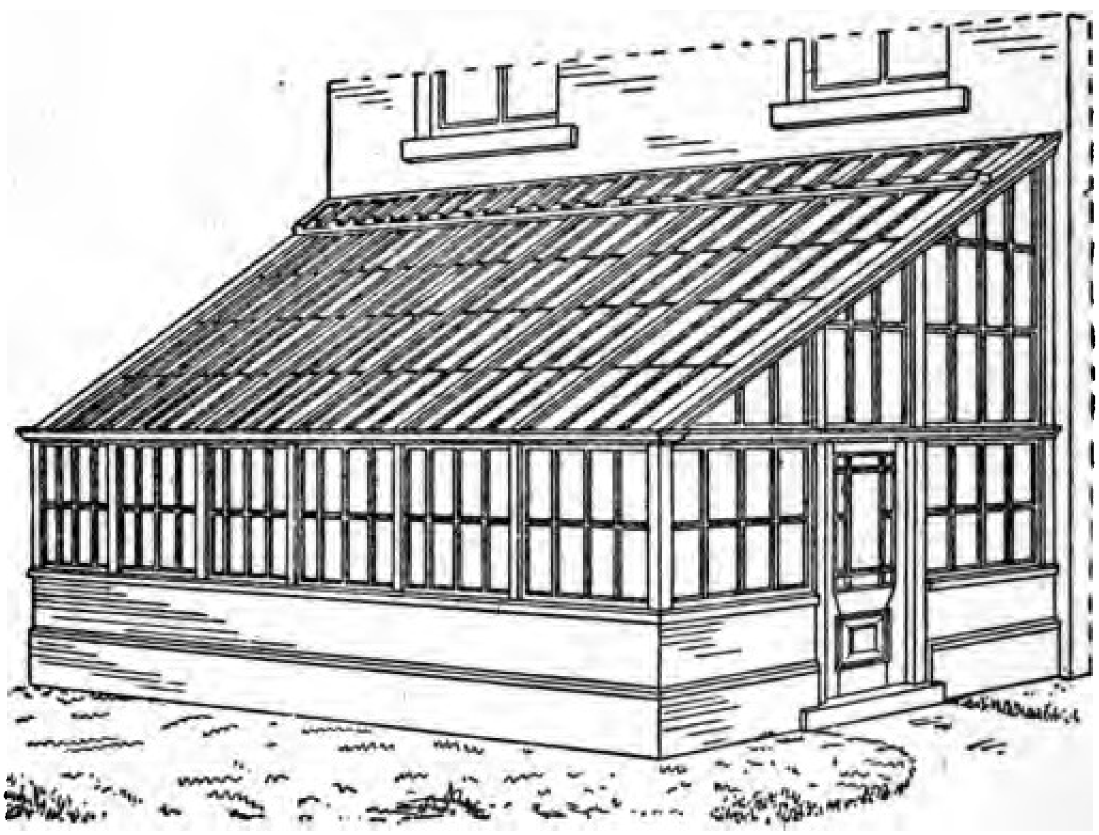


Fig. 115.

A Bay Window with solid frame and casement lights is shown in Fig. 116 to 118. Two methods of fitting such a window, with folding shutters, are given in this plan. In the half plan at C, the shutters fold into a boxing projecting into the room, and at D they fold back upon the face of the wall, which is splayed to receive them. The sills of the frame are mitred at the angles, the joint cross tongued and fixed with a handrail bolt, which should be painted with red lead before insertion. The joints in the head are halved together, the mullions stub tenoned and fixed with

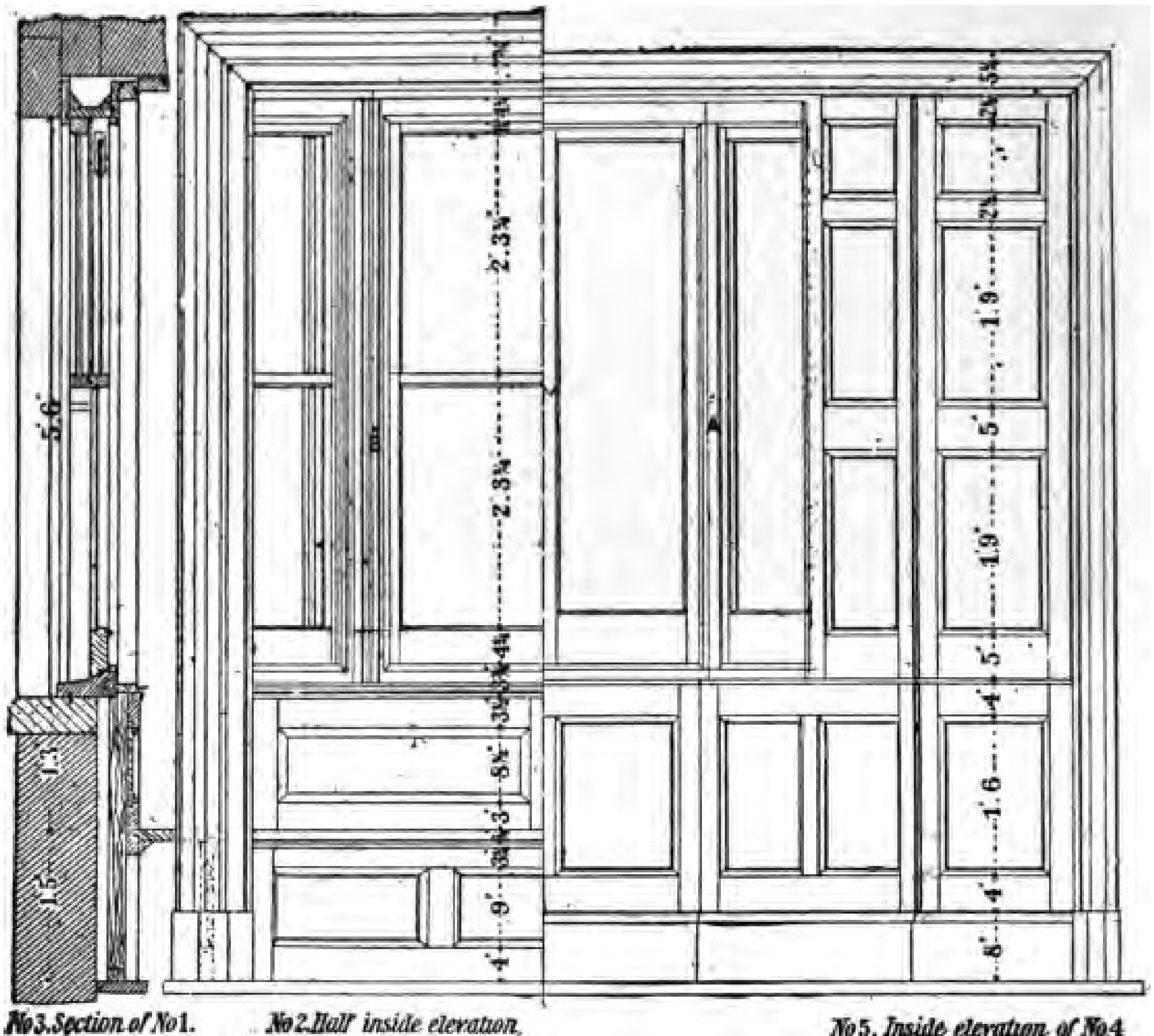


Fig. 116.

coach screws. The jambs are tenoned and wedged into the head and sill. The transom tenoned into the jambs and mullions, and secured with bolts. The mullions may be worked in one piece as shown at D, or built up as at C, and tongued and screwed together.

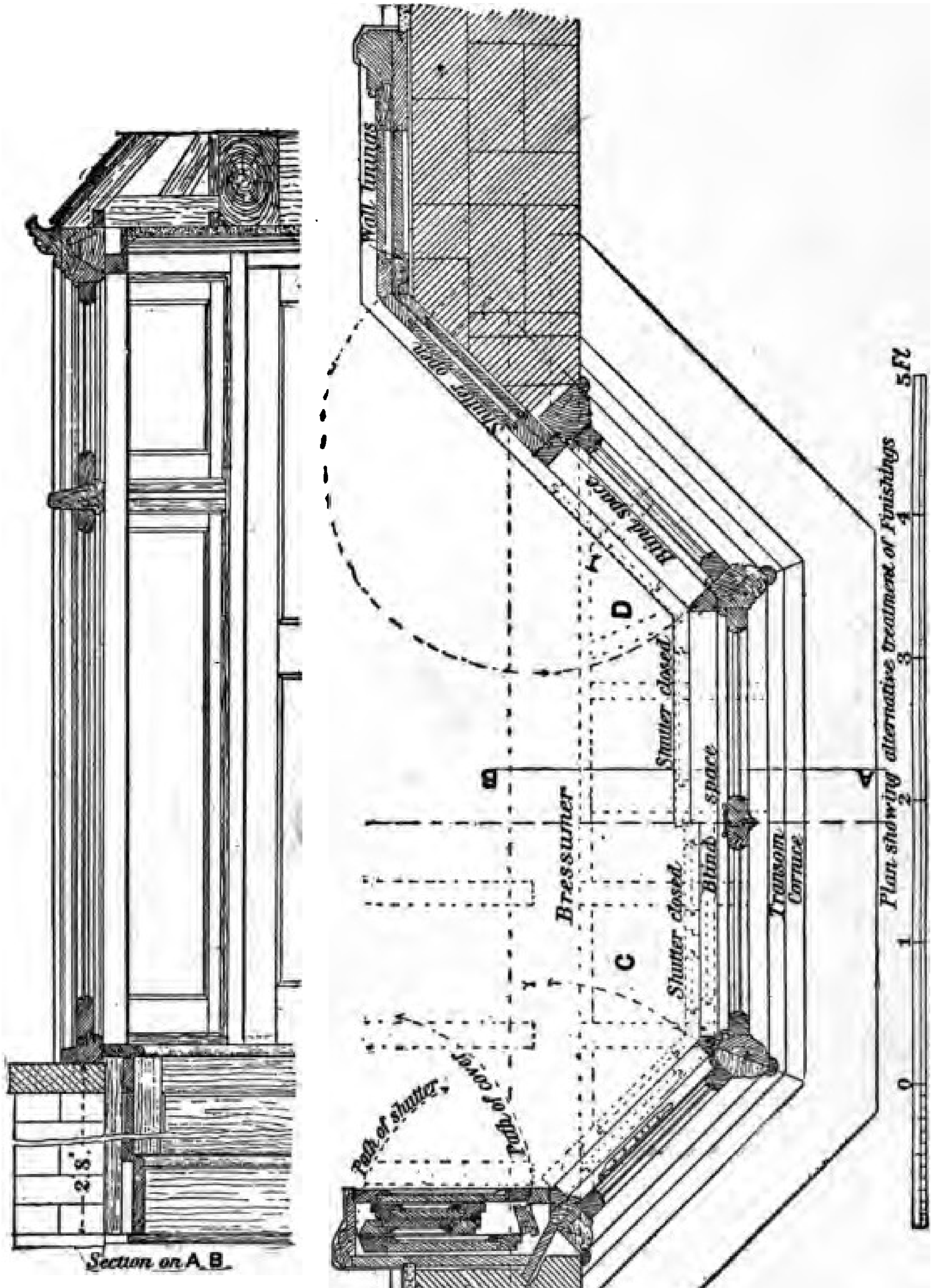

Section on A.B.
2' 8"
Wall linings
Shutter space
Blind space
D
Shutter closed
B
Bressumer
Shutter closed
Blind space
C
Transom
Cornice
A
Path of shutter
Path of cover
Plan showing alternative treatment of Finishings
0
1
2
3
4
5 Ft

Fig. 117.

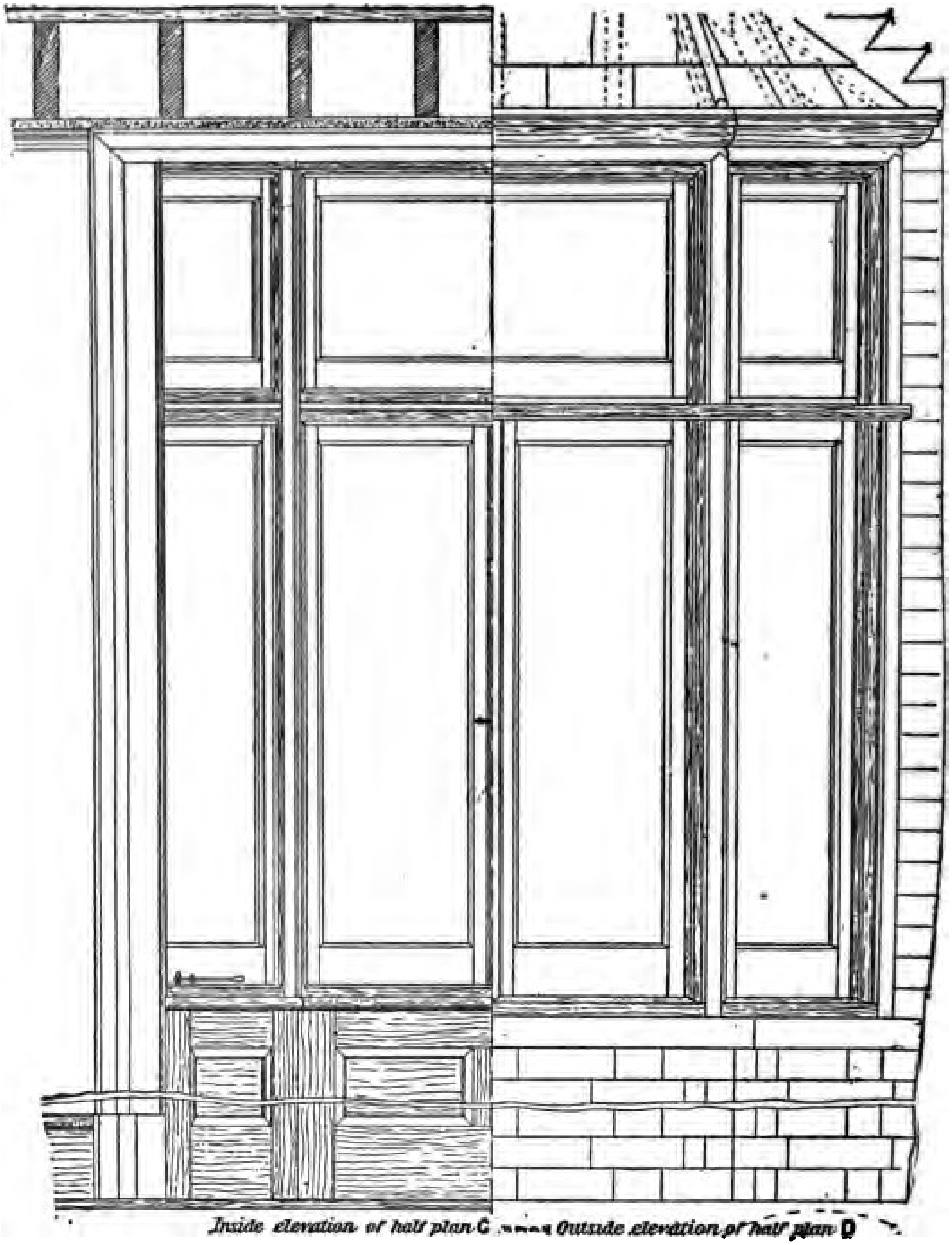
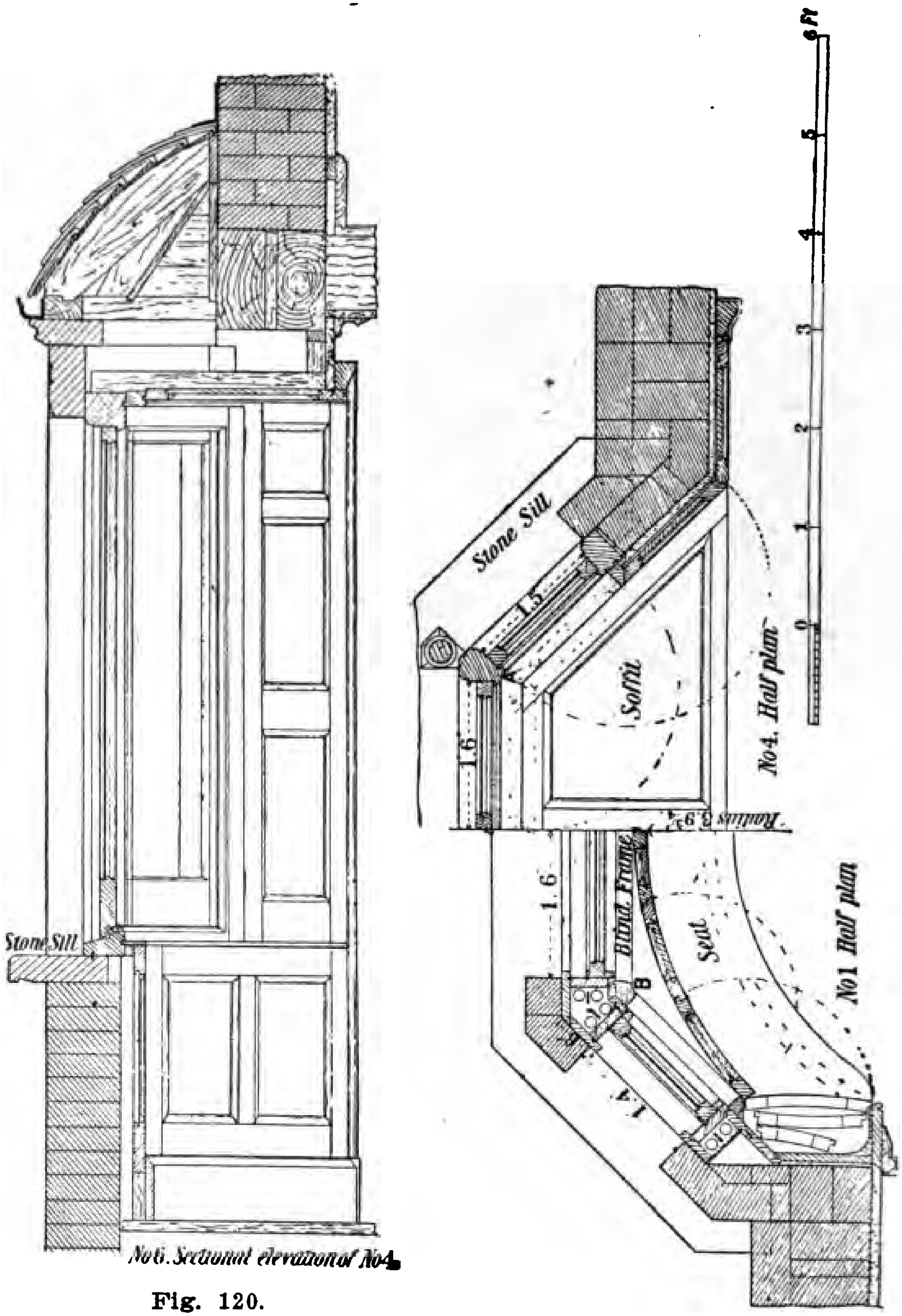


Fig. 119.

A Cased Frame Bay Window is shown in the half plan, half inside elevation, and central vertical section, Nos. 1, 2 and 3, Fig. 119. This window is composed of three ordinary sash frames, the sills connected at the angles either by halving and screwing or by mitreing them and fastening the joint with a hand rail bolt. The heads are tied together with a short piece of 1 in. stuff screwed across the top of the joints, and the joints in the linings are covered by the mullions of the blind frame B. The latter, made 2 in. wide, forms an enclosure for venetian blinds. Boxings are formed in the elbows between the sash frames and the interior face of the wall, the front of the opening being finished off with a moulded ground and architrave. These form receptacles for the folding shutters, which are curved in plan, and when opened out convert the octagonal bay into a segmental niche. The window back and the seat beneath are also curved to parallel sweeps. The window board also follows the sweep, and is rebated to receive the shutters, a shaped bead being fixed on the soffit to form a stop at the top. Nos. 4, 5 and 6 on the same plate illustrate another method of finishing a bay window. In this case the frame is solid, and is fitted with outward opening casement lights. A blind frame is provided, and the shutters fold on to the face of the jamb and wall, the outer edges passing behind the rebated edges of the architrave; the latter is continued down to the floor, and elbow linings to correspond with the shutters are fitted beneath the window board. These are fitted to the window back in the manner indicated by the dotted lines in the plan No. 4. The section No. 6 shows the treatment of the roof of the bay, which is segmental in section and covered with shaped pan-tiles. The ribs, which are elliptic at the hips, are notched into a wall plate resting on the



No 6. Sectional elevation of No 4.

Fig. 120.

stone cornice, and are nailed at the top into a shaped rib fixed on the face of the wall; the ribs are covered with weather boarding, which affords a good fixing for the tiles. The wall is carried by a breastsummer formed of two 12 in. by 6 in. balks bolted together, with spacing fillets between, and the soffit is carried by three brackets fixed to the breastsummer and the head of the window frame.

A Pivoted Light in a solid frame is shown in elevation in Fig. 122 and section Fig. 123. These are used chiefly in warehouses, lanterns, and other inaccessible positions, the lights being opened and closed either by cords and pulleys or by metal gearing. For small lights the frames are usually made out of 4½ in. stuff by 2 or 3 in. wide. Small lights are pivoted horizontally, large ones vertically. The pivot should be fixed to the frame, not the sash, and from ½ in. to 1½ in. above the centre, according to the weight of the bottom rail. The lower part of the sash should exceed in weight the upper part, just sufficient to keep it closed; its action may be easily demonstrated by inserting two bradawls in the stiles, and balancing them on the fingers. The sash is inserted and removed from the frame either by means of plough grooves in the edges of the stiles, as shown by the dotted lines in Fig. 122, or by cutting a notch through the face of the stile for the passage of the pin, which is concealed when in use by the guard beads. This latter is the better method, as it does not reduce the strength of the sash, as does the former, by cutting away the wedging. The stop beads at the sides are cut in two, one part being fixed to the frame, the other to the sash. Their joints can be at any angle

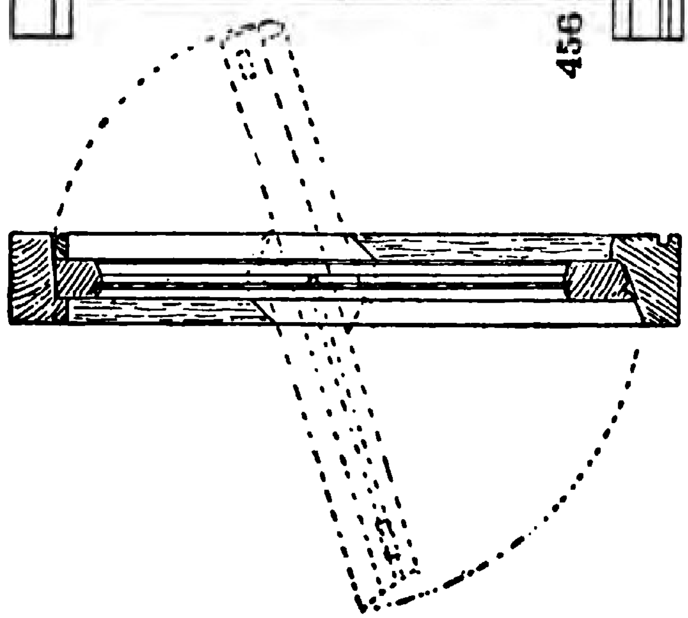


Fig. 122.

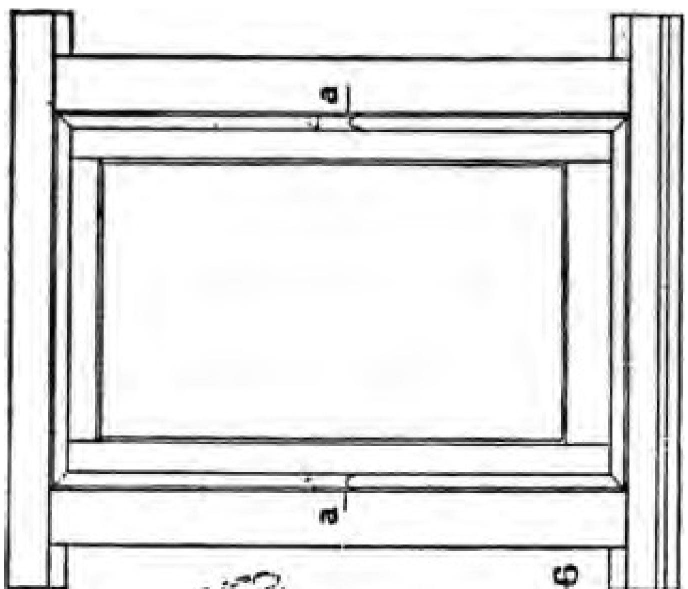
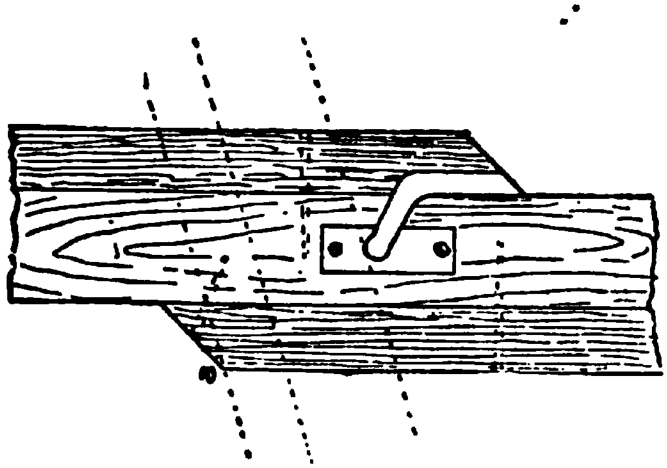


Fig. 123.

Elevation and Section of a Pivot Hung Sash.



Method of Finding Cuts in Bead

Fig. 124.

greater than that made by a line tangent to the sweep at the point of intersection a Fig. 124, but for the purpose of using the Mitre Block, they are generally made at an angle of 45 deg. A curved joint has no advantage over a straight one, except in being more expensive.

To Hang the Sash. Insert the pivots in the frame quite level, but do not screw them. Then with the try square resting on the top of the pins, square lines across the jambs. Then remove the pivots and insert the sash, which should be fitted rather tightly at first, and square the lines on to the sash. Return these on the edges, and keep the edge of the hole in the socket plate to the line, and the plate itself in the middle of the thickness. After the socket is sunk in, and the notches cut, test the sash and correct the joints, which should be a bare ⅛ in. clear all round.

To Find the Position to Cut the Beads. After fitting them round, remove them and open the sash to the desired angle, which should be less than a right angle, so that the water may be thrown off. Lay the beads upon the sash upside down for convenience of marking, and draw a line along their edge upon the jambs at the point where the line meets the faces of the frame; square over lines as at a a Fig. 123; the position is shown in Fig. 122, the outer dotted lines indicating the beads. Next replace the beads and transfer the marks to them, cutting them off the mitre block (remember that the mark is the longest point of the mitre). The upper portions outside and the lower inside are fixed to the frame, and these are shaded in the drawings. The remainder of the beads are fixed to the sash.

The above describes the method when the sash is grooved. Where the beads are slotted, a variation must be made with reference to the top cut (see Fig.

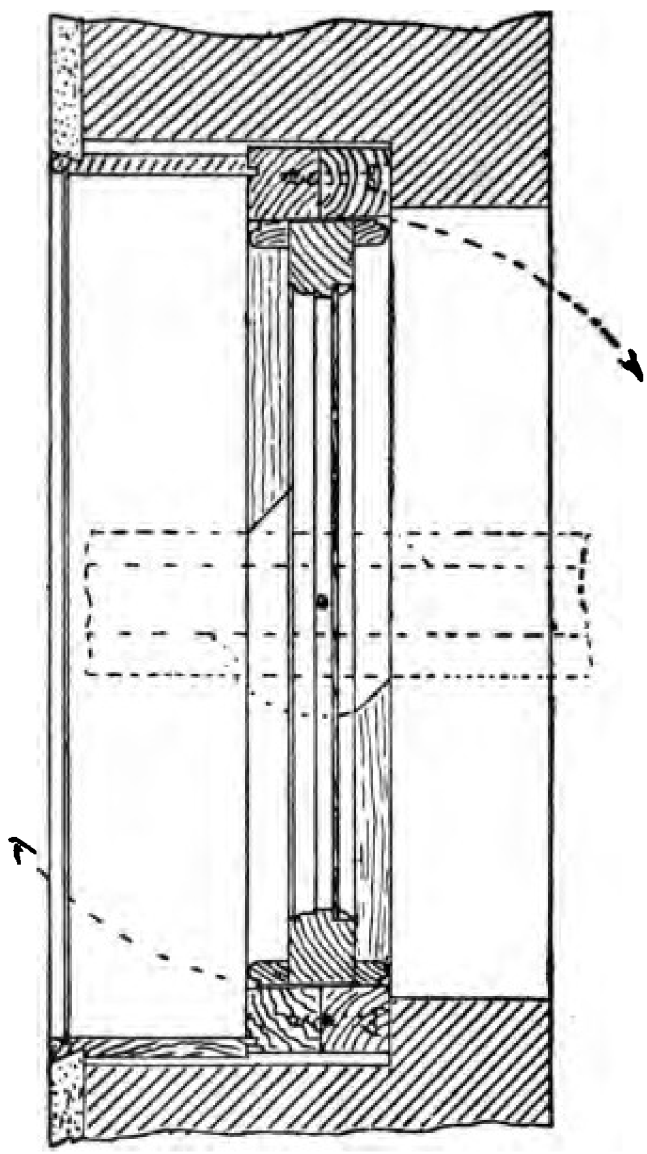


Fig. 125.

124). In this case the sash must be drawn out and rested upon the pin, then the bead laid on it and marked as before, the intersection a giving the mitre point.

A Bull's-Eye Frame with a pivoted sash is shown in Figs. 125 and 126 and enlarged detail of the joint. This frame is built up in two thicknesses, glued and screwed together, each ring being in three pieces breaking joint. The beads may be steamed and bent round, or worked on the edge of a board that has been cut to the sweep,

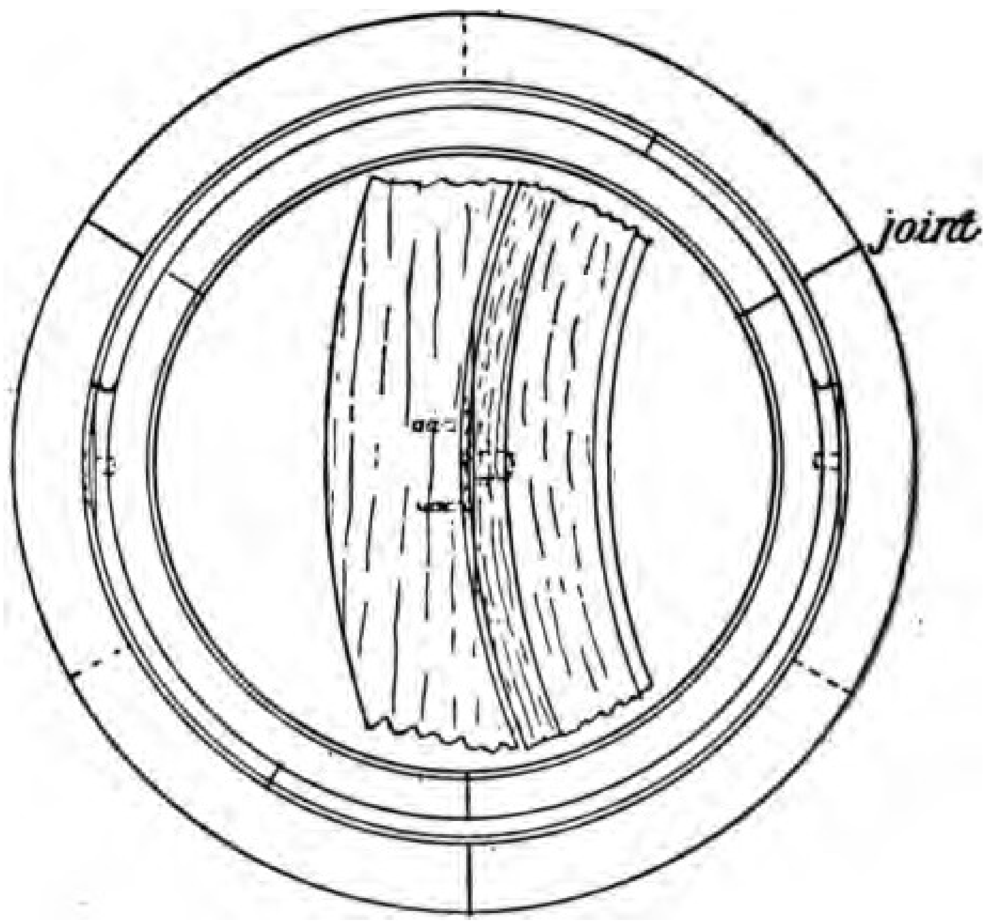


Fig. 126.

and cut off in two lengths. The sash is made in three pieces, with butt radial joints bolted together. To enable the sash to open, a plane surface must be provided at the centre, equal to the thickness of the sash and beads, as shown by the dotted lines. Having fitted the sash in, and the beads around each side, brad them temporarily to the sash, lay a straight-edge across it parallel with the centre, and square up with the set square a line at each side equal in length to the thickness, then cut the pieces so marked off with a fine saw, both beads and sash, and glue them to the centre of the frame, and fix the pivots to these frames and proceed as in a square frame.

Fig. 127.

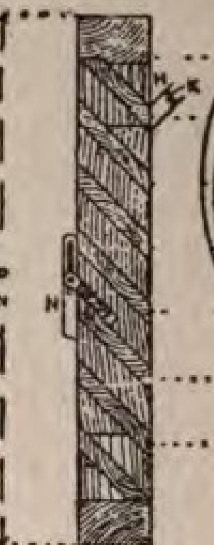


Fig. 128.

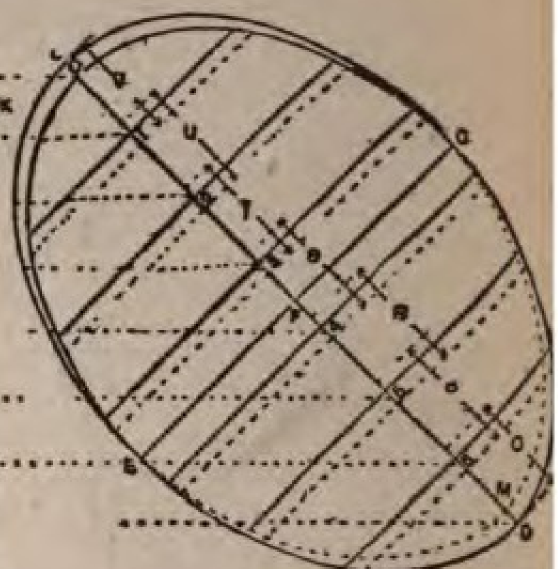


Fig. 129.

Laying Out a Circular Louvre: Suppose the frame of the louvre to be formed of four pieces, as shown at Fig. 127. These sections may be formed of one thick piece of plank, or may be built up of several thicknesses. If of one thickness, the joints may be held together with handrail screws, or dowelled and keyed. If of several thicknesses then the joints can be broken or overlapped, and the pieces either screwed or nailed together. To set out the louvre boards, make a sketch of the whole thing, full size, as shown at Fig. 127, then set up the section or side as shown at Fig. 128, then project from the quick of the bead, then draw a line C D, Fig. 129, parallel to the inclination of bevel of the louvre boards. This will give the major axis of an ellipse. Bisect this line and draw E, F, G, at right angles to C, D, making E, F, and F G each, equal to half the radius from the centre of the frame to the quick of the bead. The ellipse C, E, D, G, may be struck by any of the methods shown. As, of course, all the louvre boards are at one angle, each forms a portion of the same ellipse. From the centre of the front edge of each louvre board, project across to the major axis as shown. Now from any of the louvre boards set off, at right angles, lines from the upper and lower surfaces as shown at H, Fig. 128, then the distance K, is the amount of projection of the lower surface in front of the upper. Taking half the distance of K, measure it off on each side of the centres 1, 2, 3, 4, 5, 6. Fig. 129, then through the points last obtained draw lines parallel with the minor axis, the upper line representing the top front board or arris of the boards, and the dotted line the bottom arris. Now measure the distances K, from C to L, and from D to M, and construct the ellipse for the underside of the boards. The following will be found a simple method for making out each louvre board: Cut a thin mould equal to one-quarter of the ellipse, the edges of the louvre boards having been planed to proper bevel, as shown at N, Fig. 128, square the centre line across the piece that has to form the louvre board and lay it on the setting out at Fig. 129, the centre line of course corresponding with C D. The quarter mould which was prepared can now be laid on the louvre board, with its curve standing directly over the development, Fig. 129, the face side of the material being of course the side marked out.

The curves for the underside may be marked off on the arrises direct from the development, and the mould then applied to the other side, taking care to adjust it in the right position and to the marks made. The ends should be sawed and turned to the lines. The next proceeding will be to set out the frame for the grooves; these are represented in the conventional sketches, 4, 5 and 6. It will be noticed from Fig. 132 that the bottom louvre board is not grooved in all round; this is a bet-

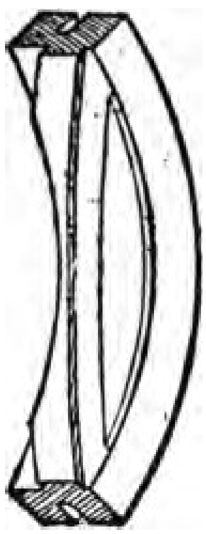


Fig. 130.


Fig. 131.

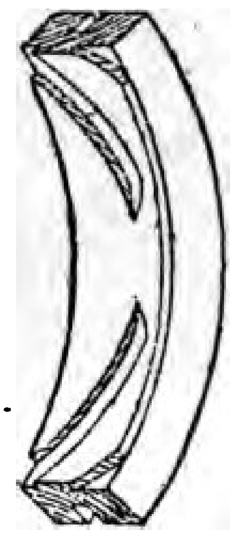


Fig. 132.

ter method than bringing it out to the front and thus destroying a part of the margin bead of the frame. The cutting of the grooves and the fitting in of the louvre boards requires careful working in order to get good joints. It must be clearly understood that Fig. 129 is not a full development of all the boards edge to edge, as that could not be represented in this space, but enough is shown to give a clear idea of how the lines are obtained. The full breadth of each is represented by the dimension lines O, P, R, S, T, U, V, and from this all the other is easy.

The Construction of Doors. Doors are named in accordance with their modes of construction, position, style or the general arrangement of their parts, and also the method in which they are hung, as **Battened,**

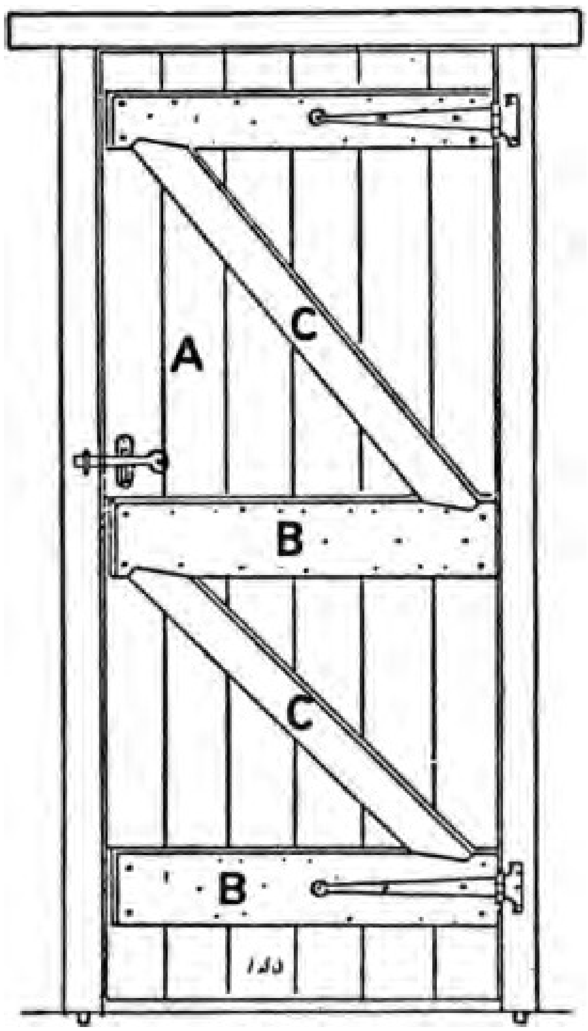


Fig. 133.

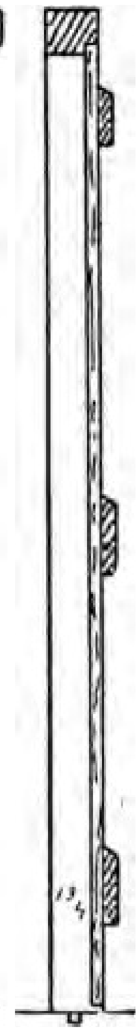


Fig. 134.

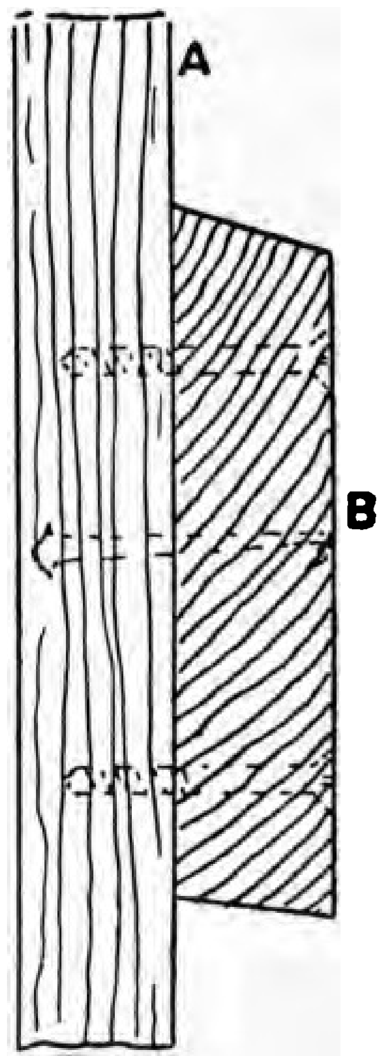


Fig. 135.

Battened Framed and Braced, Panelled or Framed, Entrance, Vestibule, Screen, Sash, Diminished Stile, Double Margin, Gothic, Dwarf, Folding, Swing, Jib, Warehouse Hung, &c. The essentials in the design and construction of doors are, for the first, that they shall have a due proportion to the building or place they have to occupy and be suitably ornamented; in the second, that their surfaces shall remain true and their parts be so arranged and connected that their shape will be unalterable by the strains of usage and the effects of weather. The various example illustrated will indicate the points to be considered in designing doors for sundry situations, and the methods of construction herein described will supply the necessary information to meet the constructive requirements.

Battened and **Battened Framed** and **Braced Doors** are shown in Figs. 133 and 134. These doors are suitable for positions where one or both sides are exposed to the weather. Little or no attempt is made to ornament them—economy of cost, strength and utility being the chief requirements of this class of door, which are fitted to coach-houses, W. C.'s and outhouses generally.

The plain **Battened Door** (Fig. 133) is composed of battens A, from ¾ to 1¼ in. thick, ploughed and tongued in the joints with straight tongues which should be painted before insertion, nailed to three ledges, B from 1 in. to 1¼ in. thick, usually with wrought nails long enough to come through and be clinched on the back side. The ends of the ledges are better fixed with screws, and their top edges as well as those of the braces C should be bevelled to throw off the water, as shown in the detail, Fig. 135. The lower edges may be throated or bevelled under, as shown. The braces should be placed so that their lower ends are at the hanging side, for if in the opposite direction, they will be useless to prevent the door racking. Their ends should be notched into the ledges about 1 in.

deep and 1½ in. from the ends, with the abutment square to the pitch of the brace. Narrow doors are sometimes made without braces, but they seldom keep "square." These doors are hung with wrought-iron strap hinges called cross garnets, which should be fixed on or opposite the battens, whether placed on the face or back of the door.

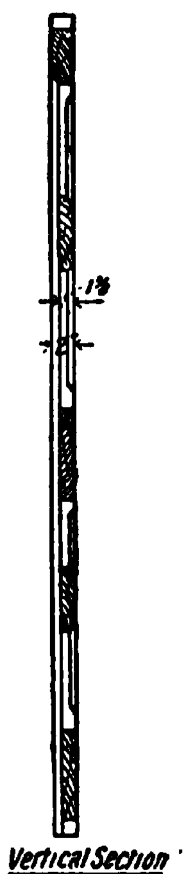


Fig. 136.

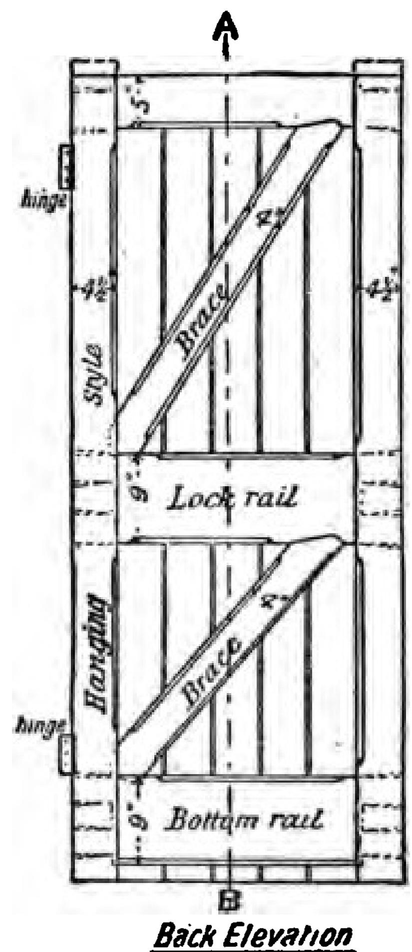


Fig. 137.

The Framed Battened and Braced Door (Figs. 136 to 137) differs from the former in that the battens and ledges are enclosed on three sides by a frame of a thickness equal to the combined thickness of the battens and ledges, so that it is flush on each side with them. The boards are tongued into the frame at the top and sides, and the ledges are framed into the stiles with barefaced tenons. The braces should not be taken into the angle formed by the stile and rail, but be kept back from the shoulder about 1 in., as shown. If the brace is placed in the corner, the strain thrown on it has a tendency to force off the shoulder, unless the door is very narrow, when the brace will be nearly upright. These doors, as in fact all framed work exposed to damp, should be put together with a quick drying paint instead of glue in the joints, because ordinary glue has such an affinity for water that it will soften in damp situations releasing its hold, and also be the means of setting up dry-rot in the timber. The battens in these doors should be made 1/16 in. slack for each foot of width to allow for subsequent expansion, or otherwise the shoulder will be forced off. The framework of these doors is first made and wedges up, then the battens folded in and driven up into the top rail and nailed to the ledges, after which the braces are cut tightly in and nailed to the battens in turn, and the whole cleaned off together. In large gates of this description it is usual to stub tenon the braces into the rails, in which case they must be inserted first and wedges up with the framing.

Framed or **Panelled Doors** are of several kinds, distinguished by the number or treatment of their panels, or by the arrangement of the mouldings, as follows:

Two to Twelve Panel Doors.

Square and Sunk. When a thin panel is used without mouldings, as shown at A in the elevation diagram of a **Framed** or four-panel door (Fig. 138).

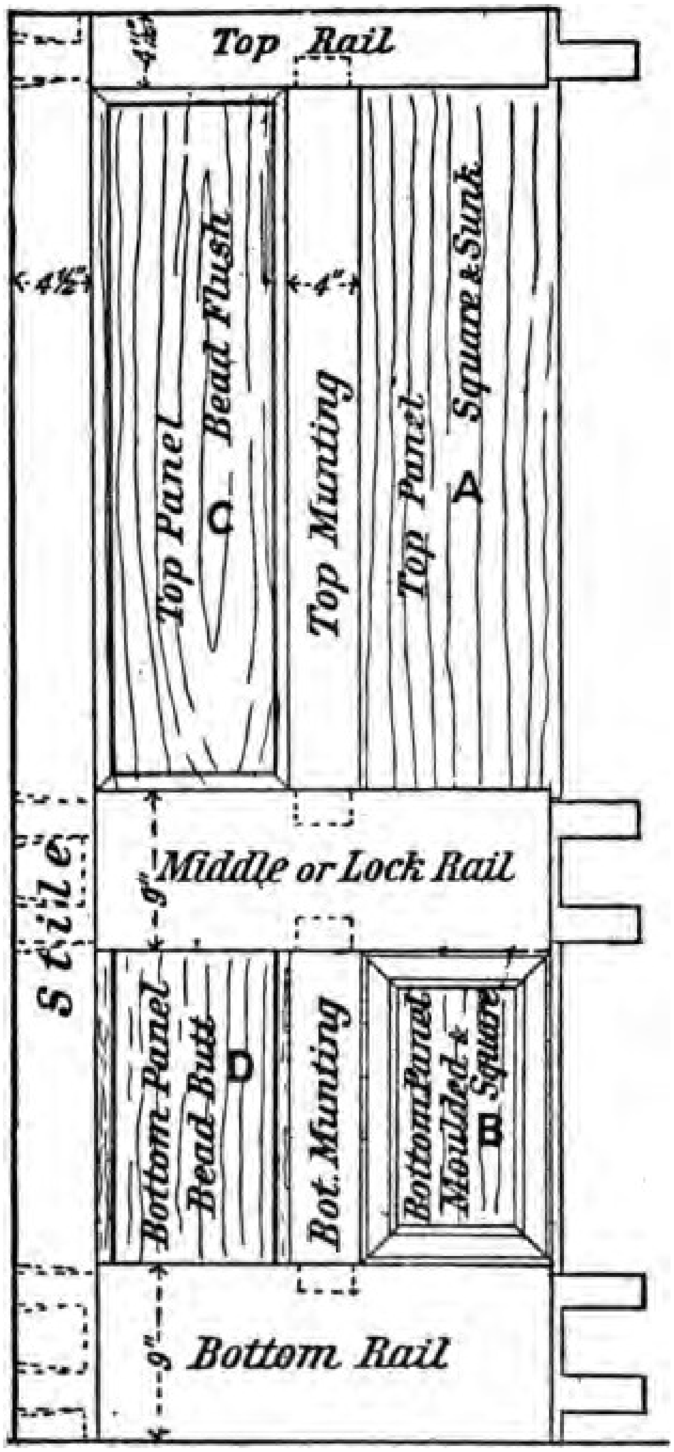


Fig. 138.

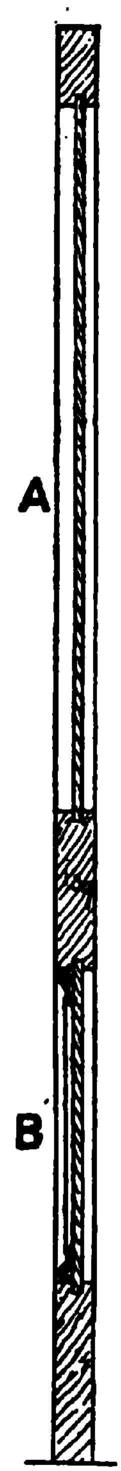


Fig. 139.

Moulded and Square. When one side of the panel is moulded and the other plain, as at B.

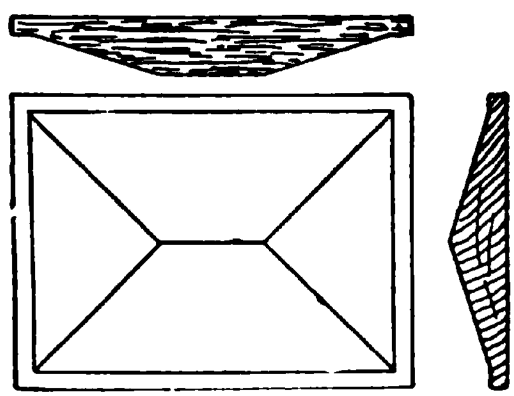
Bead Flush. When one side of the panel is flush, or nearly so, with the frame, and with a bead worked round the edges to break the joints, as at C.

Bead Butt. When the bead is worked only on the two sides of the panel, as at D.

Raised Panel. When the centre part of the panel is thicker than the margin. There are four varieties of raised panels:

Fig. 140.

1. **The Chamfered.** In this the panel is chamfered down equally all round, from the centre to the edge when square, or from a central ridge if rectangular, as shown.



Plan and Sections of a Chamfered Raised Panel.

Fig. 142. Fig. 141.

2. **Raised and Flat or Raised and Fielded.** When a chamfer is worked all round the edge, leaving a flat in the centre, as at A, Fig. 142.

3. **Raised, Sunk and Fielded** (as at B, Fig. 142). When the chamfer starts from a marginal sinking below the face.

4. **Raised, Sunk and Moulded** (as at C, Fig. 143). When the edge of the sinking is moulded.

Stop Chamfered. When the edges of the framing are chamfered and stopped near the shoulders.

Bolection Moulded. When the panel moulding stands above, and is rebated over the edges of the framing, as shown at Fig. 142.

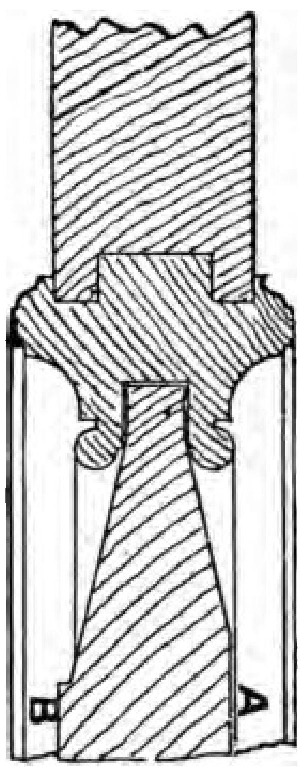


Fig. 143.

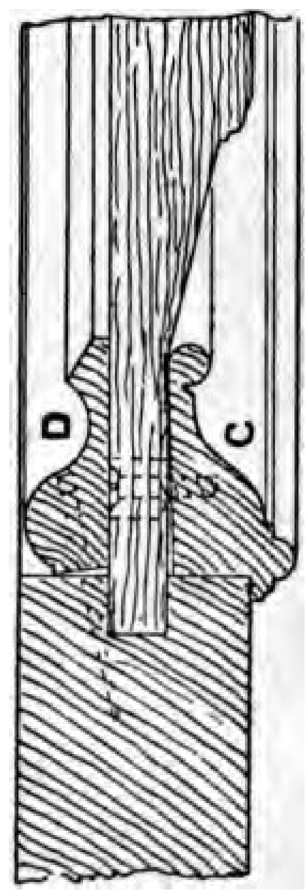


Fig. 143½.

Double Bolection Moulded. When the moulding on each side of the door is made in one solid piece, grooved to receive the panel, and is itself grooved and tongued into the framing. This variety is shown at Fig. 142.

Constructive Memoranda. The outside vertical members of doors (in common with all framed work) are called stiles. The one the hinges are fixed to is called the hanging stile, the one containing the lock the striking stile. In a pair of doors the two coming together are called the meeting stiles. The inside vertical members are the mountings, or more commonly muntings. The horizontal members are rails, respectively, top, frieze, middle, or lock, and bottom. The panels are

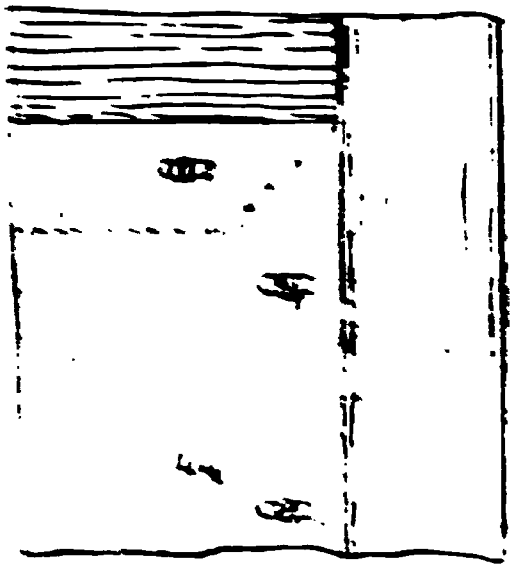


Fig. 144.

Fig. 145.

named similarly. When the grain of these run horizontally, they are said to be "laying" panels; when vertically, "upright." Doors are called **Solid-Moulded** when the moulding is wrought or "stuck" in the substance of the framing itself, as is shown at Fig. 145; and **Planted** when the moulding is worked separately and bradded around the frame, as shown at a, Fig. 148 and D, Fig. 148½. These are also called sunk mouldings.

Bead Flush Panels are commonly made as shown at a, Fig. 146, but such panels will, unless made of thoroughly seasoned stuff, inevitably split when drying. The correct way to obtain the effect of bead flush panelling is to work the beads upon the edges of the framing, as shown in Figs. 147 and 148.

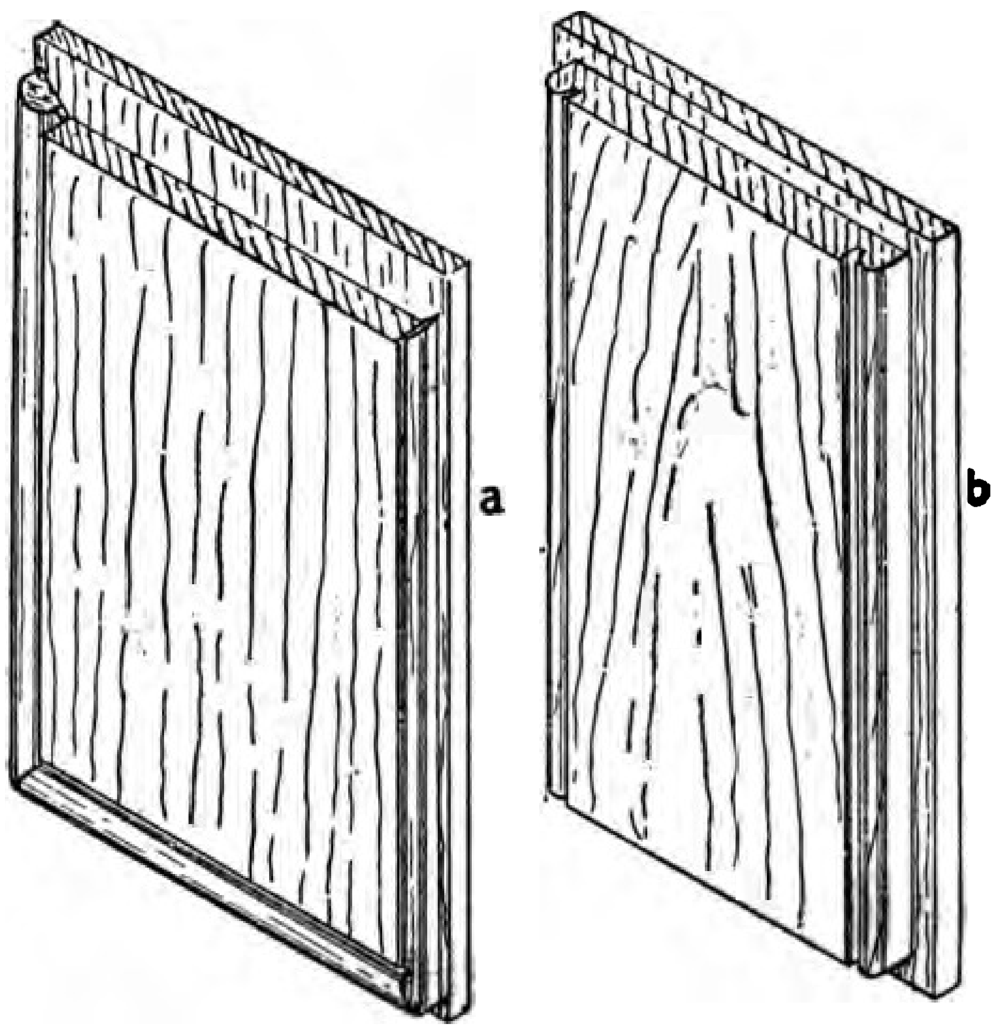


Fig. 146.

Bead Butt Panels are better kept about 1/32 in. below the framing, as a truly flush surface is difficult to prepare through the yielding of the panel, and when produced, will seldom last, as the shrinkage of the panel and frame is unequal. Planted-in "sunk" mouldings should be fixed to the framing, not to the panels, as shown in Fig. 143, D; for if fixed to the panels, when the latter shrinks the moulding will be drawn away from the frame, leaving an unsightly gap. The back edge of the moulding should be bevelled under as shown, so that when bradded in, the front edge will keep close down to the panel. As it is not permissible

Fig. 147.

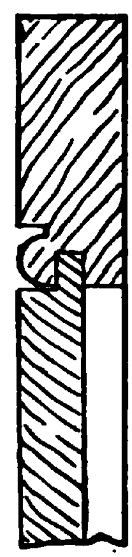
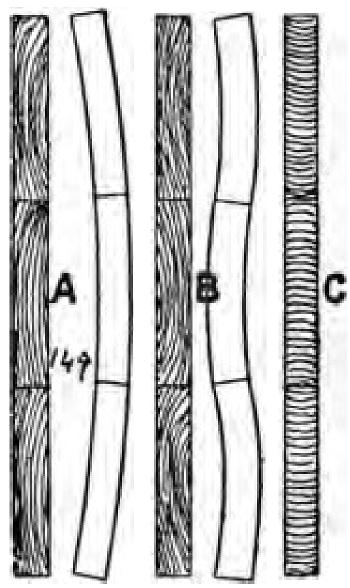


Fig. 148.

to brad polished mouldings, except in the case of inferior work, these are usually glued to the frame, and their back edges should of course be square. The panel should be polished before the moulding is planted in, so that in case of shrinkage a white margin will not be shown. When, however, the moulding is wide and thin, it is unavoidable that it be fixed to the panel to keep its front edge down, and to overcome the difficulty of shrinkage. The stiles in vertical and the rails in laying panels are prepared with a second shallow groove as shown, and the moulding, made with an extra wide quirk, is cut in and pressed back into the groove. It is glued to the panel, and shrinks with it without showing a gap. The cross pieces across the ends of the panel have their quirks shot off to the proper width, and are sprung in after the side pieces are in place. These are dowelled but are not glued, or when there is a moulding at the back, the face side is slot screwed.

Bolection Mouldings are intended to be fixed to the panels, which is rendered necessary by their great width and thickness, and they are rebated over the edge of the framing to prevent the interstices produced by the shrinkage showing. From 1/8 to 3/16 in. is sufficient for this purpose, and more should not be given or the edge will be liable to curl off. To prevent the panel being split by the fastenings when it shrinks, the moulding is fixed by slot screws, as shown in Fig. 143, the slots being cut across the grain. See the back view, Fig. 144. The moulding should also be screwed together at the mitres, and the latter may be grooved and tongued with advantage, and dropped into the panel as a complete frame, where it is fixed as described, the heads of the screws being covered by the interior moulding, which, if a sunk one, is glued to the frame, and if a bolection, dowelled to the panel, or, as is sometimes done in inferior work, bradded, and the holes filled up with colored stopping. It is not good construction to glue the moulding to the panels in any case, as the alteration of size in the latter, due to the state of the atmosphere, is very liable to cause them to split, if fixed immovably, or when swelling, to disarrange the mitres. The bradding in of mouldings is less likely to do this if the brads are not placed too thickly, as they yield slightly to the pressure. Framing is usually grooved ½ in. deep for the panels, and the latter given ⅛ in. play sideways, but fitting close lengthways of the grain. This is sufficient for panels up to 2 ft. wide. Over that width the grooves should be ⅝ in. deep, and the panel enter ½ in. at each side. Ordinary dry stuff will eventually shrink about ⅛ in. to the foot, and will swell equally if exposed to damp. When wide panels are used they will warp less if glued up in several pieces, as the pull of the fibres is lessened by the cutting, and the effect of the warping is diminished in the same ratio as their width. Much can be done to ensure the permanent flatness of panels by paying attention to the way the boards have been cut from the tree. The direction of the annual rings on the end will indicate this, and the various pieces should have their similar sides placed together. What is meant by this will be rendered plain by an examination of Fig. 149. When a panel is glued up with the hollow or heart sides of the rings all on one face as at A, and the board warps, it will case in one continuous curve, as shown in the unshaded diagram, whilst if glued up with the heart sides reversed alternately, as shown at B, it will assume the serpentine shape shown in the unshaded diagram. Boards cut radically or with the annual rings perpendicular to the surfaces, as at C, will swell less than the others, and will not warp perceptibly.

Proportions. The size of doors depends so much upon the scale and design of the buildings they occupy, that no definite data can be given, within reasonable limits, for important doors; but it may be pointed out that very large doors not only tend to dwarf a building or a room, but they also take up a great deal of space in opening, and the difficulty of preserving their accurate fitting increases in direct ratio with the size. The following may be taken as an indication of the more usual dimensions given to ordinary good class dwelling house doors: **Entrance Doors,** from 7 ft. to 8 ft. 6 in.



illustrating the Effect
of Position on the Parts
of a Panel.

Fig. 149.

high by from 3 ft. to 4 ft. 6 in. wide, 2 to 2½ in. thick. **Reception Rooms,** 7 ft. by 3 ft. 3 in. by 2 in. **Bed-Rooms,** 6 ft. 8 in. by 2 ft. 8 in. by 1½ in. Details of interior doors; stiles and top rails, in common work, out of 4½ in., muntings and frieze rails 4 in., middle and bottom rails 9 in. Superior doors vary much, but generally stiles and rails are somewhat wider than the above, muntings and frieze rails narrower. Height of lock rail usually 2 ft. 8 in. to its centre. This is a convenient height for the handle, which is generally placed in the middle of depth of rail. When an entrance door is approached from a step the middle rail is kept about 6 in. lower, to bring the height of the handle convenient.

Common Doors, both internal and external, are made of "yellow pine" or Georgia pine throughout. A better class of interior doors have yellow pine frames and white pine panels. The latter wood should not be used for external work, as it is far too soft and will not stand wet. **Superior Internal Doors** are made throughout of Honduras mahogany, black walnut and oak; also of pine and baywood, veneered with Spanish mahogany. **External Doors** of oak, teak, walnut and pitch pine.

In constructing doors of any of the above mentioned figure woods, great attention must be paid to the arrangement of the members, so as to balance the figure, and this may also well be studied in the conversion of the plank. For instance, two stiles, each having pronounced figure at one end, and the other end plain, should have the figured ends placed at the bottom. This gives the effect of solidity, whilst the reverse would make the door look top heavy. Similarly the upper rails should be plain, the lower figured. It must be understood the above only applies when the wood is a mixed lot. When the wood is handsomely figured throughout, the point of most importance is the effect of its position upon the figure, and this is so great that in some of the light-colored woods, wainscot and baywood for instance, a piece that in one position will appear richly figured will in another show quite plain and dull. The best way to judge the effect is to prop the pieces up in the approximate positions they will occupy when finished, facing a top light. Then when standing a few feet off, the play of light on the fibres will be observed. Deep-colored woods, such as teak and Spanish mahogany, may have their figure brought out by slightly oiling them, which will facilitate their arrangement. Panels also require balancing, the more

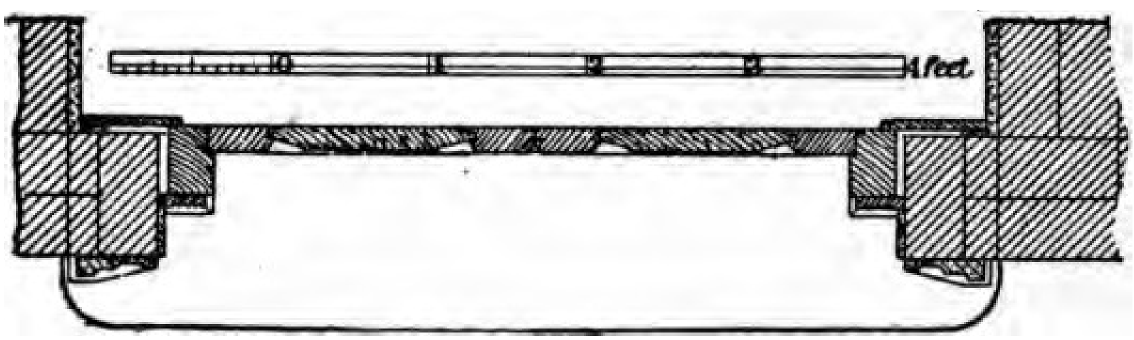


Fig. 150.

heavily marked ones being placed below plainer ones, and symmetrically arranged, either in pairs, or figured in the centre and plain outside. In all cases where the figure is coarse, taking a truncated elliptic shape, the base or wider part should be kept downwards. The panel at B is upside down from an artistic point of view. This arrangement is known in the workshop as "placing the butts down," although as a matter of fact the width of the figure is not due to its being towards the butt end of the tree, but merely to the accidental position the surface of the board occupies with relation to the annual rings, which are more or less

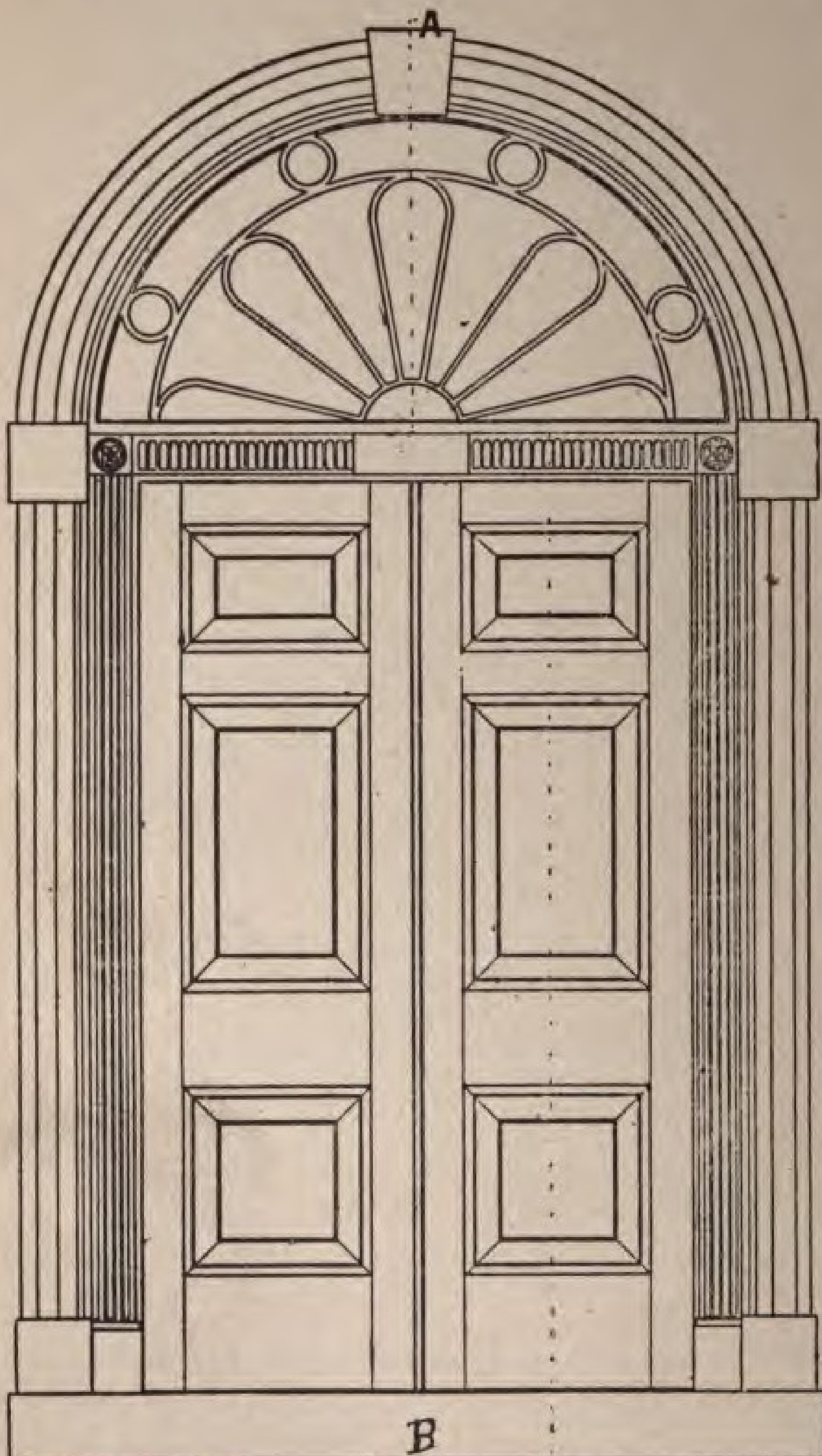


Fig. 151,

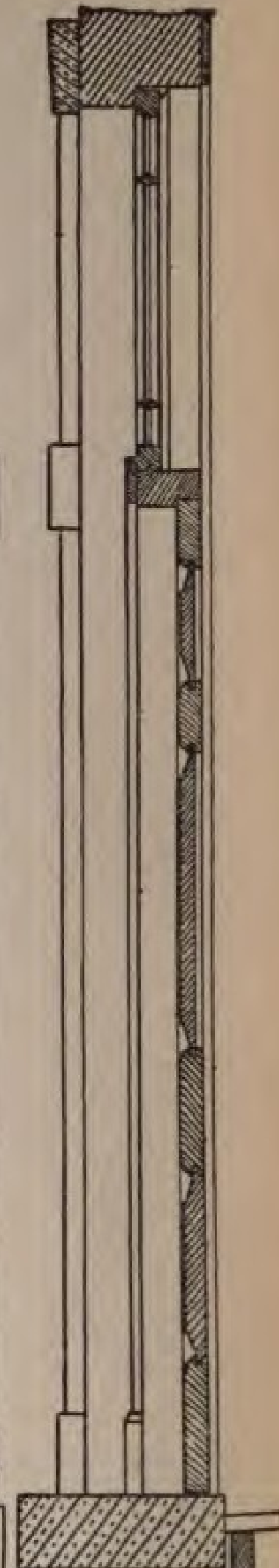


Fig. 152,

waved in length, due to crooked growth, and the board passing through them in a plane, their edges crop out on the surface in irregular elliptic shape.

Double Margined Doors are wide single doors framed to appear as pairs of doors. They are used in openings too wide proportionately for a single door, but where half the opening would be rather small for convenient passage. Figs. 150 to 154 show elevation and sections of a **Double Margined Entrance Door,** typical of the style in vogue in the latter part of the eighteenth and early part of the nineteenth centuries.

These doors are made in two ways. In the earlier method the central imitation stile, which in this case is really a munting, is made in one piece and forked over the top and bottom rails, which are continuous. The intermediate rails are stubtenoned to the central, and through tenoned and wedged to the outside stiles, but unless the stub tenons are fox-wedged, the shoulders are very liable to start, for which reason the method of construction now to be described is generally preferred. The door is composed of two separate pieces of framing, each complete with two stiles and a set of rails that are tenoned through and wedged up. The two portions are then united by a ploughed and tongued and glued joint, which is hidden by a sunk bead in the centre, as shown in the diagram, Fig. 155, and the parts keyed together with three pairs of hardwood folding wedges. The door is sometimes further strengthened by having flat iron bars sunk and screwed into the top and bottom edges. The actual process of putting the door together is as follows: After the various rails and panels have been duly fitted and marked, each leaf is taken separately and the stiles knocked on. The one intended for the meeting stile having been

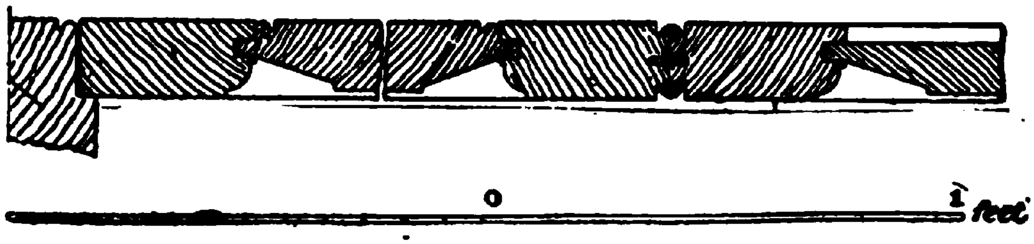


Fig. 153.

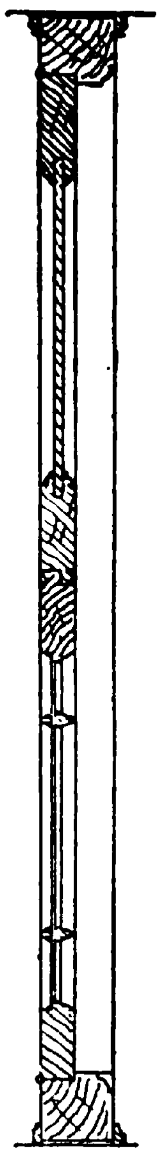


Fig. 154.

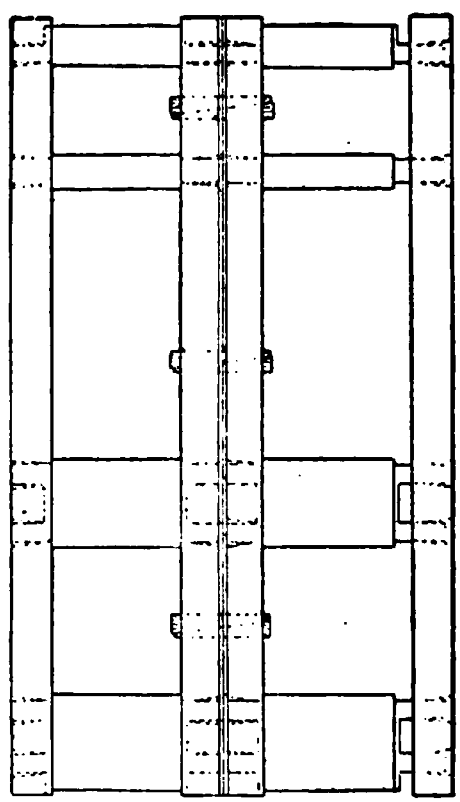


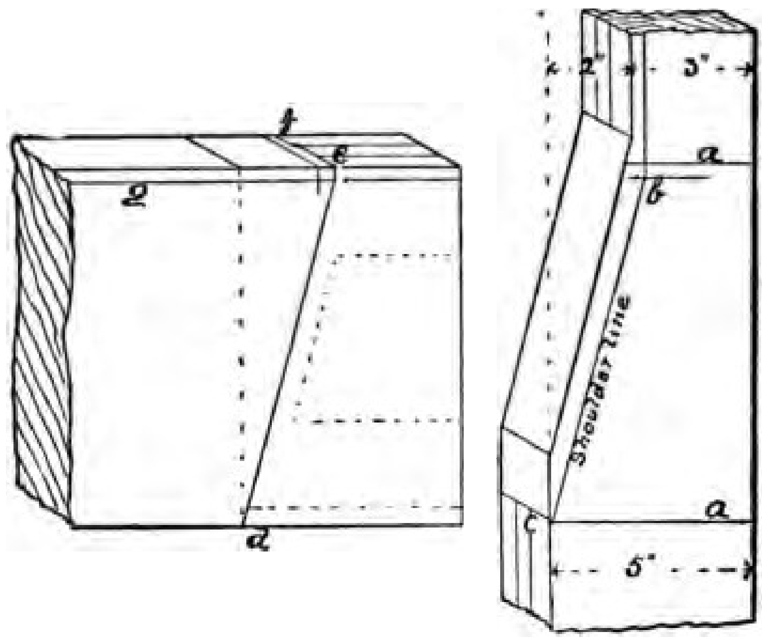
Fig. 155.

Fig. 156.

glued, is cramped up and wedged. Then the meeting stiles are made to a width, grooved, jointed, and rebated for the beads, as shown in the detail, Fig. 150. The ends of the tenons and wedges should be cut back 1/8 in. to prevent them breaking the joint when the stile shrinks. The mortises for the keys will have been made when the mortises for the rails were done, and cross-tongues are next glued in, the joint rubbed, the two stiles pinched together with handscrews, and the oak keys, well glued, driven in. At this stage the frames are stood aside to dry, after which the projecting ends of the keys are cut off, the panels inserted, and the two outside stiles glued and wedged in the usual manner. After the door is cleaned off, the grooves to receive the beads are brought to their exact size with side rabbit and router planes. Should iron bars be used, they are inserted in grooves made after the door has been shot to size. The bars should be about 1/2 in. shorter than the width of the door, so that the ends are not visible.

Diminished stiles are sometimes cut out in pairs from a board or plank. When this is done, the back or outside edge is shot straight and the setting out made thereon, the two portions being gauged to width also from the back, but the method more suitable for machine working. Here the stile may be cut parallel to the full width, the face edge shot in the usual way and the setting out made upon that, the diminish being gauged from inside. In this method the mortises are made before the diminished part is cut out, to render that operation easier for the machinist. He should not, however, mortise right up to the sight lines on the diminished part, because if the chisel is at all out of upright when the waste is cut away, the mortise will be found beyond the sight line, which will be a serious defect.

To Set Out the bevelled shoulders of the stile and rail. Taking the stile first, having as described in the first method gauged and faced up the inside edges, and set out the width of the rails and mortises on the back edge, square over on each side the sight lines of the middle rail as at a a, Fig. 156. Then draw a second line representing the depth of sticking of the rail on the face side, and the rebate on the back side, as at b.

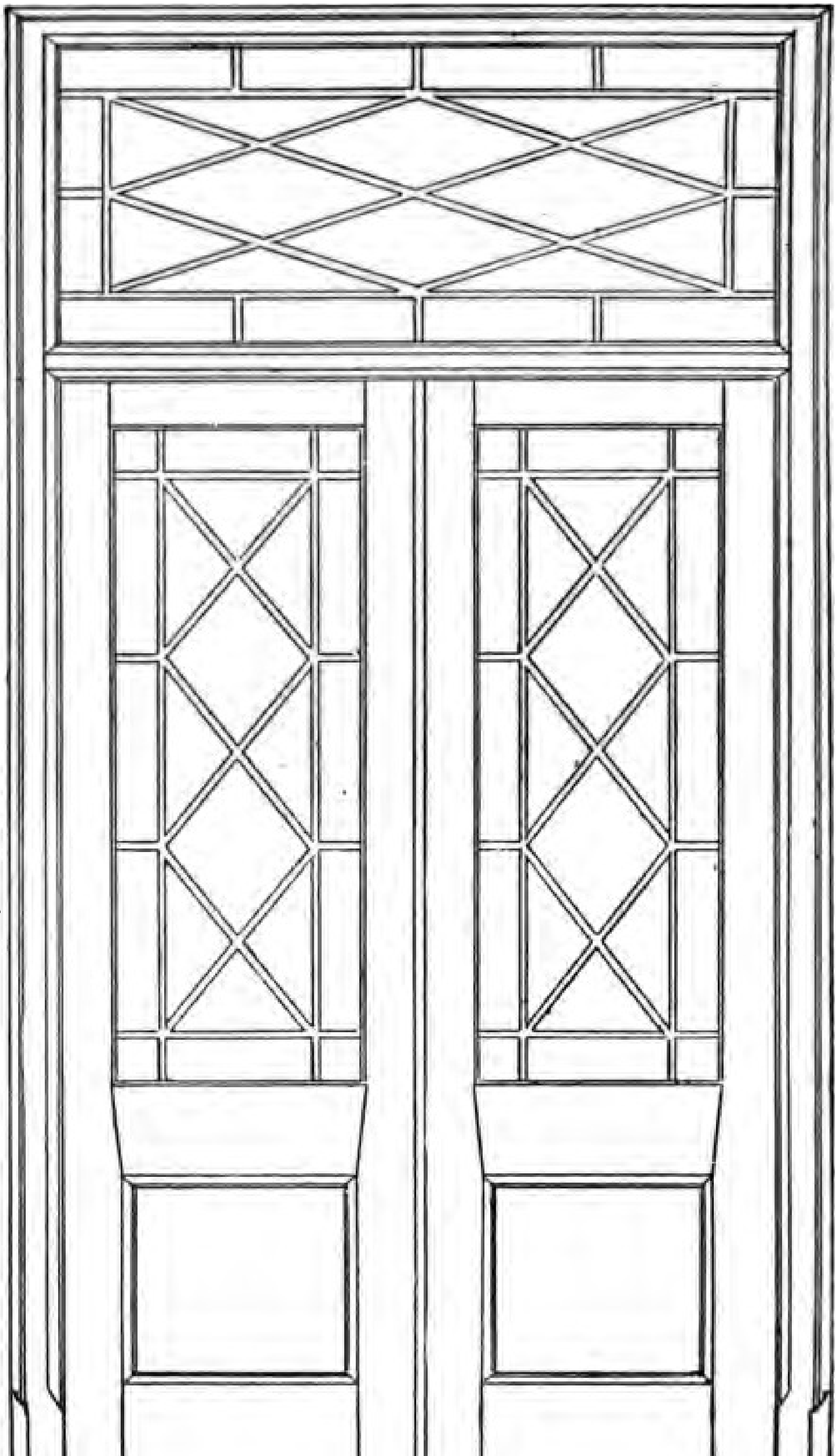


Stile and Rail of a Diminished Stile Door.
Method of Obtaining Shoulder Lines.

Fig. 156 ½.

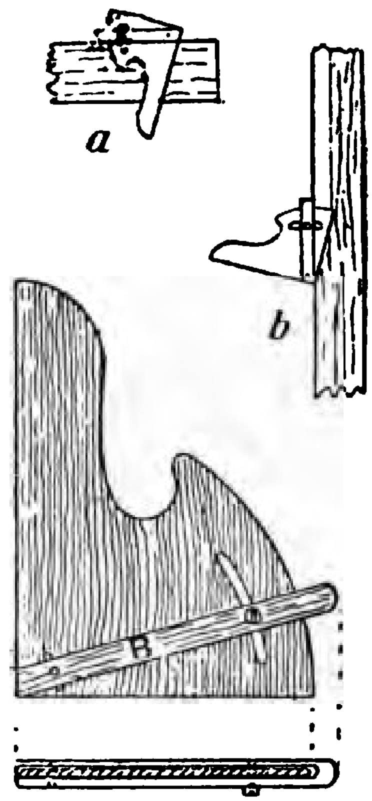
Next run gauge lines down on each side of the diminished part as working lines of rebate and sticking, and from the points of intersections of the stickings and rebates respectively, draw in the shoulder lines to the sight line of the lower edge of the rail at c. The only difference to be made when the stile is prepared by the second method is that the sticking and rebate gauges would be run from the original face edge instead of the actual diminished edge, and as before stated, the sight lines would be marked on the face edge instead of the back, and squared down to the intersections.

To Set Out the Rail. Mark on the bottom edge the "width" or sight lines of the widest part of the stiles. Square this point on to the upper edge as shown by dotted line in the sketch, Fig. 156, and set off therefrom the amount of the diminish on the stile, as shown by the dotted line on the stile in the example: this is 2 in. This line, knifed in on the edge, is the "sight line" of the upper part of the stile. Again set off beyond this line the amount of the sticking and rebate shown in the sketch by the lines e and f. Next run the sticking and rebate gauges cn front and back sides, as shown at g, and square down the lines e and f to meet them. Then draw the shoulder lines from the intersections to the point d on each side. Having thus found the shoulder line upon one rail, bevels may be set to them and used to mark any number. A contrivance sometimes used when a large number of similar shoulders have to be set out is shown in Fig. 157. This is known as a **Shoulder Square.** It consists of an ordinary set square provided with a movable fence or bar B, which is slotted to pass on both sides of the square, and is pivoted near the right angle. A set screw near the outer end of the bar passes through a concentric slot, and fixes the fence in any desired position. The pivot works tightly in a small slot to allow the lower edge of the bar to enter the right angle, and the outer edge of the square is also made a concentric curve to permit the



A PAIR OF VESTIBULE DOORS. *(Eighteenth Century Style.)*
Fig. 157.

easy passage of the end of the bar. The theory upon which the action of the tool is based is that the angle between the parallel bar B and either of the edges of the square is the complement of the remaining angle, the two combined forming a right angle which is the desired angle between the edges of a rail and stile. Its application is shown in the upper part of the figure, a



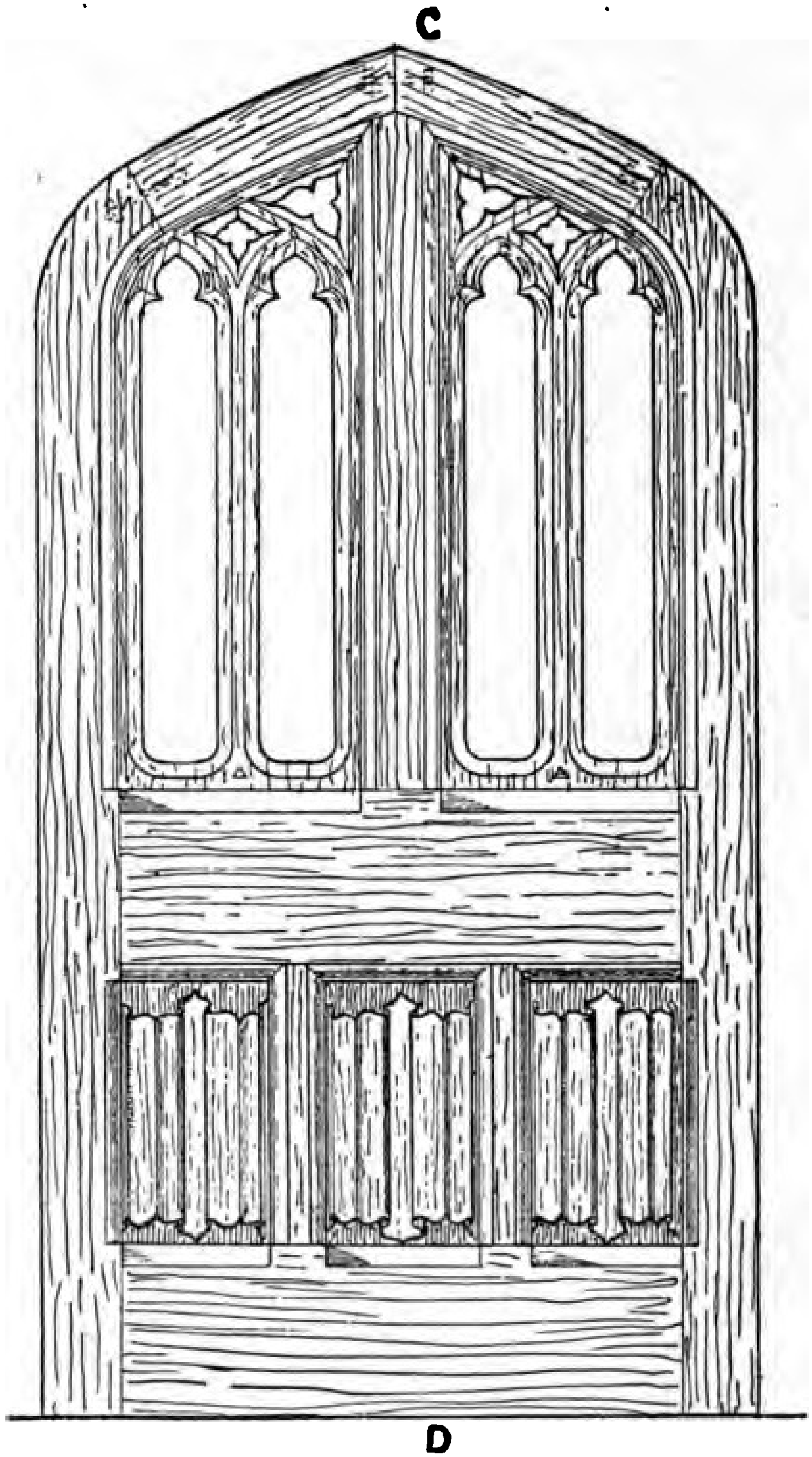
An Adjustable Square
for Bevel Shoulders.

Fig. 157½.

being the rail, b the stile. Either a rail or stile is first set out as described above, the edge of the square set to the shoulder line, and the bar brought up to the face of the work and fixed with the set screw when it is ready for application to the other piece. The moulding upon a diminished stile should not be mitred but continued on to the shoulder, and the rail scribed over it, which will prevent an open joint occurring should the rail shrink. When doors, after knocking together, are stored for a second season, a slight difference will have to be made in the setting out of the shoulders of the middle rail. The wider part of the stile will shrink more than the narrow part, and consequently if the shoulders are set out accurately at first as described above, when they are refitted the shoulders will be found short at the lower ends. To prevent this, allow about 1/32 in. extra on the lower part of the shoulder at each end of the rail.

A Gothic Door of the Tudor period is shown in the elevation in Fig. 158 and section in Fig. 159. The head is four-centred. The upper panels are pierced tracery, and the lower ones carved drapery. The mouldings in this type of door are invariably stuck solid, and those on the stiles stopped at the sight lines of the rails. The mouldings on the latter are also frequently stopped at the muntings as shown, especially in the earlier work. Many of the doors, however, of the Tudor period have the upper ends of the muntings mitred. In modern work of this style, when the mouldings are not stopped, it is usual to scribe them at the intersection. Chamfers, however, are always stopped to obtain a square-built shoulder for the munting, as a shoulder scribed over a chamfer soon gets faulty through the shrinkage.

Mediaeval doors were always constructed of oak, but **pitch** or **Georgia pine** is now much used in this



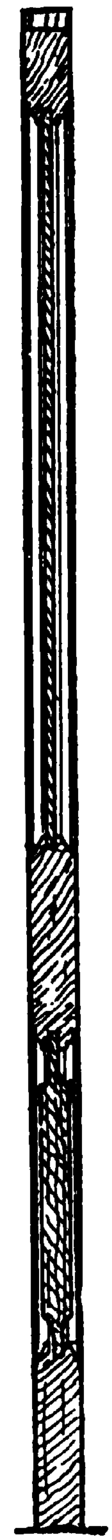


Fig. 158. Fig. 159.

style of work. The older examples are mortised, tenoned, and pinned together, wedging and glueing being a modern invention. The joints at the head are usually slip tenons, pinned, or with dovetail keys inserted. These joints in a modern door would be secured either with a hammer-headed key or handrail bolts.

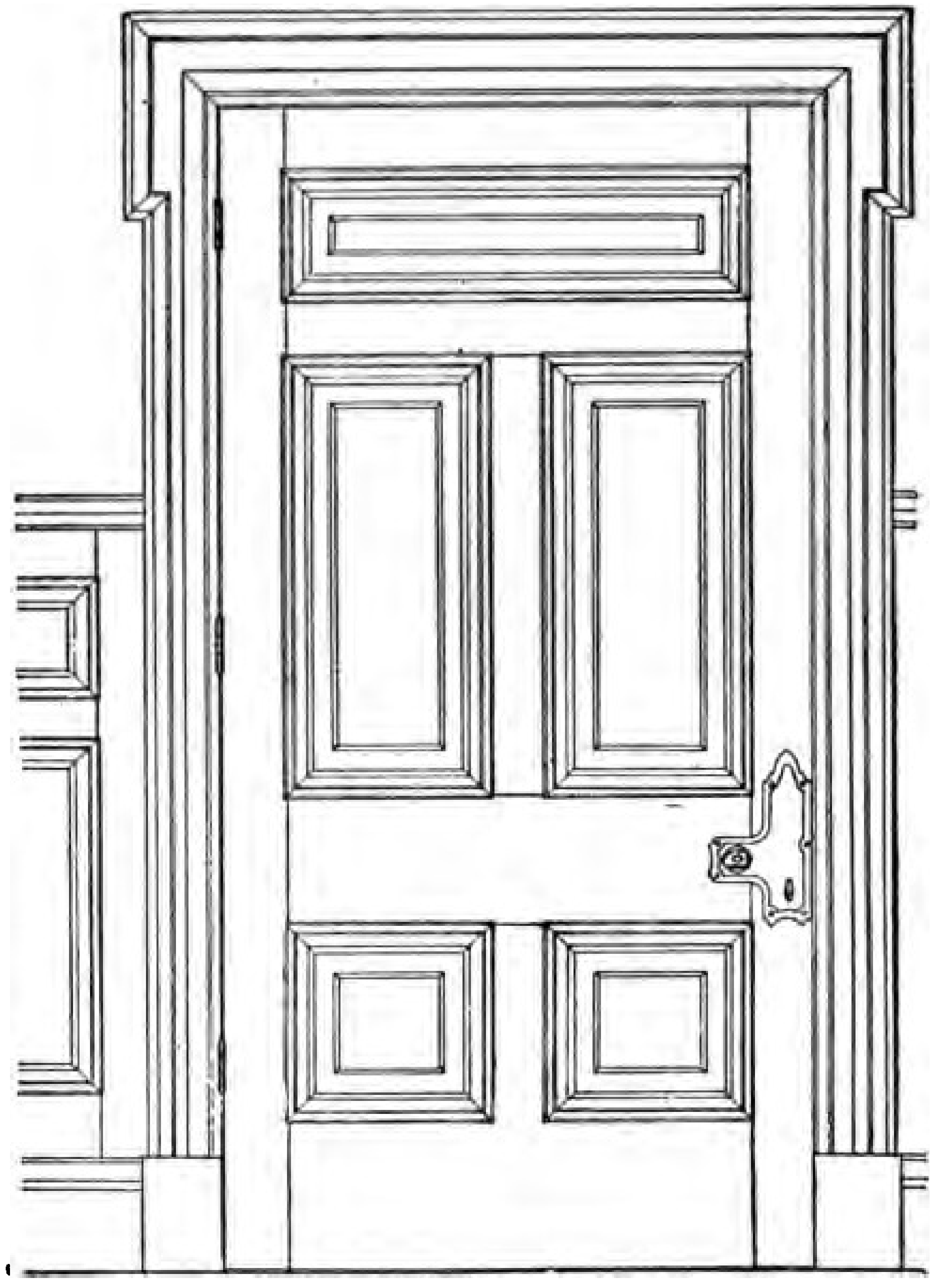


Fig. 160.

Fig. 160 shows the elevation of a superior five-panelled interior door with its finishings. Fig. 161 shows a vertical section through the opening, and Fig. 162 is

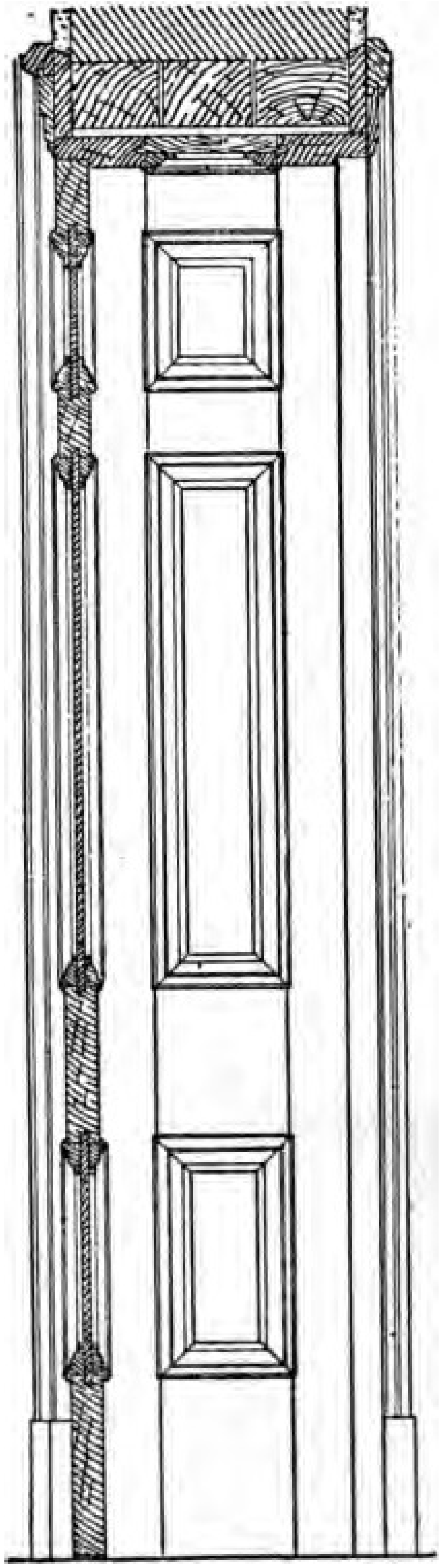


Fig. 161.

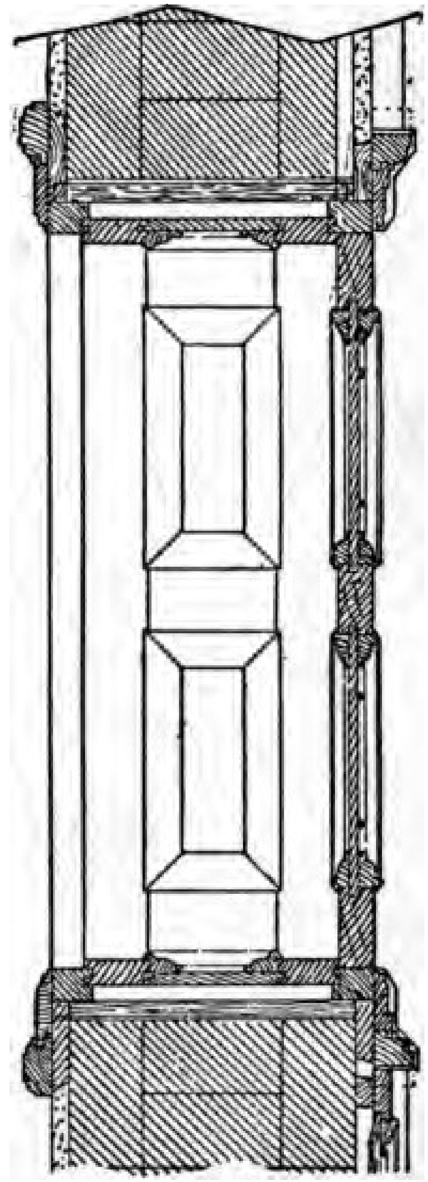


Fig. 162.

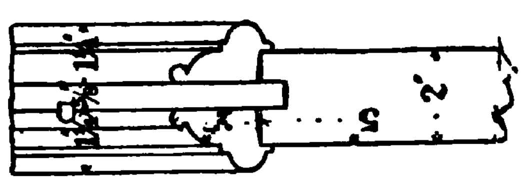
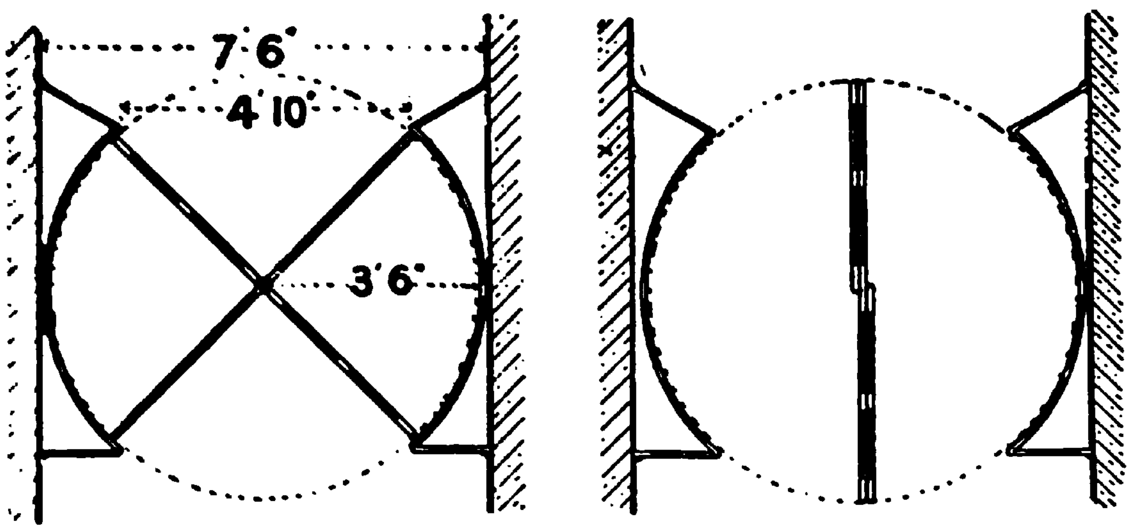


Fig. 163.

a plan showing in outline the framed soffit lining. Fig. 163 is an enlarged section of one stile and part of panel, mouldings, etc.

Revolving Doors.—An arrangement of vestibule doors, suitable for banks, hotels, etc., is shown in plan in Fig. 164. The doors are arranged at right angles to each other, and revolve around a vertical axis like a turn-stile. Curved side frames, each a little wider than a quarter of a circle, are fixed on each side of the doorway. A suitable width for the doors is 3′ 6″. The ad-



Plans of Revolving Vestibule Doors.

Fig. 164. **Fig. 165.**

vantages of such an arrangement is that it is noiseless and draughtproof, the latter feature being obtained by having an india-rubber tongue fixed in the outer edge of each door. The doors are so hung that alternate doors can be folded back against the adjacent ones (Fig. 165), and thus give an uninterrupted passage when required.

Other Panelled Framing.—Framework filled in entirely with wooden panels, or with wooden panels in the lower part and glass in the upper part, is also required in the fittings for offices, for school partitions, and for screens in churches, business premises, etc. The arrangement of the framing is similar to that of doors, and the same terms are used to describe the various parts, the only difference being the proportions of height and width; these are, of course, governed by special requirements.

Superior Doors.—In superior work, where the doors and surrounding framework are made of ornamental hardwood, it is often necessary to construct a door which shall be of one kind of wood on one side of the door and an entirely different kind on the other side. This would be necessary, for example, with a door opening from an entrance hall fitted entirely with oak into a room, the fittings of which must all be of walnut or mahogany. Such a door may be constructed in two thicknesses, each of the respective kind of wood, and each of a thickness equal to one-half of that of the finished door. The two parts are then secured together by tapering dovetailed keys, and the edges of the door are afterwards veneered to match the side of the door to which they correspond. Figs. 166 and 168 give details of this kind of door.

Grounds.—The architraves surrounding an opening are nailed to the lining, or where possible to the frame. In the best class of work, however, it is usual not to fix the door frames until the plastering is finished. Rough wooden battens or **Grounds,** of thickness equal to that of the plaster, are fixed to the walls around all door and window openings. These serve as a guide to the plasterer, and the door frames and the surrounding architraves are secured to them. When it is not desirable to have any nail holes visible in the

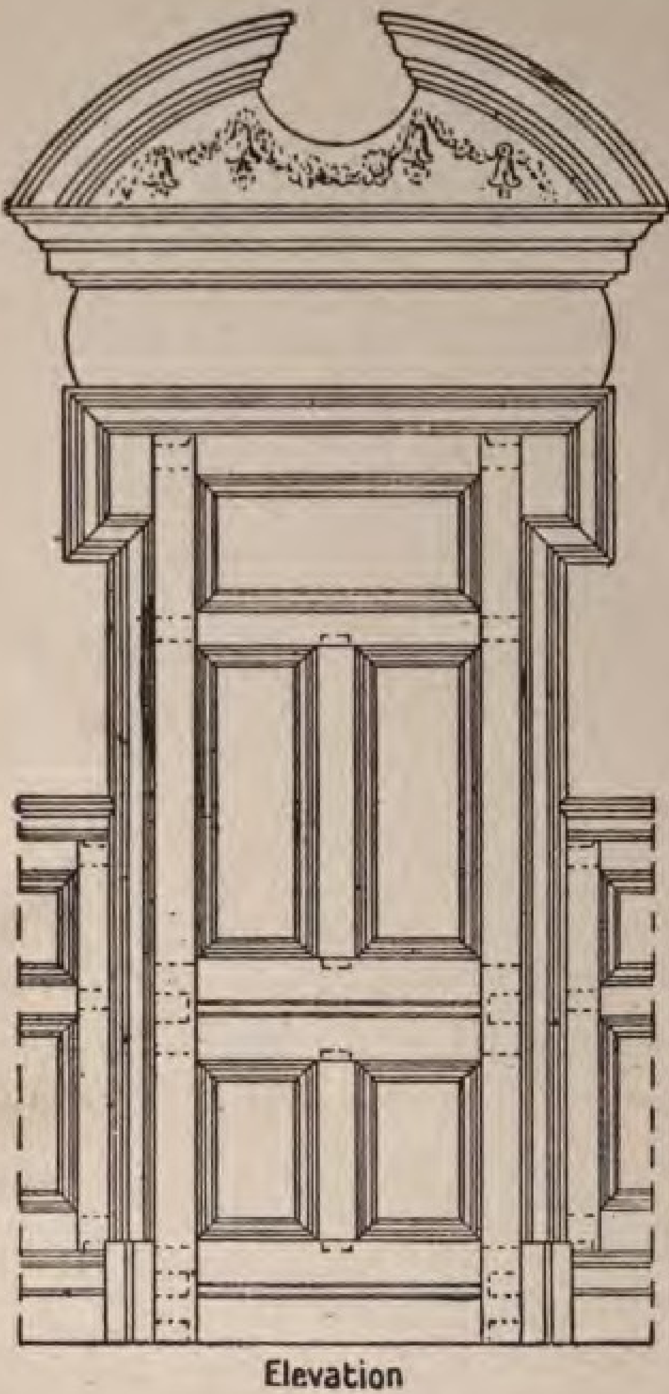


Fig. 166.

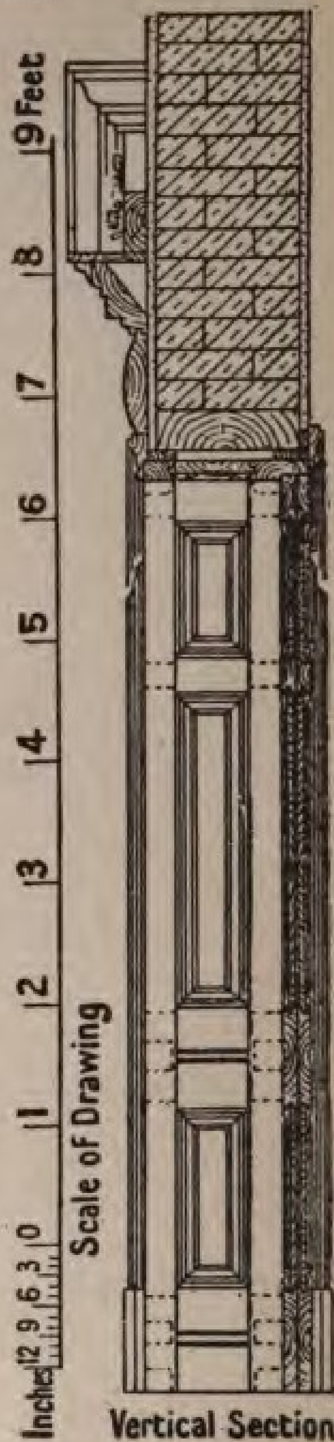


Fig. 167.

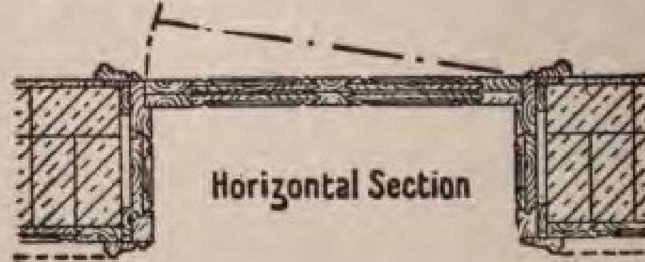


Fig. 168.

finished surfaces, the door frames and architraves are fixed by screws.

The fixing of the architraves around such a doorway affords a good example of **Fixing by Secret Screwing.** The mitres of the architraves are first glued and secured with dovetail keys or slip feathers. Stout screws are turned into the grounds about 12″ apart, being left so that the head of the screw projects about half-an-inch in front of the surface. On the back side

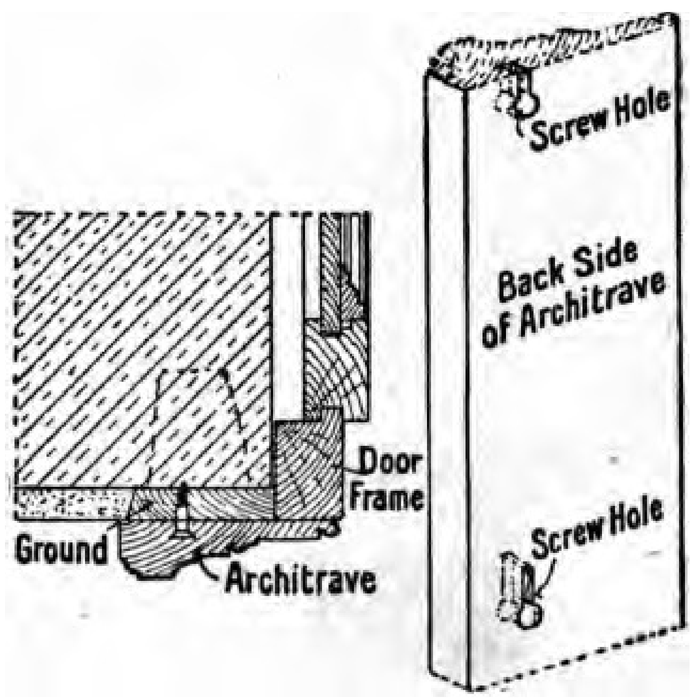
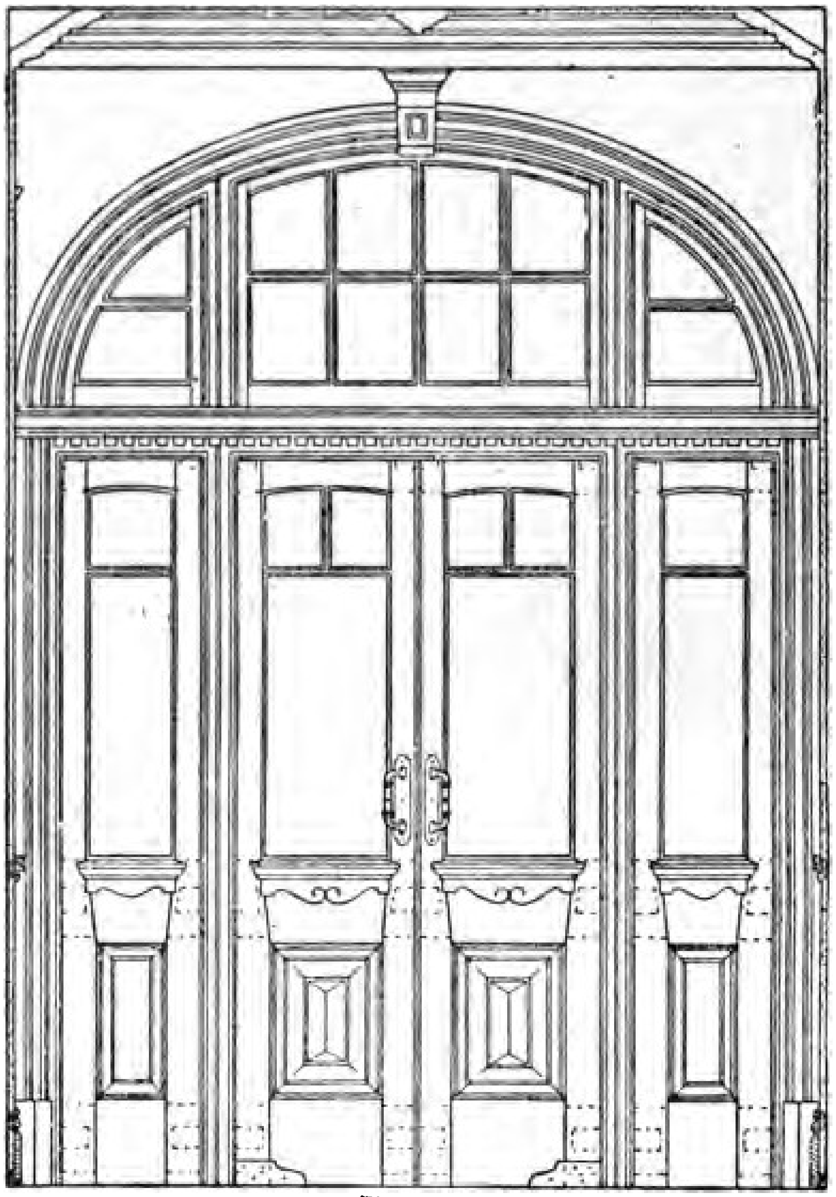


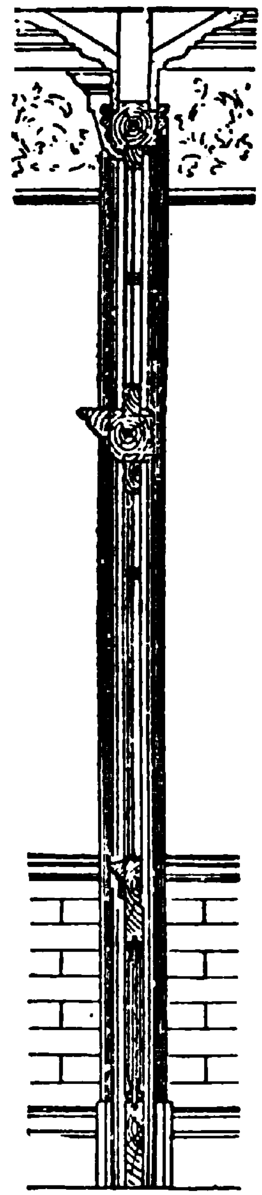
Fig. 169.

of the architrave, exactly opposite the screw heads, small holes—equal to the size of the shanks of the screws—are bored; and about three-quarters of an inch below these, larger holes—of size equal to the heads of the screws—are bored. Each small hole is connected to the large one adjacent to it by a slot, the depth of which is slightly greater than the projection of the screws. The architrave is fixed by placing it against the wall with the larger holes fitting on the screws, and then carefully driving it down so that the heads of the screws hook into the fibres behind the



Elevation

Fig. 170.

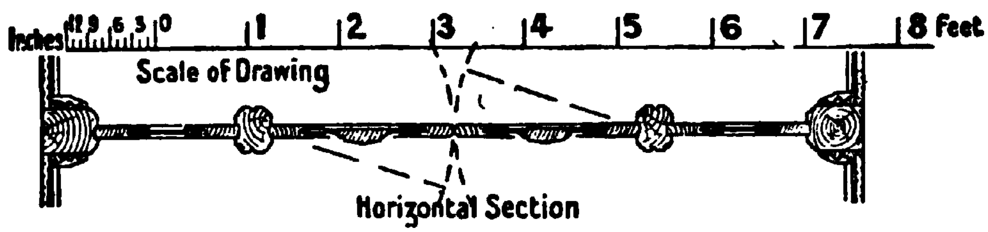


Vertical Section

Fig. 171.

slots. By placing the screws so that they are slightly inclined, the tendency is to draw the architraves closer to the wall. Fig. 169 shows the explanatory detail.

The above remarks upon door frames, linings, etc., apply especially to the doors of dwelling-houses. Door frames for warehouses, workshops, outbuildings, etc., do not as a rule require linings or architraves, a small fillet being nailed into the angle between the door frame and the wall instead. Vestibule doors are often hung to swing both ways, and the door frames have a hollow rebate or groove in the middle of the width of the frame, to receive the rounded edge of the door (Figs. 170, 171 and 172). Many of the heavier kinds of framed and ledged doors are not provided with wooden frames but are hung with bands and gudgeons, or arranged to run on pulleys as described elsewhere.



Details of a pair of Vestibule Doors with Side-lights.

Fig. 172.

Fig. 173 shows an ordinary sash door with three panels below. Fig. 174 shows a section of door and frame. Fig. 175 shows plan of door and part of cross section of frame. Fig. 176 shows the height and width rod of a door, which guides the workman in laying out his work.

External Doors are invariably hung in **Solid Frames.** Internal doors, chiefly to build up casings or **Linings,** of comparatively thin substance. In certain positions, such as vestibules and shop-fronts, where there are no wall openings to line, solid frames are also used for

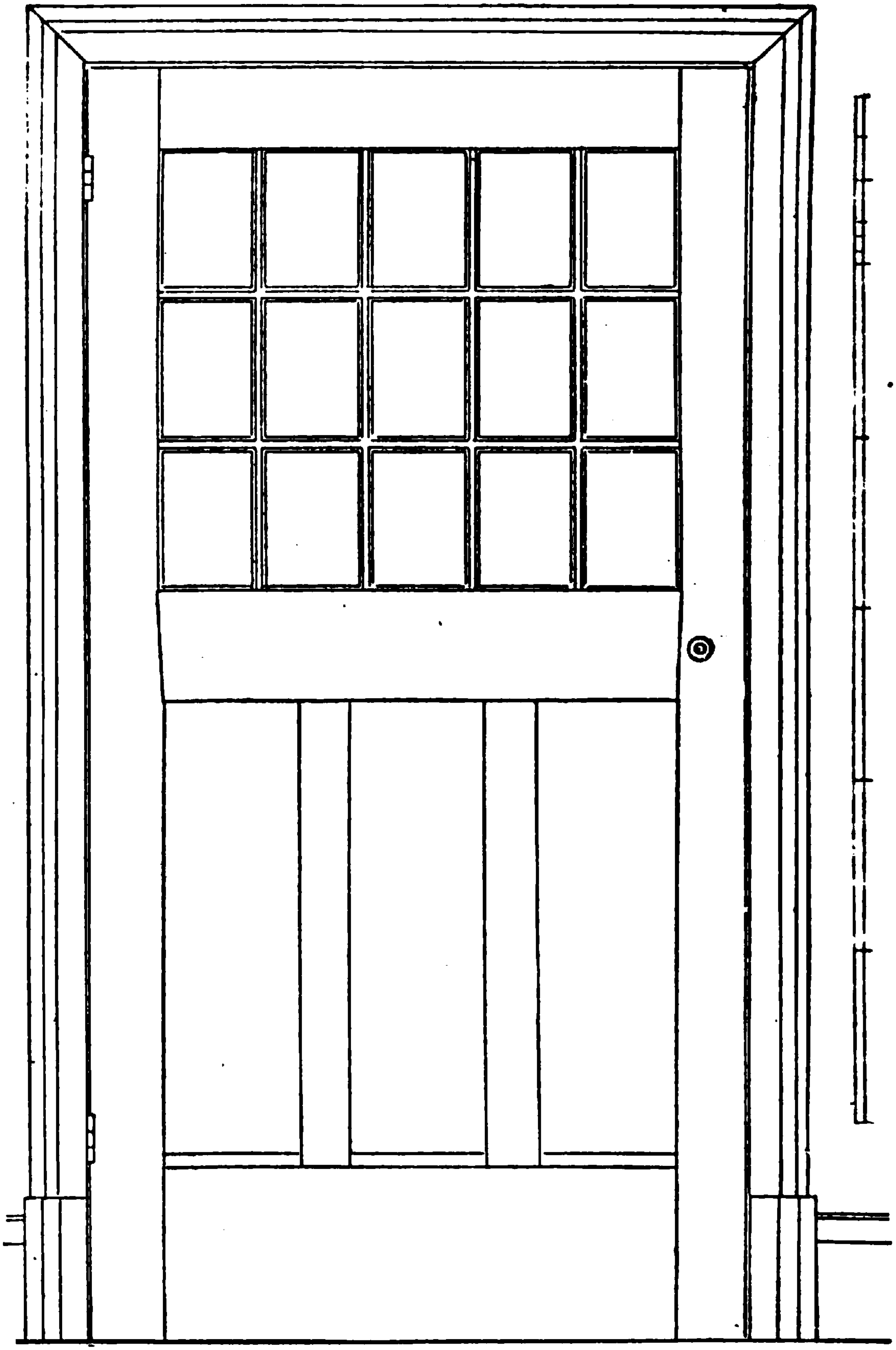


Fig. 173.

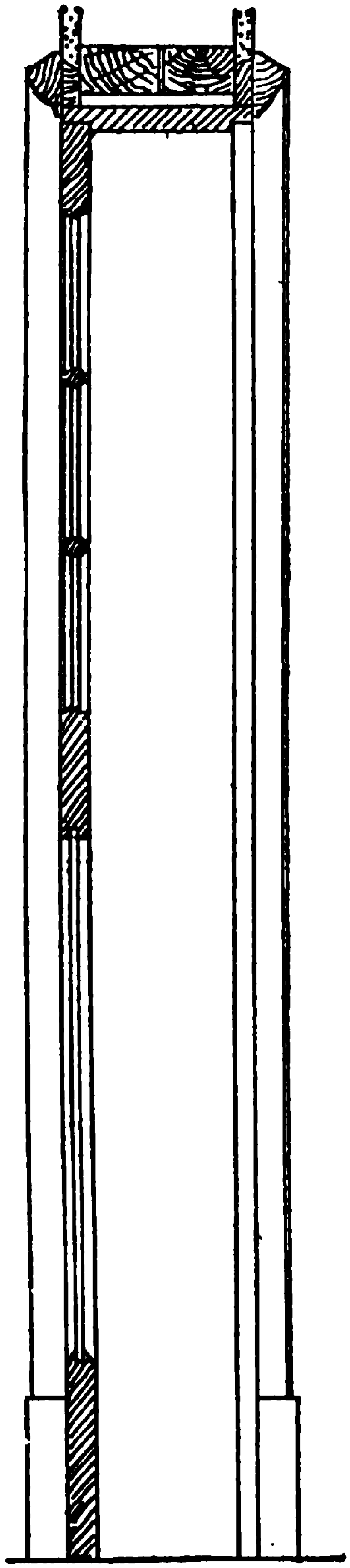


Fig. 174.

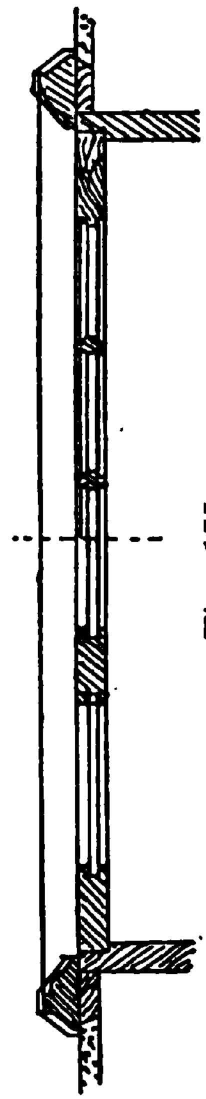
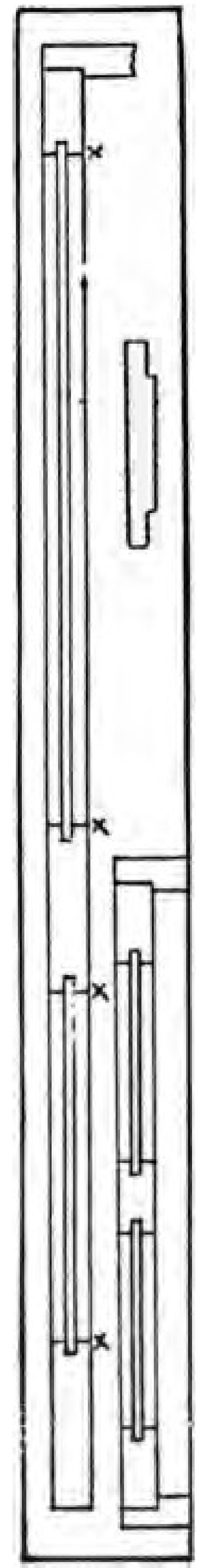


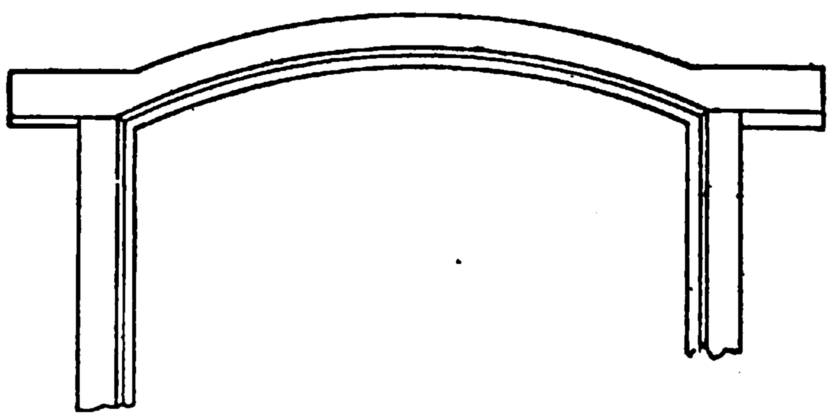
Fig. 175.



Height and Width Rod of a Door.

Fig. 176.

interior doors. The members of solid frames are usually made of square section or slightly thicker than wide; this arrangement may, however, be varied to meet the necessities of the design; the rebates, stops, mouldings, &c., are worked in the solid. The outer vertical members of these frames are called josts or jambs; interior ones, mullions. The horizontal members are sill, transom and head. The jambs are framed between the head and sill, chiefly that the ends or horns of the latter may run beyond the frame, and so provide fixings that can be built into the wall; and



A Segment-headed Frame.

Fig. 177.

also because the shoulders of the post form a better abutment for carrying any load that may be thrown on the head than the edge of a tenon would. Transoms are cut between the jambs and also between the mullions when these are used.

A Segment-headed Frame is shown in Fig. 177, and enlarged details of the joints in Figs. 178, 179, 180. The heads of these frames are cut out of the solid when the rise will permit of their being cut from deals of ordinary width. When this cannot be done, they are made in two lengths, jointed at the crown, and fastened with a handrail bolt. The horns are taken out level at the springing line, and the back is made roughly parallel to the shoulders for convenience in fixing. When the frame is 4 in. and upwards in thickness, double tenons should be used, as shown in Fig. 179; and if the position in which the frame is to be fixed does not admit of the horns being left on, the mortises should be haunched back, as shown in Fig.

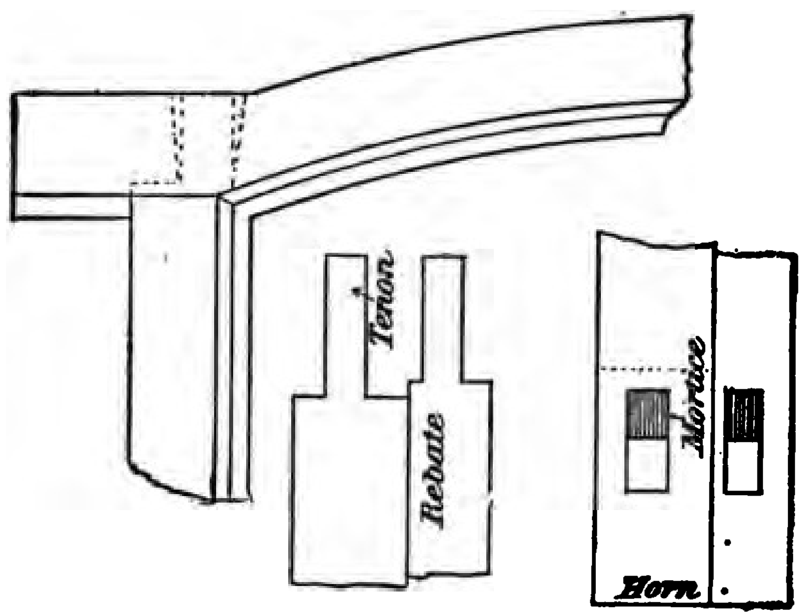
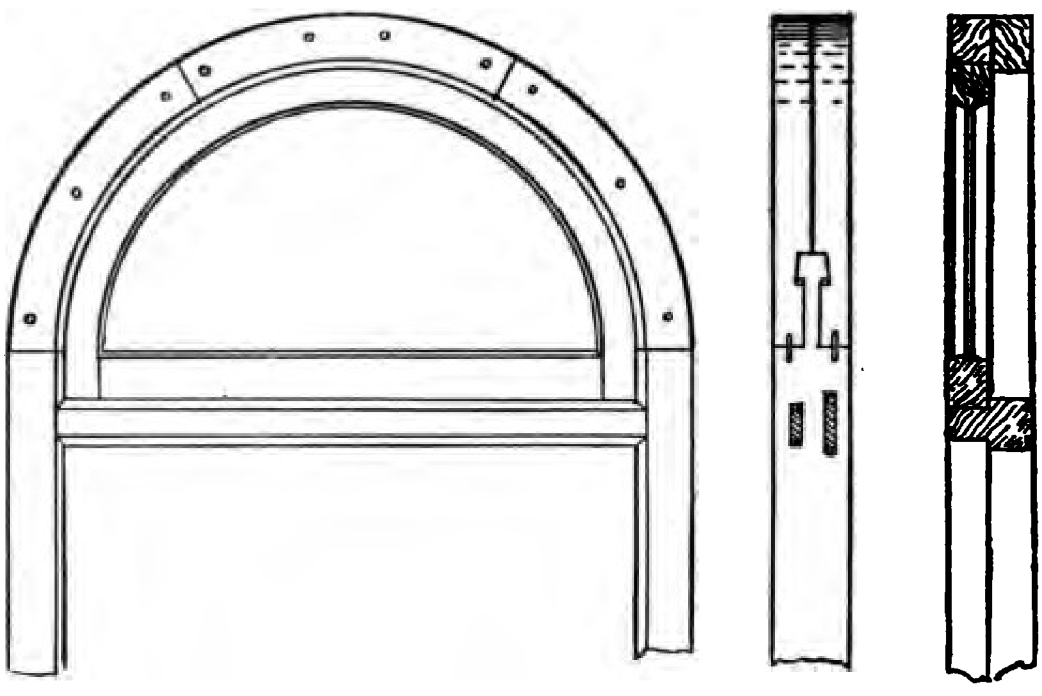


Fig. 178. Fig. 179. Fig. 180.

180, although the horns would not be cut off until the frame was fixed, as they would be required for the purpose of cramping up the frame.

A Semi-headed Frame may have its head cut in two or three lengths and bolted together, but is frequently built up as shown in Figs. 181, 182, 183. The jambs and transom are worked solid, but the head is formed in two thicknesses glued and screwed together, one layer being in two lengths, the other in three, so as to break joint. This is both a strong and economical way of forming a head, because the grain is less cut across than it would be in a head cut out of one thickness, and the labor of rebating is also dispensed with, the inner ring being kept back ½ in. to form a rebate. The head is fastened to the jambs by hammer-head tenons and shoulder tongues, as shown in Fig. 182, and double tenons are used for the transom to



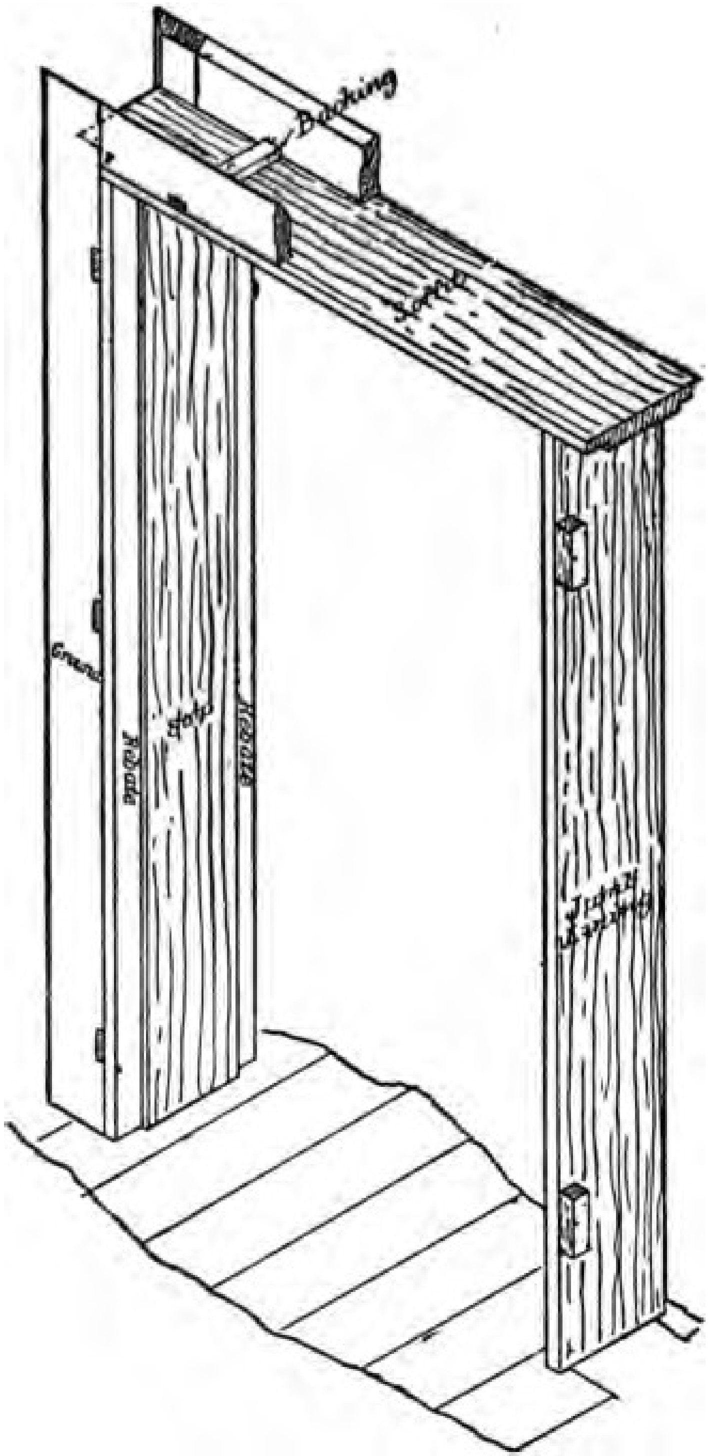
A Semi-headed Solid Frame.

Fig. 181. Fig. 182. Fig. 183.

avoid cutting the root of the head tenons. The transom is better kept about 3 in. below the springing as shown, to ensure a strong joint. But if the exigencies of the design necessitate its being placed at the springing, then the jamb should be carried above the springing, and a portion of the curve worked upon it, because if the two joints are made together, the connection will be very weak.

There are four varieties of door casings or **Jamb Linings** as they are also termed, viz., **Plain, Framed, Double Framed,** and **Skeleton Framed,** these names defining the method of construction. Other sub-names are also used denoting the nature of the ornamentation, as in doors, with which they agree in the general arrangement of their parts. The term **Plain** is applied to any wall lining however it may be treated, if it is made of one flat board or surface.

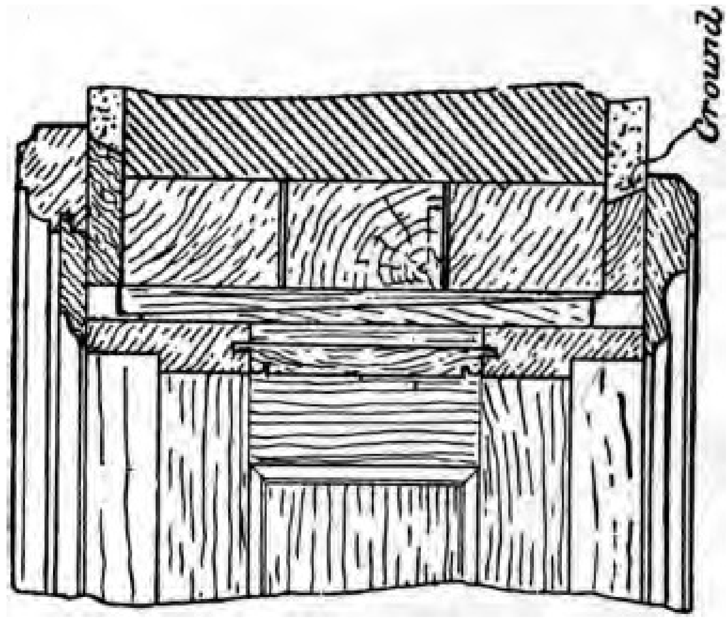
A "Set" of Linings comprise a pair of jambs and a head or soffit lining. The flat, against the edge of which the door rests when closed, is called the **Stop,** and in common work these are merely nailed upon the surface of the main lining, being kept back from the edge sufficiently far to form a rebate for the door. In better work the rebate is worked in the solid, the lining in such case being thicker. Not less than 1½ in. stuff should be used for any lining to which a door has to be hung, as the rebate takes ½ in. out of the thickness, leaving only 1 in. for screw hold for the hinges. This, however, may be supplemented by hinge blocks glued to the back of the lining just behind where the hinges will be inserted, as shown on one side of the isometric sketch of a set of **Plain Jamb Linings** (Fig. 184). When the lining is rebated on both edges, it is said to be "double rebated." Plain linings are not suitable for walls thicker than 14 in., in consequence of the amount of shrinkage which disarranges the finishings, and even in 14 in. work they are better framed.



Isometric Sketch of a Double Rebated Set of Plain Linings with Double Set of Grounds.

Fig. 184.

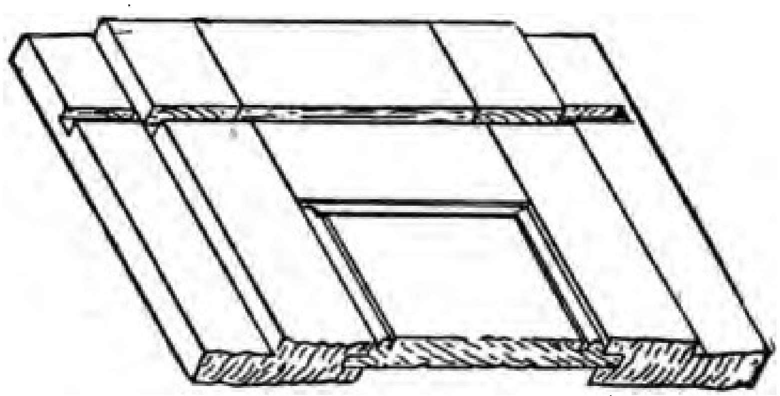
Grounds.—Figs. 188 and 189 are light frames forming a boundary to all openings in plastered walls, their



Head of a Framed Jamb Lining.

Fig. 185.

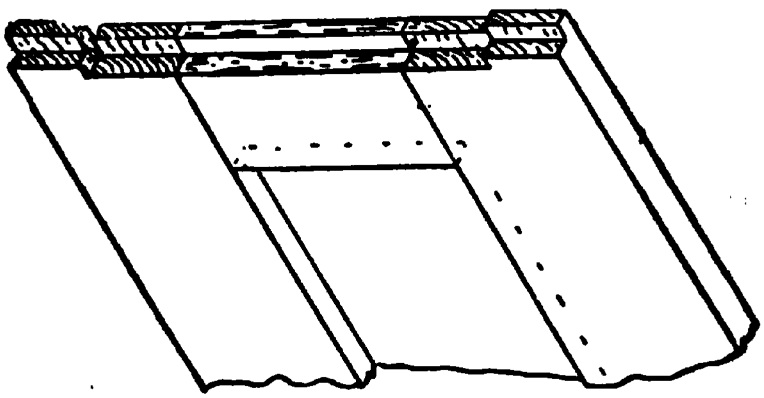
purpose being to act, as the name implies, as a groundwork for the linings, architraves, &c., and also as a



End of Soffit showing Housings.

Fig. 186.

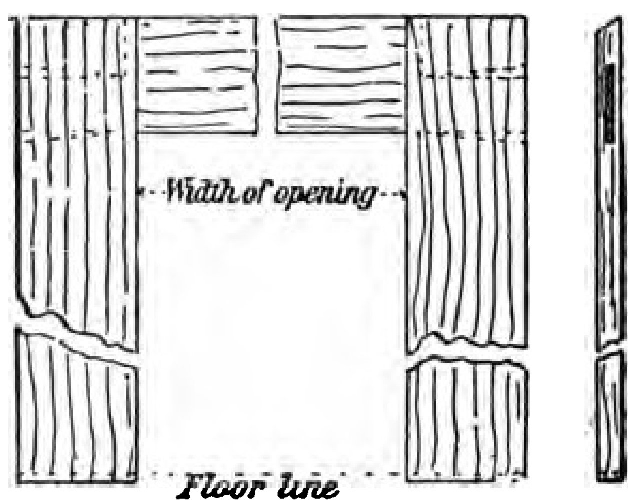
margin and gauge for the thickness of the plaster. They are either bevelled or grooved on the back edges to form a key for the plaster, and should be framed up perfectly square, and with faces and inside edges true, straight and square, to ensure good fittings in the finishings. They are fixed by nailing to plugs or wood



End of a Jamb Lining showing Tongues.

Fig. 187.

bricks in the walls, and should be secure, plumb and out of winding. They are usually prepared out of 1 in.



Elevation and Edge View of a Set of Grounds.

Fig. 188. Fig. 189.

stuff, their width depending upon the width of the architraves, which should overhang them about 3/8 in. More cover than this is not advisable, or the fixing for the outside of the architrave will be lost, and for the same reason much bevel should not be given to the back edge. A ¼ in. will provide quite sufficient key for the plaster, and also, as the grounds have to be bevelled before glueing up, a very thin edge is liable to be crushed in cramping them up. The sets of grounds on each side of an opening are connected across the jamb by 1 and 2 in. slips called Backing pieces, dovetailed, and nailed to the edges. Grounds

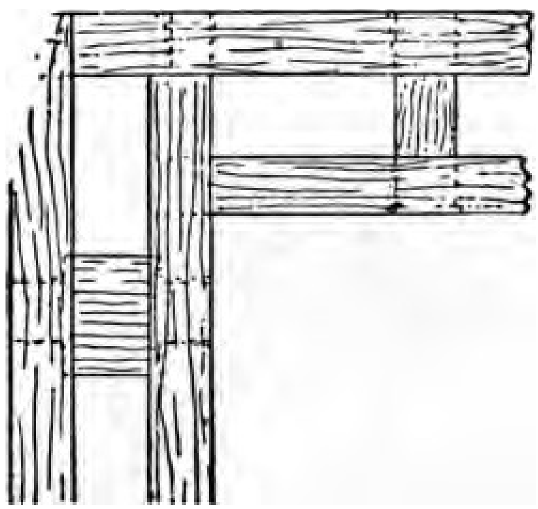
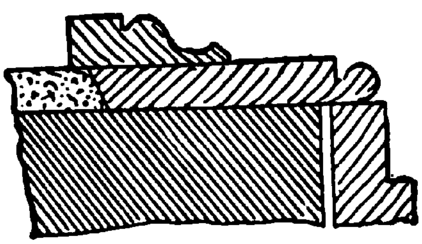


Fig. 190.

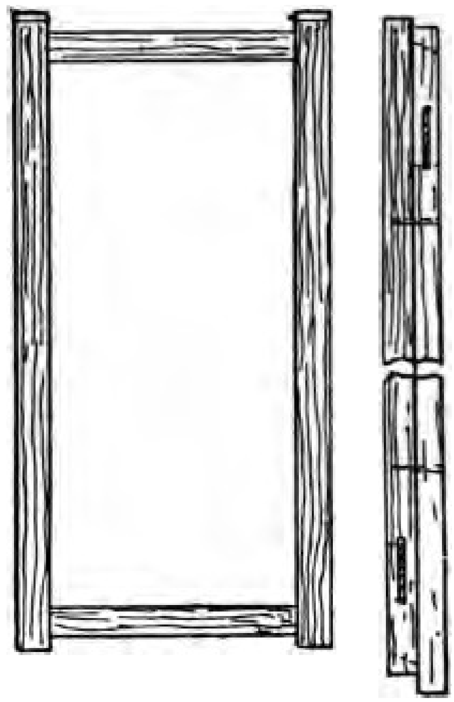
are, however, very often fixed flush with the brickwork, which does not permit of the use of backings; it is a very inferior method, the linings either touching the walls, or if made smaller, having to be fixed with folding wedges and firring pieces, which do not afford so firm a fixing. When the architrave is of such width that the sides of the ground are required more than 5 in. wide, it is not advisable to make them in one piece, because from their position they are very liable to cast, and so spoil the appearance of the finishings. In such cases, the grounds should be formed as skeleton frames with rails and stiles, where required from 3 to 5 in. wide (see Fig. 190).

Grounds are sometimes moulded or beaded on their inside edges, as in Fig. 191. They are then called **Architrave Grounds** or **Moulded Grounds**, and are fixed to the edges of the linings, which in such cases must



A Moulded Ground.

Fig. 191.



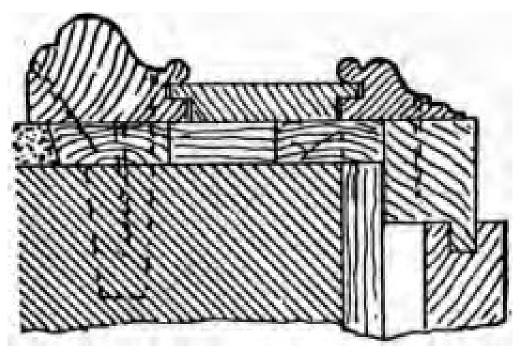
Method of Arranging "Grounds" for Glueing Up.

Fig. 192.

be fixed first. The moulded ground is more frequently used in window and shutter openings than in door openings. All joinery finishings are fixed to grounds, but these are seldom framed, except in the cases of door, window and shutter openings, all other work being fixed to rough grounds, which will be illustrated in their appropriate place.

Framed Grounds are usually wedged up and sent out from the shop in pairs, as shown in Fig. 192, Fig. 193 being an edge view to larger scale. Two sets are lightly nailed face to face, and with heads reversed, in which position they can be knocked apart and glued, then cramped, squared, and wedged up with facility.

Architraves are the moulded borders or frames to window or door openings. When square at the back, as in Fig. 191, they are termed **Single-faced**; when moulded at the back, as in Fig. 185, **Double-faced.** No

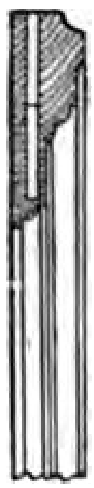


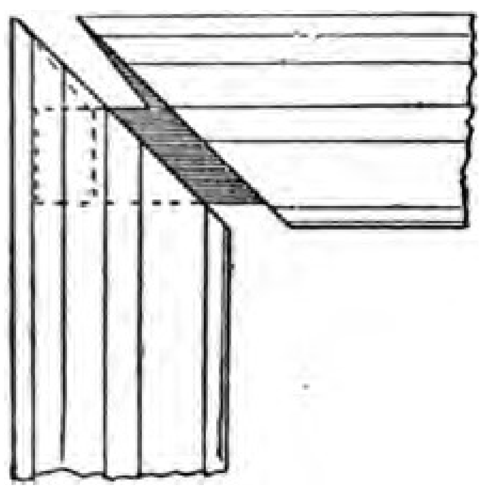
Framed Architrave
and Grounds.

Fig. 193.

architrave should be made wider than 6 in. in one piece, as there is great danger of its splitting when shrinking, being fixed necessarily at both edges; but when this size is exceeded, the moulding should be made up of two or more members, grooved and tongued together at some convenient point, as shown in Figs. 185 and 193, the latter being an example of a very wide so-called **Framed Architrave**; then, if fixed at both edges, it will be free to shrink in the middle.

Heavy Architraves are ploughed and cross tongued in the mitres, and in some cases framed or stub mortised and tenoned, as shown in Figs. 194 and 195. These are glued and fixed with screws turned in from the back. Small handrail bolts are also used to draw up the mitres in thick mouldings. Sets of architraves so treated are framed up on the bench, provided with stretchers at the bottom end, and braced to prevent

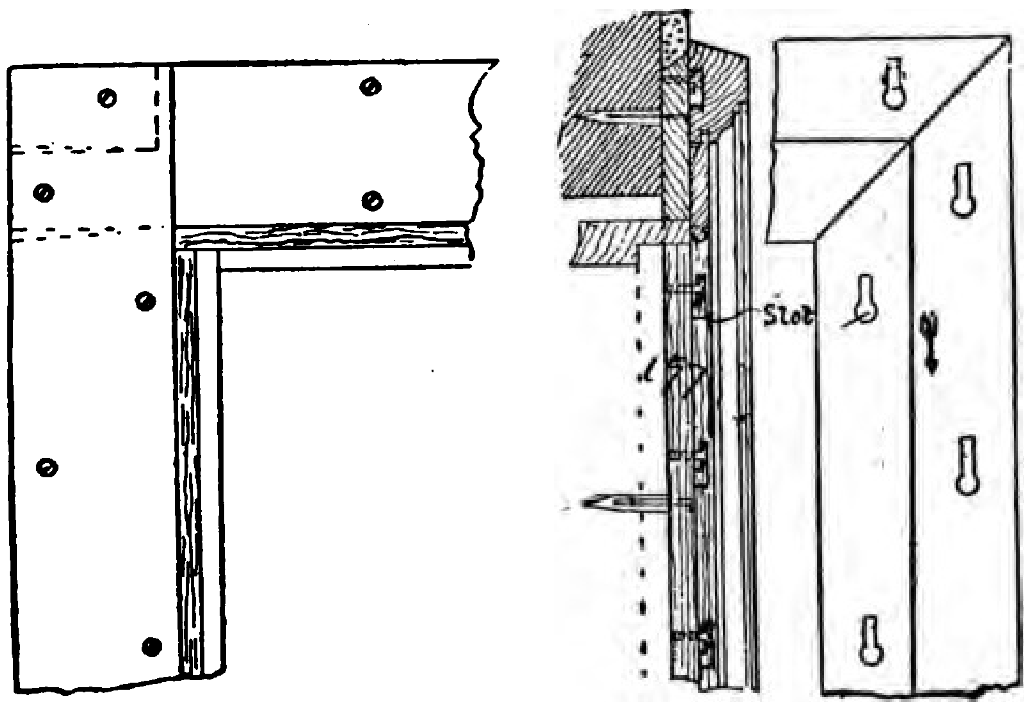




An Architrave Tenoned and Mitred.

Fig. 194. Fig. 195.

racking during transit, and are fixed complete. Smaller ones are mitred and fixed piece by piece. In the best class of polished work, secret fixings are used for securing the architrave to the grounds. These are shown in Figs. 196, 197 and 198. A number of stout screws are turned into the grounds with their heads projecting uniformly about ¼ in. Corresponding holes and bevelled slots are made in the back of the moulding, which is dropped on the screws, and carefully driven down in the direction of the arrow. The architrave should be driven on to the screws dry in the first place; two workmen being employed in the operation, one upon a scaffold carefully drawing down the architrave

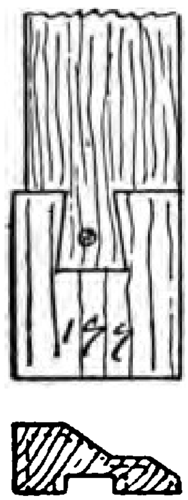


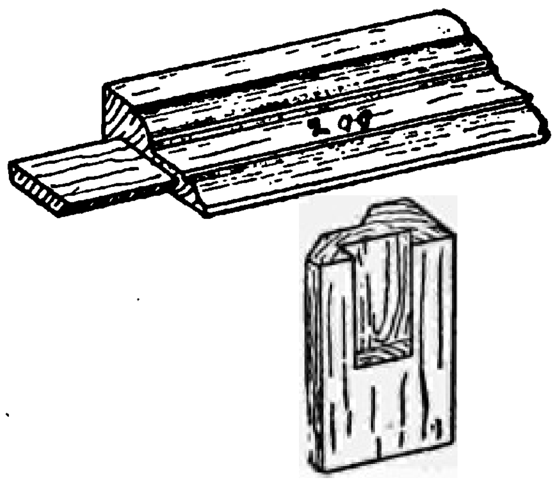
Elevation of Back of Architrave. — Section of Architrave. and Elevation of Face of Grounds.

Fig. 198. Fig. 197. Fig. 196.

equally on each side; the other pressing the moulding tightly towards the grounds, and striking a piece of wood with a heavy hammer over each screw as his fellow strikes the top. After being duly fitted the architrave may be knocked up again, and the front edges only lightly glued before replacing.

Foot or **Plinth Blocks** are used at the bottom of all architraves in good work, and are secured in two ways (see Figs. 199 and 200). The latter is the better way, as there is less difficulty in fitting the shoulder, and also less danger of splitting the plinth, when driving the dovetail tenon in; this is glued and secured with





Method of Fixing Plinth to Architrave.

Fig. 199. Fig. 200.

a screw turned in the back. A handscrew may be used to grip the sides of the block whilst driving the latter on to prevent its splitting.

Door and Jambs Complete.—Figs. 201 and 202 show a door, with jambs and linings complete. The details for constructing and fastening in place are all shown. Each member of the door is named, also base and grounds.

Figs. 202 to 220 show a number of typical doors, some of which will be found suitable for any particular purpose.

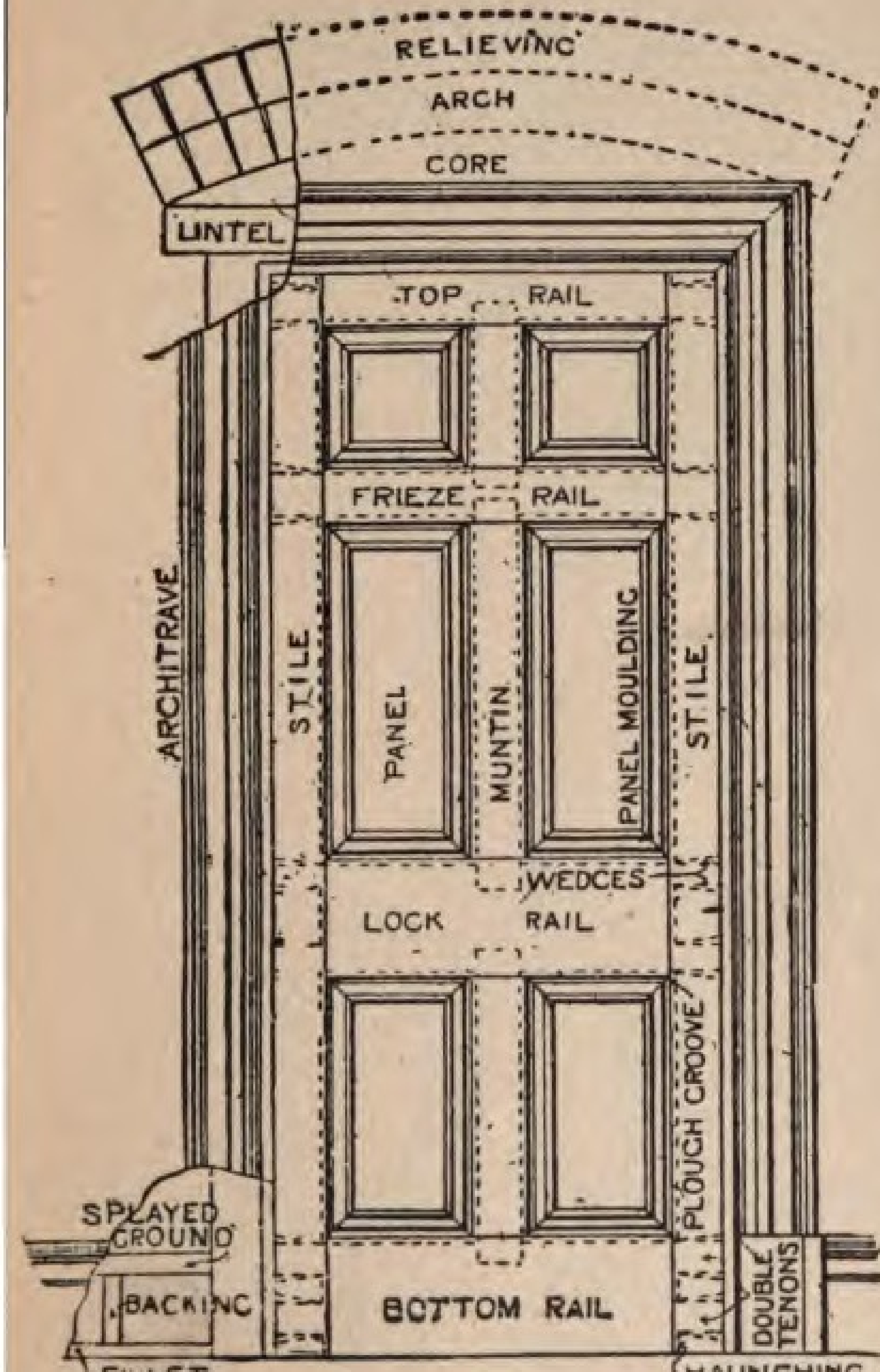


Fig. 201.

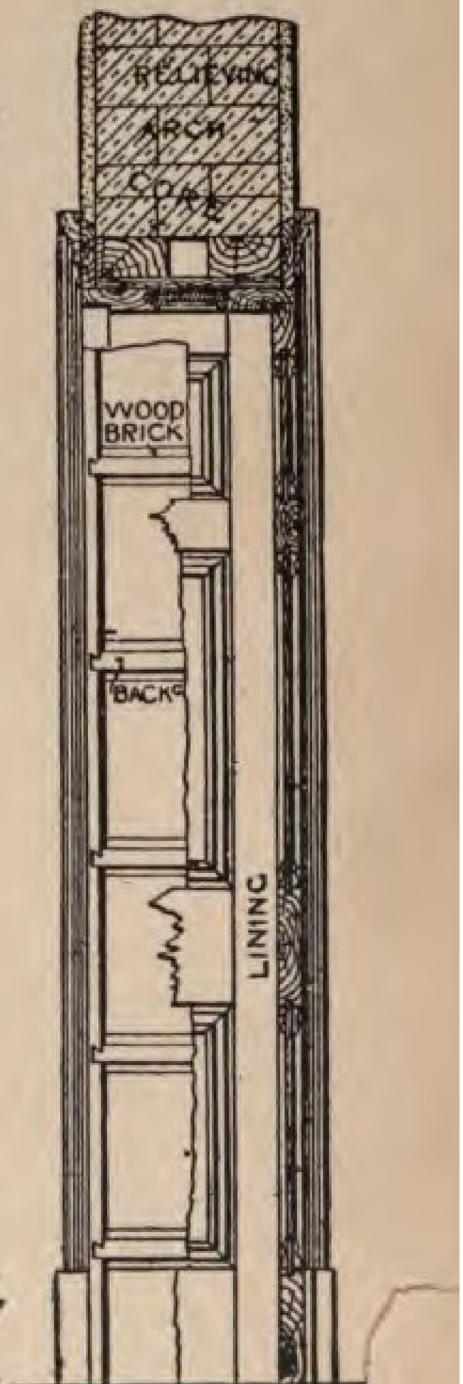


Fig. 202.

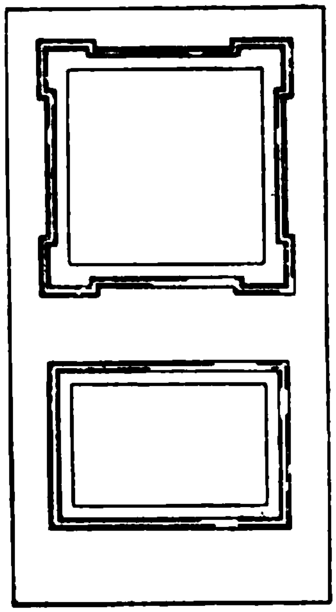


Fig. 203.

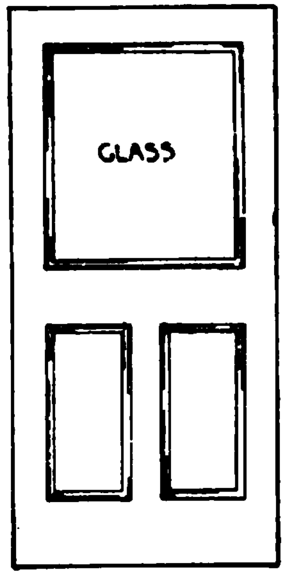


Fig. 204.

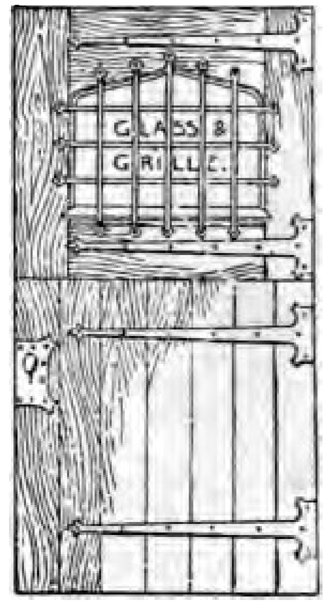


Fig. 205.

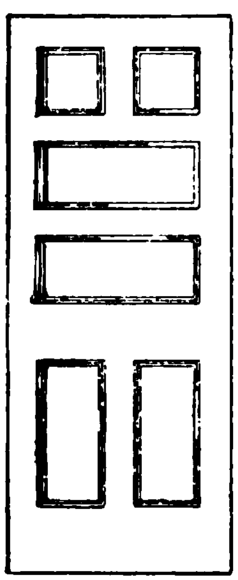
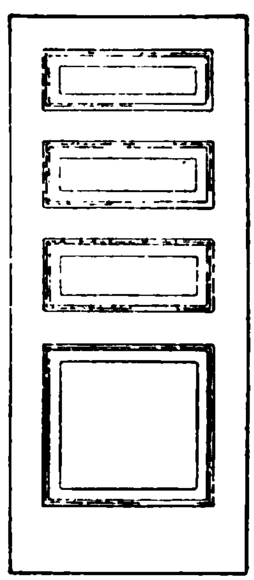


Fig. 206.



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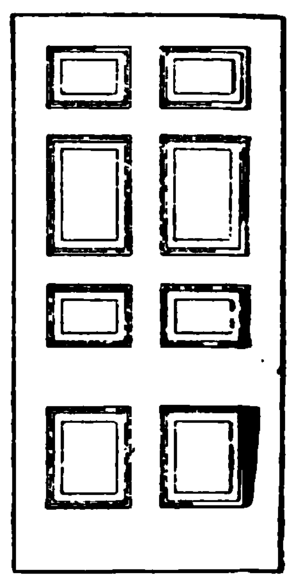


Fig. 208.

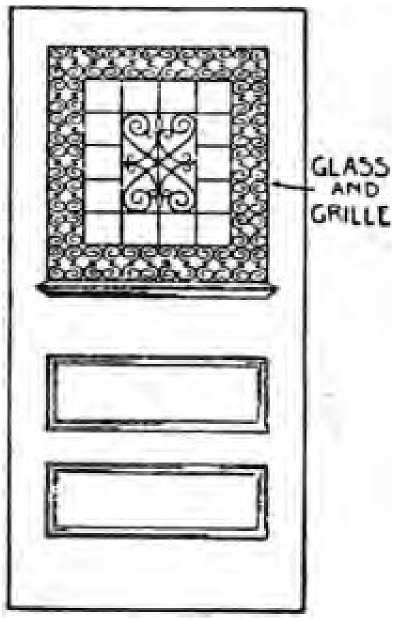


Fig. 209.

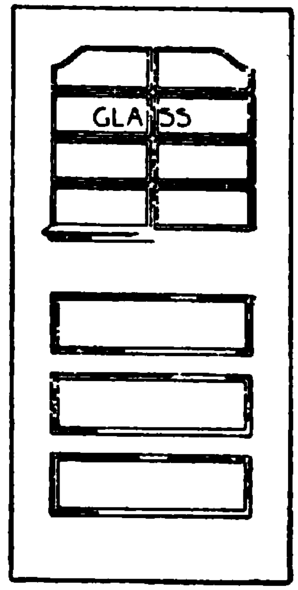


Fig. 210.

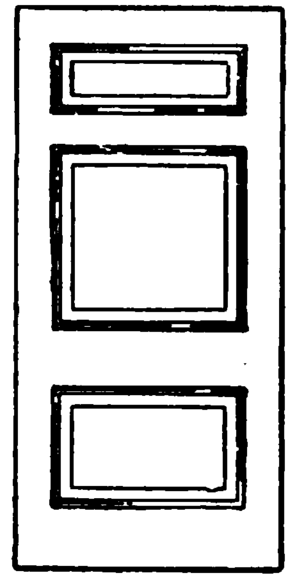


Fig. 211.

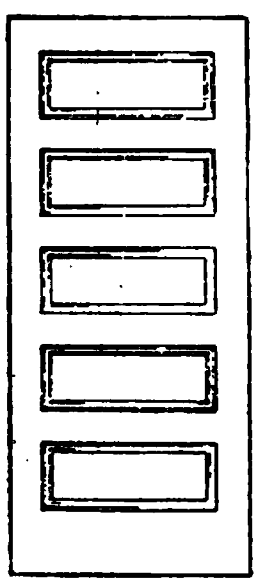


Fig. 212.

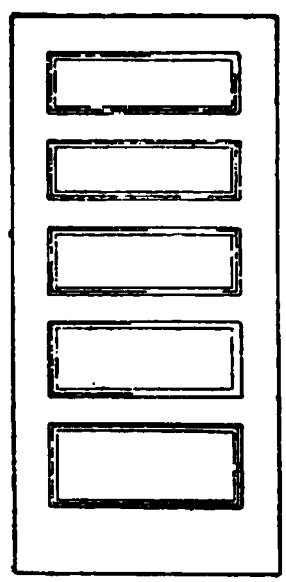


Fig. 213.

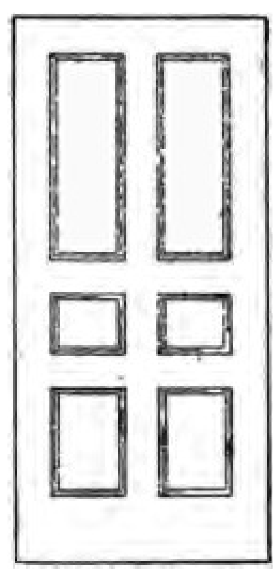


Fig. 214.

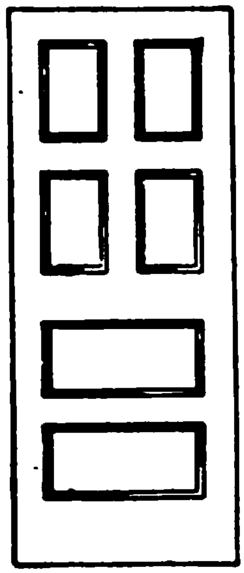


Fig. 215.

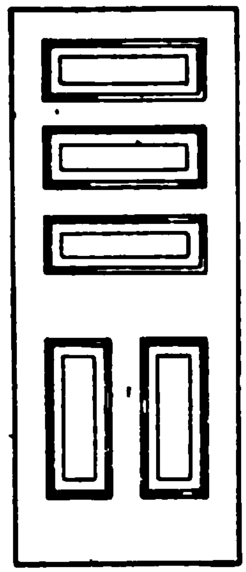


Fig. 216.

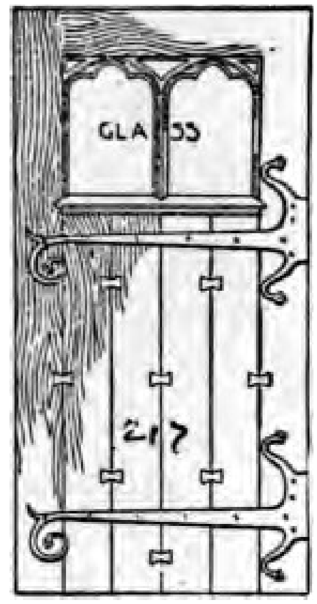
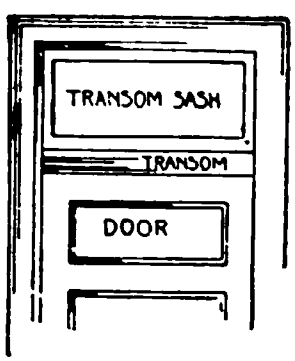


Fig. 217.



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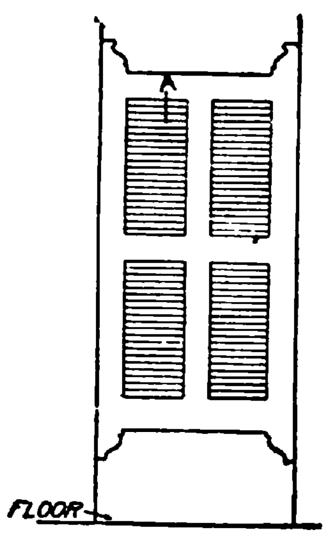


Fig. 219.

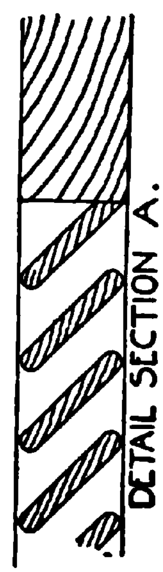


Fig. 220.

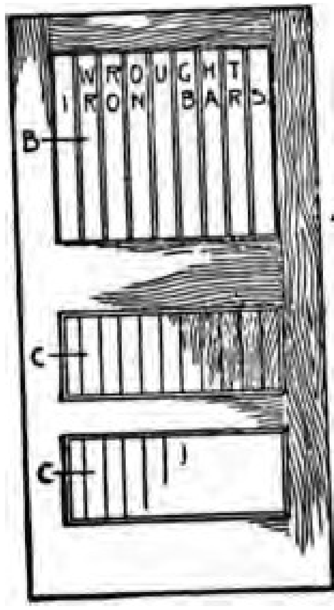


Fig. 221.

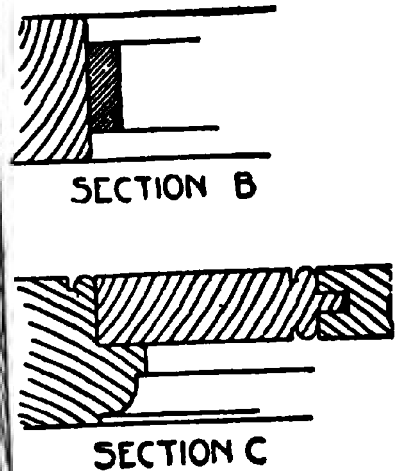


Fig. 222.

PART III.

MECHANICS OF CARPENTRY.

In this part I intend to give the reader some practical instructions in "The Mechanics of Carpentry," with the methods of calculating the strength of trusses, and timbers for structural purposes. All the rules shown have been taken from the best authorities, as Tredgold, Barlowe, Hurst, Riley, Trautwine, Hatfield, Kidder, Nicholson and others. I have, in a few instances, corrected, and added to some of the propositions, which I have found in practice, to be necessary.

INTRODUCTION TO THE MECHANICS OF CARPENTRY.

It is well known that some members of a framed structure must be made stronger than others. The reason is that the weights or other forces acting on a truss differ from each other in magnitude and direction. It is obviously necessary, therefore, to be able to estimate the various forces acting, so that the members may be made of the required strength without undue waste of material. The general principles underlying the measurement of forces may with advantage now be considered briefly.

The Nature of Force. Force may be defined as that which moves, or tends to move, a body at rest, or which changes, or tends to change, the direction or rate of motion of a body already moving. A familiar example of force is met with in gravitation, whereby an object has a tendency to fall to the ground. In order to support it, an upward force equal to the weight of the object must be exerted. The phrase "equal force" implies that forces can be measured. In this country they are usually measured in terms of weights in lbs., cwts., etc. Any one who has seen a pulley, or lever, at work knows that the direction of application of a force can be changed. Evidently, then, forces can be represented graphically. Lines drawn to scale are employed and these can be arranged to exhibit at the same time both the magnitude and the direction of the forces. Thus, a weight of 10 lbs. acting vertically downwards can be represented by a vertical straight line 10 units in length. If the unit of length be 1/8", the line will measure ten times 1/8"=1¼"; whereas, if the unit of force be represented by a length of 1", the graphic representation of the force will be a vertical straight line 10" long.

Resultant of two or more Forces. (1) When two or more forces together act at a point in the same direction and in the same straight line, the **resultant** force is equal to the sum of the **components.**

Example. (a) If two 10 lb. weights attached to a cord are hung upon the same nail, the resultant weight acting upon the nail is 10+10=20 lbs.

(2) If two equal forces together act at the same point in opposite directions, but in the same straight

line, they neutralize each other, and the forces are said to be in **equilibrium.**

Example. A spring balance carries a weight of 6 lbs. The index finger of the balance shows that the spring exerts an upward force equal to the downward force—the weight; and a state of equilibrium is obtained.

If the two unequal forces together act at the same point in opposite directions, but in the same straight line, the resultant force is equal to the difference between the forces, and is in the direction of the greater.

It is evident, then, that the directions of the forces, and therefore the angles they make with one another, must be considered in determining the forces acting at any given point.

If a flexible string be attached to a weight, and then passed over a frictionless pulley, there will be the same tension in every part of the string, irrespective of any change of direction caused by using the pulley.

To illustrate these facts clearly, suppose that two 7 lb. weights, connected by a cord, hang over a smooth peg as shown in Fig. 1. The total weight on the peg, neglecting the weight of the cord (which may thus be any length), is 14 lbs., the sum of the two weights.

Again, suppose three such pegs in a horizontal straight line, and the cord and weights to be passed over them as shown in Fig. 2. Evidently the weight on the central peg is nothing. Now, suppose the outside pegs to be lowered slightly, as shown by dotted lines in the figure; the central peg will now carry a small proportion of the weight, and the more the outside pegs are lowered, the more weight will be thrown on the central peg, until, as shown in Fig. 1, it carries all the weight, i. e., 14 lbs. Therefore the weight upon the central peg varies according to the direction of the forces acting on it—from nothing in Fig. 2 to 14 lbs. in Fig. 1.

The magnitude and direction of the resultant force acting upon the central peg, and upon each of the outside pegs, can be determined by the parallelogram of forces.

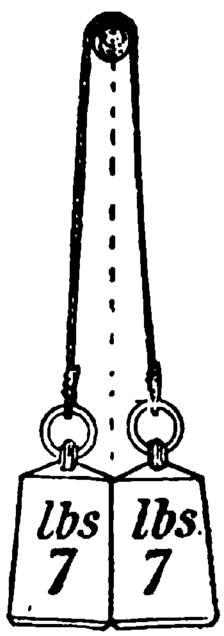


Fig. 1.

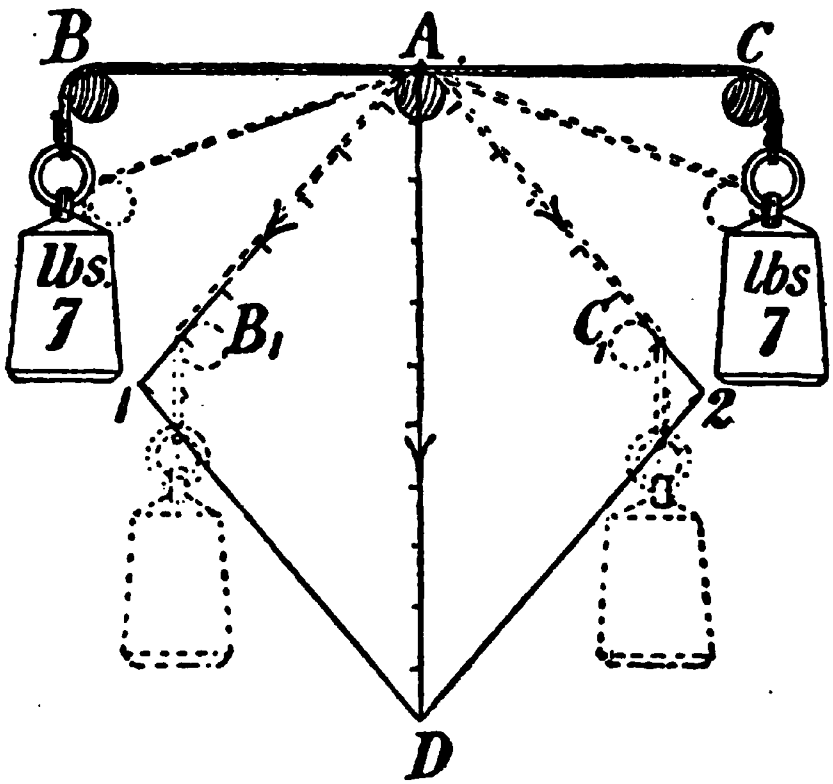


Fig. 2.

The Parallelogram of Forces.—If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through the point.

Example 1. The angle at A, when the cord passes over the pegs B_1, A,C_1, shown by the dotted lines in Fig. 2, is given. Determine by the parallelogram of forces the stress on the peg A, i. e., the single force acting through the point A, which shall be equal in effect to the forces AB_1, AC_1 acting together.

Produce AB_1 and AC_1, and mark off on each line 7 units, measuring from A. Then A1 and A2 represent in magnitude and direction the forces caused by the loads. Complete the parallelogram by drawing 1D parallel to A2, and 2D parallel to A1. The length of the diagonal A D, measured in the same units as the lines A1 and A2, represents the magnitude of the resultant force—i. e., the stress on the peg A. The direction of the force will obviously be downwards. A force represented in magnitude and direction by D A would evidently counterbalance the force A D, and would therefore counterbalance A1 and A2 acting together. **Forces which balance each other are said to be in equilibruim.**

Example 2. Determine the magnitude and direction of the single force which will replace the two forces exerted by the cord and weight on the peg B_1 (Fig. 2).

Draw ab 7 units long (Fig. 3) and parallel to the cord A1 in Fig. 2. From b draw bc also 7 units long and parallel to the cord below the peg B_1. Complete the parallelogram by drawing dc and ad parallel to ab and bc respectively. Then the diagonal bd gives the magnitude of the required force, the direction of which is from b to d.

Example 3. Fig. 4 shows the application of the parallelogram of forces to determine the resultant force on the peg B.

In the above examples no allowance has been made for the weight of the cord or for the friction on the pegs. It is assumed in each case that the forces are acting at the point of intersection of the straight lines produced.

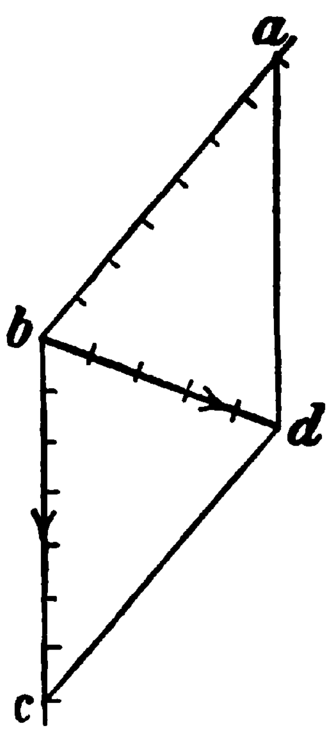


Fig. 3.

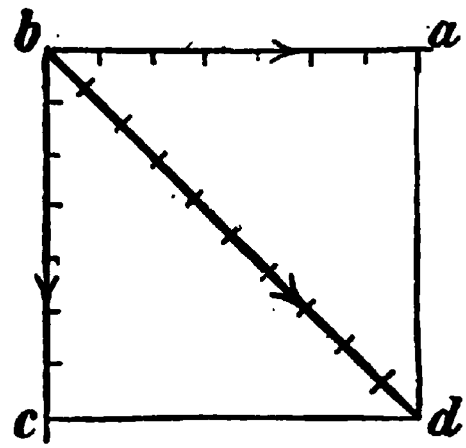


Fig. 4.

Example 4. Two forces of 10 and 6 lbs. respectively act from a point and in directions which are at right angles to each other. Determine the magnitude and direction of the single force which can replace the two forces.

Let the line A B (Fig. 5) represent in magnitude a force of 10 lbs. acting at the point A in the direction indicated by the arrow, and A C a force of 6 lbs. acting at right angles to A B. Complete the parallelogram A C D B. Then the length of the diagonal A D represents the magnitude of the resultant force, and the direction in which it acts will be from the point A, as shown by the arrow.

It must be understood clearly that a resultant is a force which can take the place of, and will produce the same effect as, two or more forces. To maintain equilibrium, the resultant force must be counterbalanced by an equal force acting in the opposite direction. The force so acting is called the **equilibrant.**

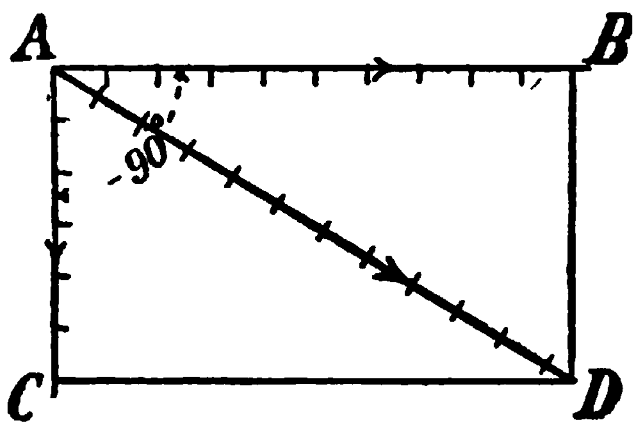


Fig. 5.

Example 5. Figs. 6 and 7 show the magnitude and direction of the resultant force when forces of 9 and 6 lbs. respectively act at angles of (a) 120°, (b) 45°.

The simple apparatus shown in Fig. 8 clearly illustrates the principle of the parallelogram of forces. On a vertical board are fixed two small pulleys by means of screws, so that they revolve with as little friction as possible. By making a three-way string, passing it over the two pulleys, and adding varying weights to each of the three ends of the string, it can be demonstrated clearly how the three forces act. In Fig. 8 the weights are respectively 5, 6, and 4 lbs. By drawing the parallelogram A B D C, such that A B equals 5 units in length, and A C equals 4 units, the diagonal D A is found to measure 6 units, and to represent the magnitude of the middle weight. If other weights are attached to the ends of the strings, different results will, of course, be obtained.

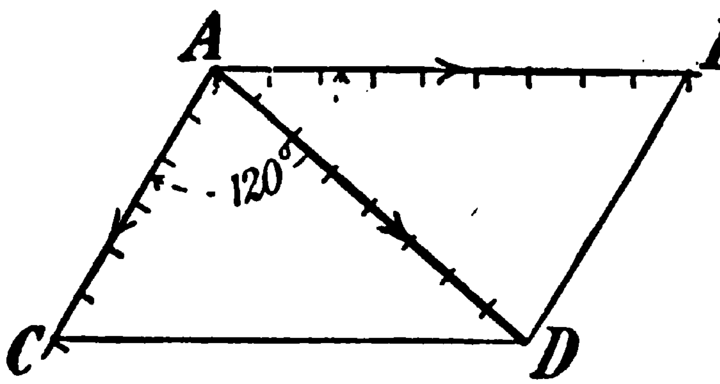


Fig. 6.

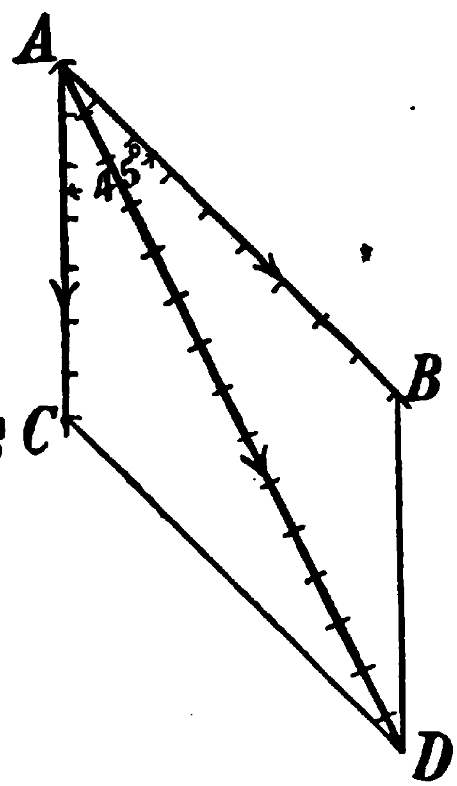


Fig. 7.

Triangle of Forces. The triangle of forces is used to determine the magnitude and direction of any three forces which balance each other. The rule may be stated as follows: **If three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the three sides of a triangle taken in order.**

Example 1. The forces acting upon A (Fig. 8) are in equilibrium.

Since the length of the line A B=5 units, and the line B D is parallel and equal in length to A C=4 units, and the diagonal D A is in a line with the direction of the middle vertical weight and equal in length to 6 units; then the sides A B, B D, and D A of the triangle A B D represent both in magnitude and direction the forces acting at the point A.

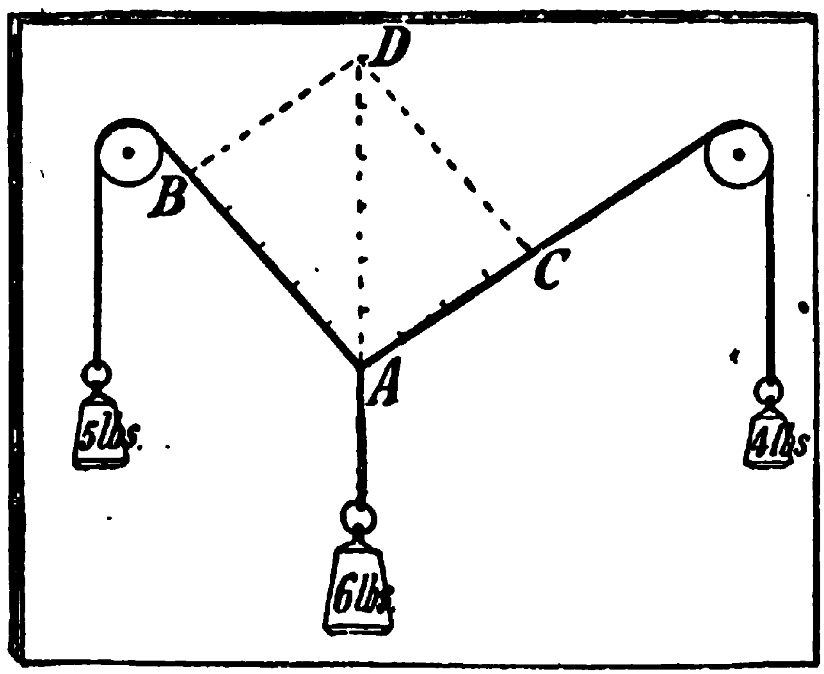


Fig. 8.

To save confusion it is usual, however, to draw a separate triangle to illustrate these forces. A somewhat different system of lettering also simplifies the consideration of the examples. This is known as **Bow's notation.** In it the two letters denoting a force are placed one on each side of the line representing the force, that is, in the spaces between such lines. Thus in Fig. 9 the three forces acting at the point o are referred to as A B, B C, C A respectively.

Example 2. Given the magnitude (9 lbs.) and the direction (indicated by the arrow) of A B, and the angles which the directions of the three forces make with each other, it is required to find the magnitude and direction of B C and C A when the forces are in equilibrium.

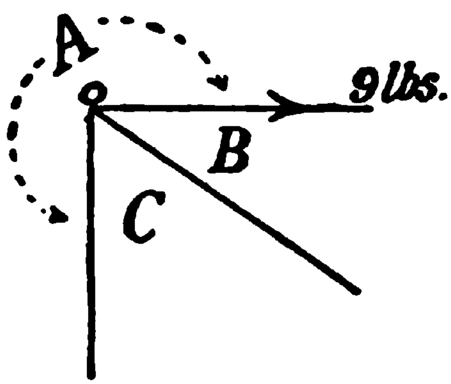


Fig. 9.

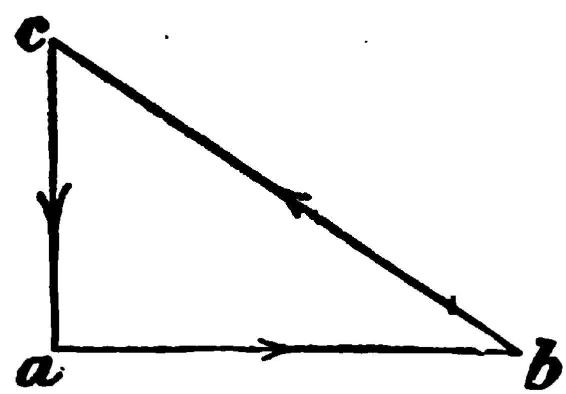


Fig. 10.

Draw the line a b (Fig. 10) parallel to the direction of action of the force A B, 9 units long, and in the direction shown by the arrow. From b draw b c parallel to B C until it meets a c drawn parallel to C A. Then the triangle a b c is the triangle of forces, and the direction of the forces B C and C A can be found by taking the sides of the triangle in order, viz., a to b, b to c, c to a; and these directions give also the directions of action of the forces represented by the lines parallel to a b, b c, and c a respectively. Thus A B acts from the joint o; B C acts towards o; and C A acts from o.

The following examples show the application of these principles to simple practical questions.

Example 3. A rope bears a tensile stress (pull) of 30 cwts. Find the magnitude of the stress in each of two other ropes which make an angle of 60° with each other, and together balance the stress in the first rope, supposing the second and third ropes are equally stressed.

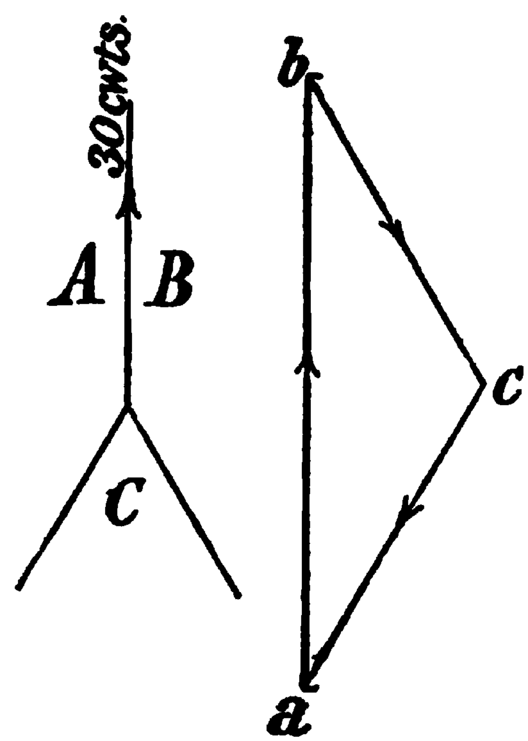


Fig. 11.

Fig. 11 shows the application of the triangle of forces to the solution of this question, the answer giving the stress in each rope as 17.32 cwts. By going round the sides of the triangle in order, it will be seen that the force in each of the three ropes acts from the joint.

Example 4. A buckling-chain is used to raise heavy blocks of stone. What is the amount of stress in the links of the chain when raising a weight of one ton, if the buckling-chain is:

(a) pulled tightly as in Fig. 12;
(b) placed loosely round the stone as in Fig. 13.

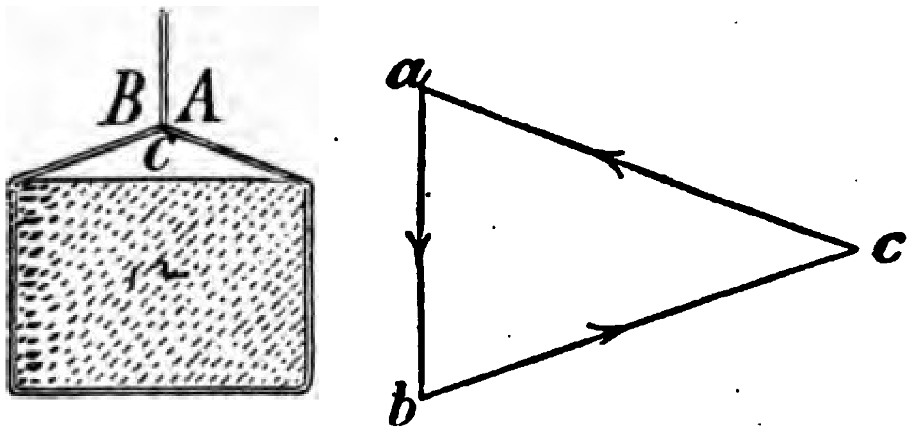


Fig. 12.

The correct solution of this question depends on (1) the weight of the stone; (2) the angle between the forces A C and B C.

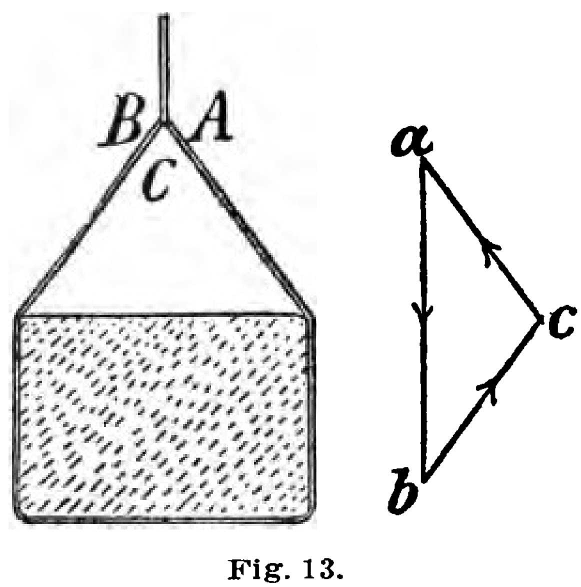


Fig. 13.

The application of the triangle of forces in each case (Figs. 12 and 13) shows that the stresses A C and B C are more than twice as great when the chain is fixed as in Fig. 12 as they are with the arrangement in Fig. 13; or, the tighter the chain—i. e., the greater the angle between the forces B C and C A—the greater is the stress on the links.

Example 5. A triangular bracket fixed against a wall, as shown in Fig. 16, has a weight of 5 cwts. suspended from the outer end o. What is the nature and amount of stress in each of the members o A and o B?

Fig. 15 is the triangle of forces used to determine these stresses, and is drawn as follows: $1_1 2_1$ is drawn

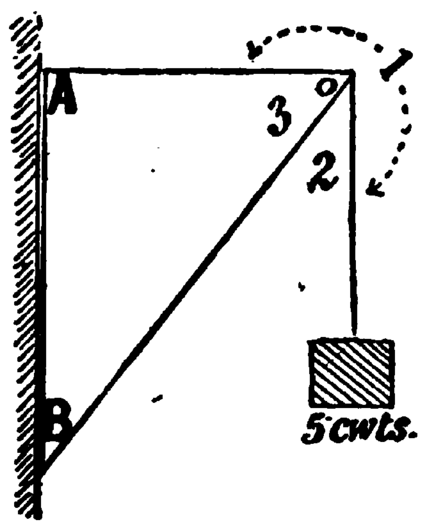


Fig. 14.

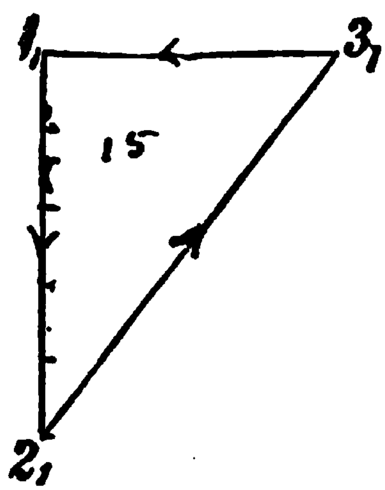


Fig. 15.

parallel to and represents the downward force (the weight of 5 cwts.) to scale. From 2_1 draw $2_1 3_1$ parallel to 2 3 in Fig. 14 until it meets $1_1 3_1$ drawn parallel to 1 3. Then the triangle $1_1 2_1 3_1$ represents the magnitude of the forces.

By going round the triangle in order as shown by the arrows, we find that 2 3 acts towards the joint o and is therefore a compression stress or thrust, and 3 1 acts from the joint and is therefore a tension stress or pull.

Fig. 16 shows a somewhat modified design of triangular wall-bracket, and Fig. 17 is the triangle of forces by which the stresses in the various members are ascertained.

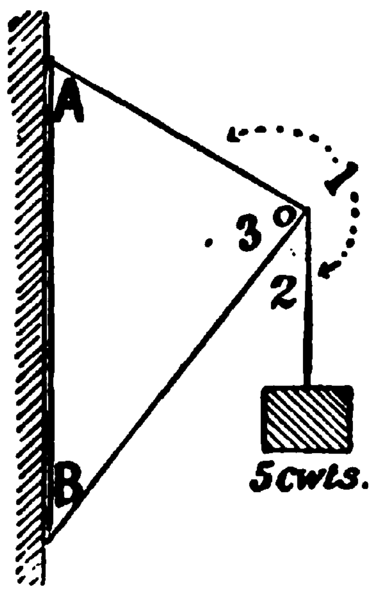


Fig. 16.

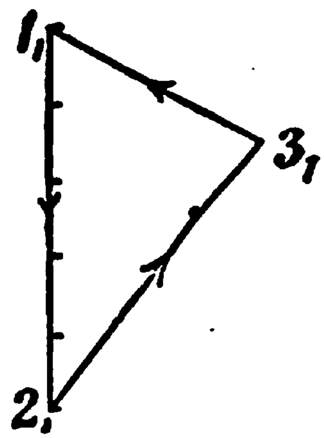


Fig. 17.

Example 6. What is the nature and amount of stress in each of the members A B and A C (Fig. 18) caused by the weight of 10 cwts. acting as shown?

This example may be taken as typifying a simple kind of roof-truss with the weight taking the place of the ridge piece. Re-letter or figure the diagram according to Bow's notation. Draw the vertical line $2_1 3_1$, equal in magnitude and direction to the weight 2 3. Complete the triangle by drawing lines parallel to the members A C and A B, from the points 2_1 and 3_1 respectively. These lines represent the amount of stress along the members A C and A B. On taking the sides of the triangle in order as shown by the arrows, it is seen that $2_1 3_1$ act downwards; $3_1 1_1$ acts towards the joint A, as does also $1_1 2_1$; therefore each member is subject to a compression stress (thrust).

Fig. 19 shows another example of this kind with a much smaller angle between the forces.

Fig. 20 illustrates a still further example, where the two sides are of unequal inclination.

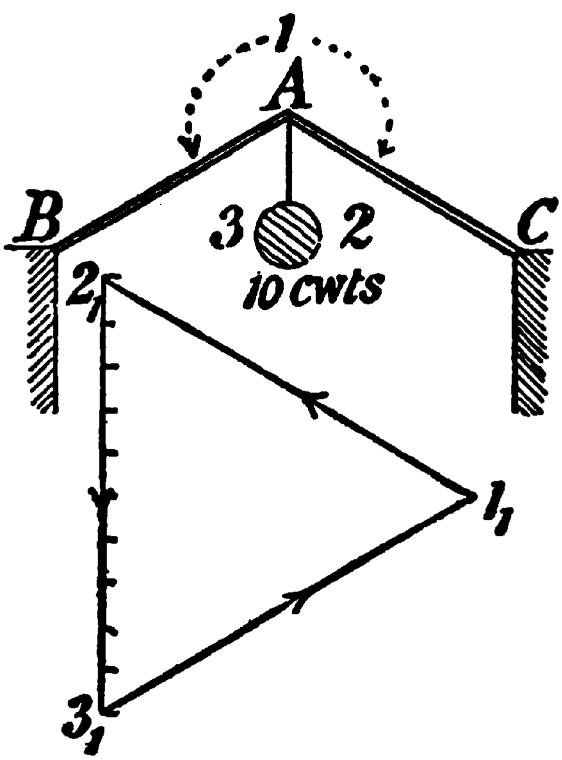


Fig. 18.

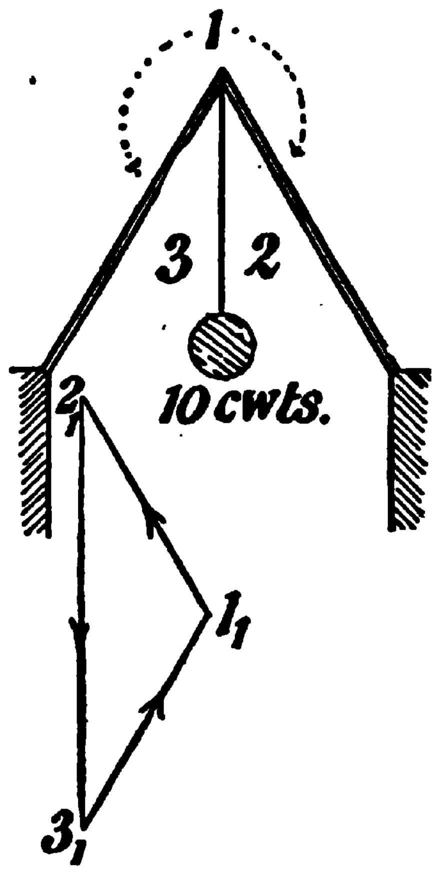


Fig. 19.

Polygon of Forces. The method of obtaining the resultant of any two forces acting at a point can be extended to three, four, or any number of forces.

Example. O A, O B, O C, O D, O E, (Fig. 21) represent the magnitude and direction of five forces acting at the point O. Determine the magnitude and direction of the resultant force.

This problem can be solved either by an application of the parallelogram of forces or by a direct construction.

(1) Determine by the parallelogram of forces, the resultant O 1 of forces O A and O B (Fig. 22). Similarly, determine the resultant O 2 of the forces O 1 and

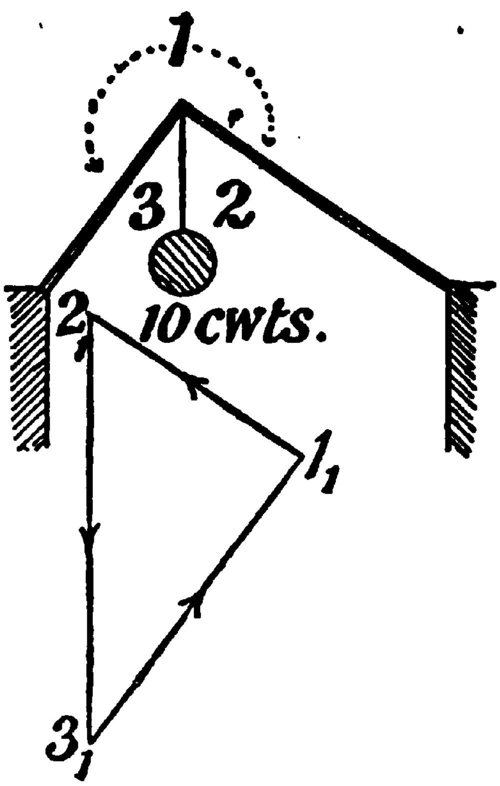


Fig. 20.

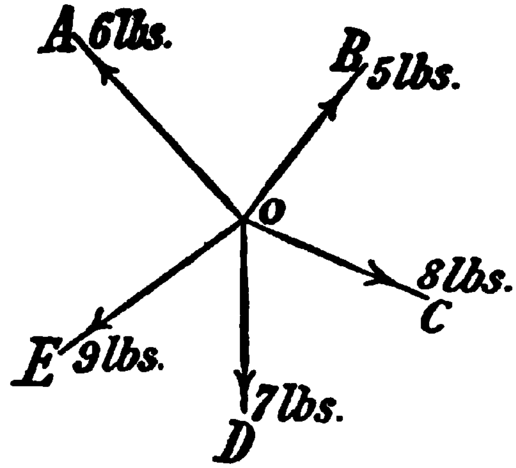


Fig. 21.

O C. Again, O 3 is the resultant of the forces O 2 and O D; and finally O 4 is the resultant of O 3 and O E. Therefore, O 4 is the resultant of all the original forces; or, in other words, a single force equal in magnitude and direction to the force O 4 will have the same effect at the point O as the five forces have when acting together. Since a force 4 O will balance O 4, a force represented in magnitude and direction by the line 4 O will, together with the five given forces, produce equilibrium at the point O.

(2) The same result may be obtained more simply as follows. Re-letter the forces as shown in italics (Fig. 22), and then, as in Fig. 23, draw a straight line a′b′

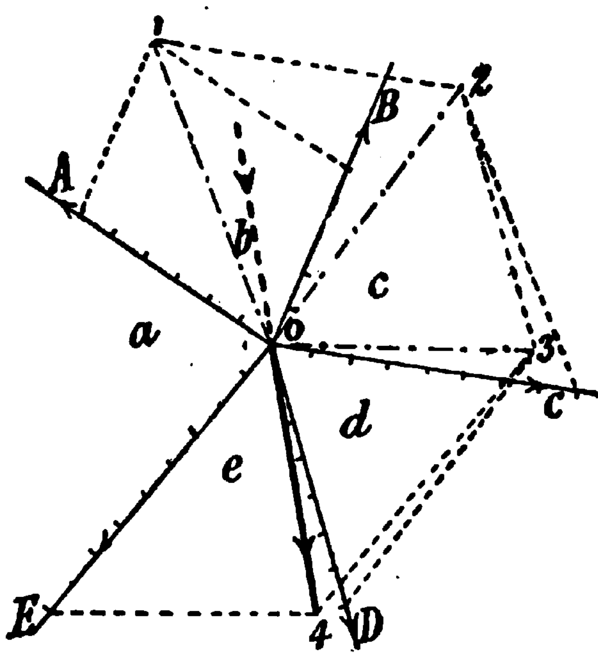


Fig. 22.

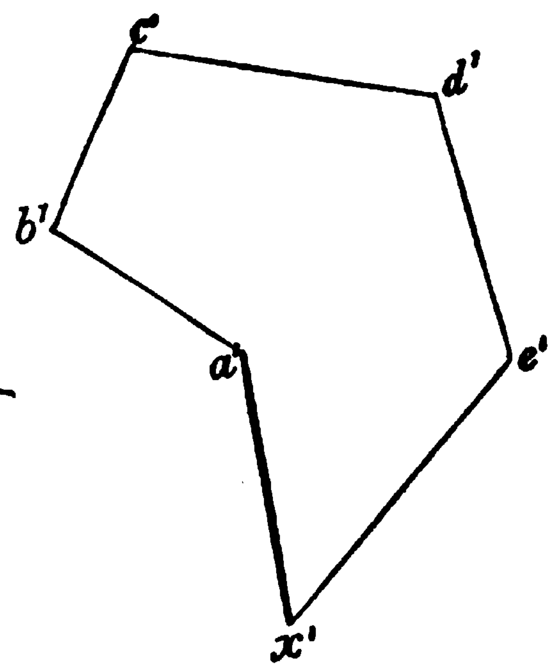


Fig. 23.

equal in magnitude and parallel to a b (Fig. 22). From b′ draw b′c′ equal and parallel to b c; continue the process, taking the forces in order. It will be found by drawing the closing line of the polygon, that is, by joining x′ to a′, that x′a′ gives the magnitude and the direction of the force required to produce equilibrium. Conversely, a′x′ is the resultant of all the original forces. By drawing the line 4 O through the point O (Fig. 22) and indexing it to scale, the required resultant—which corresponds with the one determined by the parallelogram of forces—is obtained. Its direction is indicated by the arrow.

Fig. 25 is the polygon of forces when two of the forces, b c and d e, act towards the joint (Fig. 24), the magnitude of all the forces being as in the previous example. In this case the equilibrant is determined, and is shown by the thick line in Figs. 24 and 25.

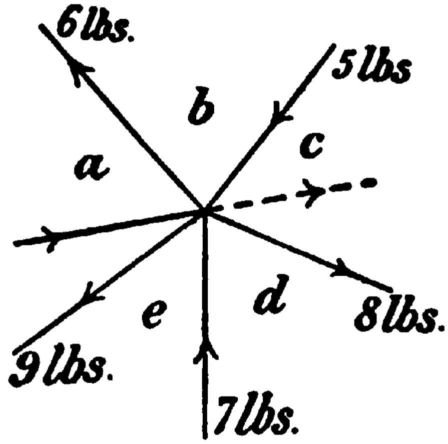


Fig. 24.

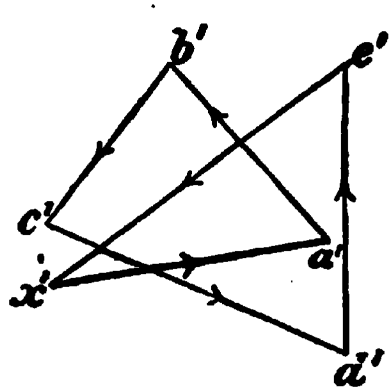


Fig. 25.

Figs. 21 and 25 should be compared carefully.

The **polygon of forces** may be stated as follows: **If two or more forces act at a point, then, if starting at any point a line be drawn to represent the magnitude and direction of the first force, and from the point thus obtained another line be drawn similarly to represent the second force, and so on until lines have been drawn representing each force,—the resultant of all these forces will be represented by a straight line drawn from the starting point to the point finally reached.**

Polygons, parallelograms, or triangles, of forces, when used to determine either the resultant or the equilibrant of stresses acting at a point, **are called reciprocal diagrams.**

Inclined Forces in one Plane but not acting through one Point. The foregoing examples deal only with forces which act at a single given point, and in these cases the resultant acts at the same point. When all the forces do not act at the same point, the magnitude and direction of the resultant is obained as in previous examples, i. e., by drawing the reciprocal diagram; the line of action, however, still remains to be determined. To determine this line of action, it is necessary to draw what is known as the **funicular** or **link polygon.** The method is as follows:

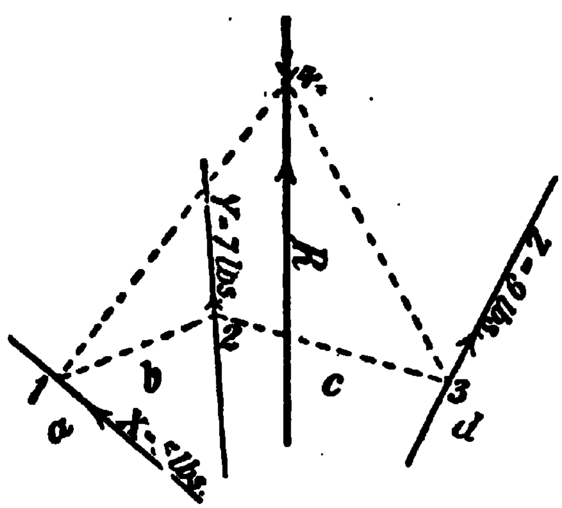


Fig. 26.

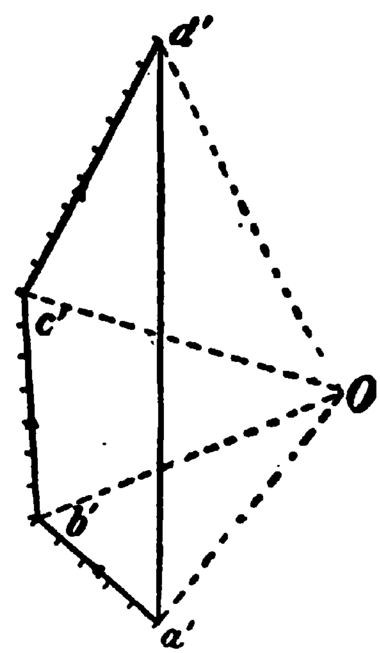


Fig. 27.

Example. Let X, Y, Z (Fig. 26) be three forces in the same plane and of the magnitude and direction shown. It is required to find the magnitude and the line of action of the resultant force.

Re-letter the forces a b c d according to Bow's notation and draw the reciprocal diagram a'b'c'd'; the line a′d′ which closes the figure represents the magnitude of the resultant. To obtain the actual line of action of the resultant, take any point or pole O and join a′O, b′O, c′O, d′O. The figure thus obtained is called the **polar diagram.** The funicular polygon is now constructed by drawing—anywhere in the space b—a line 1 2 parallel to b′O and intersecting the forces X and Y at 1 and 2 respectively. From 2 draw 2 3 parallel to c′O, intersecting the force Z in 3. Through 1 draw 1 4 parallel to a′O, and through 3 draw 3 4 parallel to d′O. Through the point of intersection 4, draw a line R parallel to a′d′. R is the required line of action of the resultant of the three given forces, and its magnitude is represented by the length of a′d′.

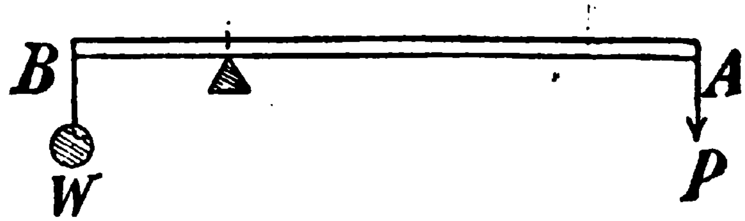


Fig. 28.

Parallel Forces. In addition to forces acting in the ways already explained, it is necessary to consider a few examples of **parallel** forces. (These must not be mistaken for those dealt with by the parallelogram of forces, as they are entirely different.)

In all the examples of parallel forces now to be considered, the forces will act vertically. As these can be shown easily both graphically and arithmetically, each example will be worked out by both methods.

The simplest examples of the equilibrium of parallel forces are found in the use of **levers.** The lever shown in Fig. 28 is a straight bar resting on a triangular block, F, called a **fulcrum.** At the ends A and B of the lever, forces P and W respectively act vertically downwards. It is plain that forces P and W will tend to rotate the lever in opposite directions around the fixed point F. The tendency of either force to rotate the lever is called the **moment** of that force; it is measured by the product of the force into the perpendicular distance (called the **arm** of the force) of the fixed point from the line of action of the force. **When the two moments are equal the lever is in equilibrium.** The conditions of equilibrium therefore are:

$$P \times AF = W \times BF.$$

If A F be 6″, B F be 2″, and W=9 lbs., then P will require to be $\frac{9 \times 2}{6} = 3$ lbs.: the moment of each force being 18 inch-lbs.

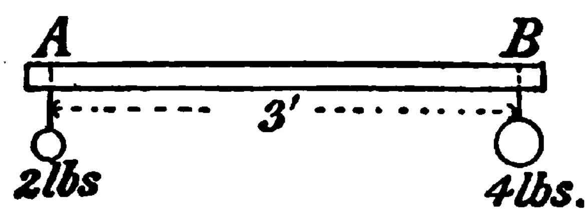


Fig. 29.

Since moments are always expressed in terms of the product of a force and a length, both these factors enter into every statement of the magnitude of a moment. If the distances be expressed in feet, and the forces in cwts., the moments will, of course, be expressed in ft.-cwts., and so on.

Example 1. A horizontal bar 3 ft. long has a weight of 2 lbs. at one end, and of 4 lbs. at the other end (Fig. 29). Find the point at which the bar must be supported so that it will rest horizontally. (Neglect the weight of the bar.)

Arithmetically.—Since the lever is in equilibrium, the total downward force of 6 lbs. is balanced by an upward force (reaction) of 6 lbs. at the unknown point of support, and the moment of the upward reaction, about any point, is equal to the sum of the moments of the downward forces about the same point. Consider moments about A:

Moment of weight at A, about A, $=2\times0$.
" " " " B " A, $=4\times AB$.
" reaction about A $=(2+4)\times AX$.

$$\therefore\ 6\times AX=(2\times0)+(4\times AB);$$
$$\therefore\ 6AX=0+(4\times3);$$
$$\therefore\ AX=12/6=2 \text{ feet}$$

Graphically.—In the consideration of these forces graphically, the polygon of forces becomes a straight line A polar diagram and a funicular polygon are required.

Fig. 30 shows the bars with the weights suspended. Letter the forces A B and B C. The polar diagram is drawn as follows: Draw to a suitable scale, a vertical line a c, 6 units long—equal to the sum of the weights. From any point O which may be at any convenient distance from a c, draw O a, O b, and O c. To construct the funicular polygon, draw, in the space B, 1 2 parallel to b o. From 1 draw 1 3 parallel to a O, and from 2 draw 2 3 parallel to O c, and produce it to intersect 1 3 in 3. Then the vertical line drawn through the point 3 will give the position of the fulcrum. If the distances from this point to the points of application of the weights be measured, it will be found that they are in inverse proportion to the magnitudes of the

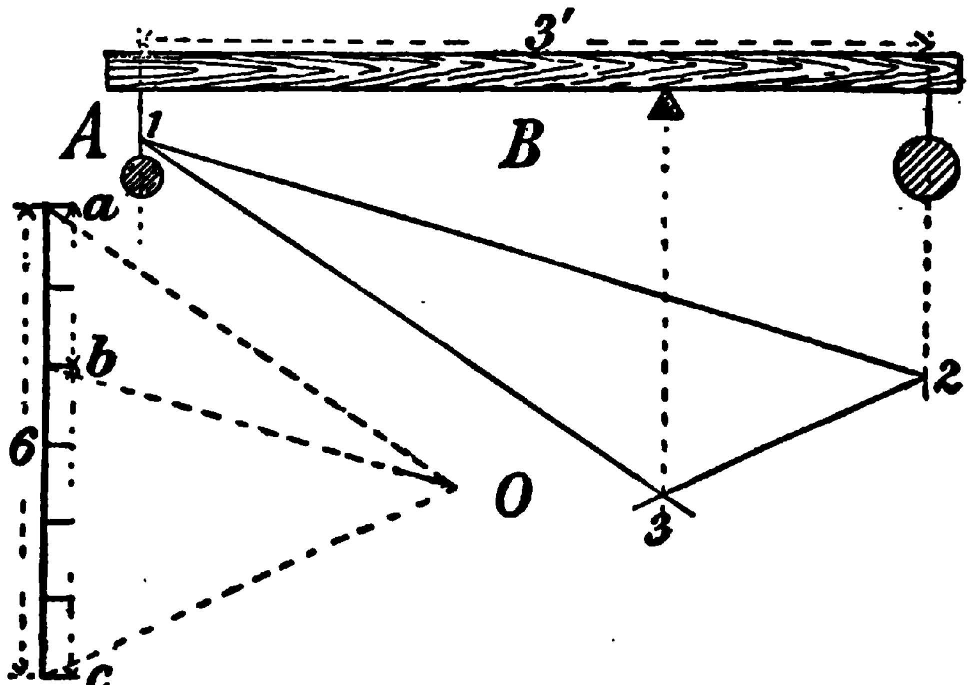


Fig. 30.

weights, and that the weight on the right hand side of the fulcrum, multiplied by its arm of leverage, will be equal to the weight on the left hand side, multiplied by the arm of leverage on that side—the arms being one and two feet respectively.

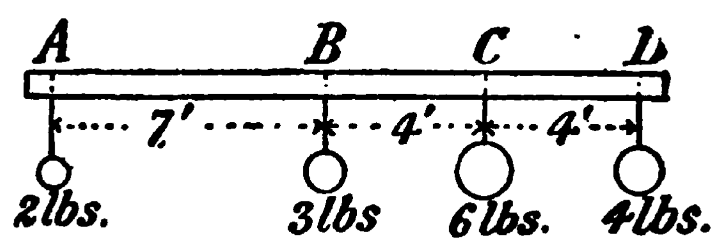


Fig. 31.

Example 2. Four weights of 2, 3, 6, and 4 lbs. respectively hang on a bar as shown in Fig. 31. Determine the point at which the bar must be supported to rest horizontally, the weight of the bar being neglected.

Arithmetically.—Let the required point of support be denoted by the letter X. When the bar is in equilibrium, the sum of the moments about A of the downward forces, must be equal to the moment, about A, of the upward reaction at the point of support.

$\therefore$ the downward moments about A

$$=(2\times0)+(3\times7)+(6\times11)+(4\times15)$$
$$=\ 0\ +\ 21\ +\ 66\ +\ 60\ =147.$$

The moment about A of the upward reaction

$$=\text{Sum of all the weights}\times AX$$
$$=(2+3+6+4)\times AX=15AX;$$
$$\therefore\quad 15AX=147;$$
$$\therefore\ AX=147/15=9\ 4/5\ \text{feet.}$$

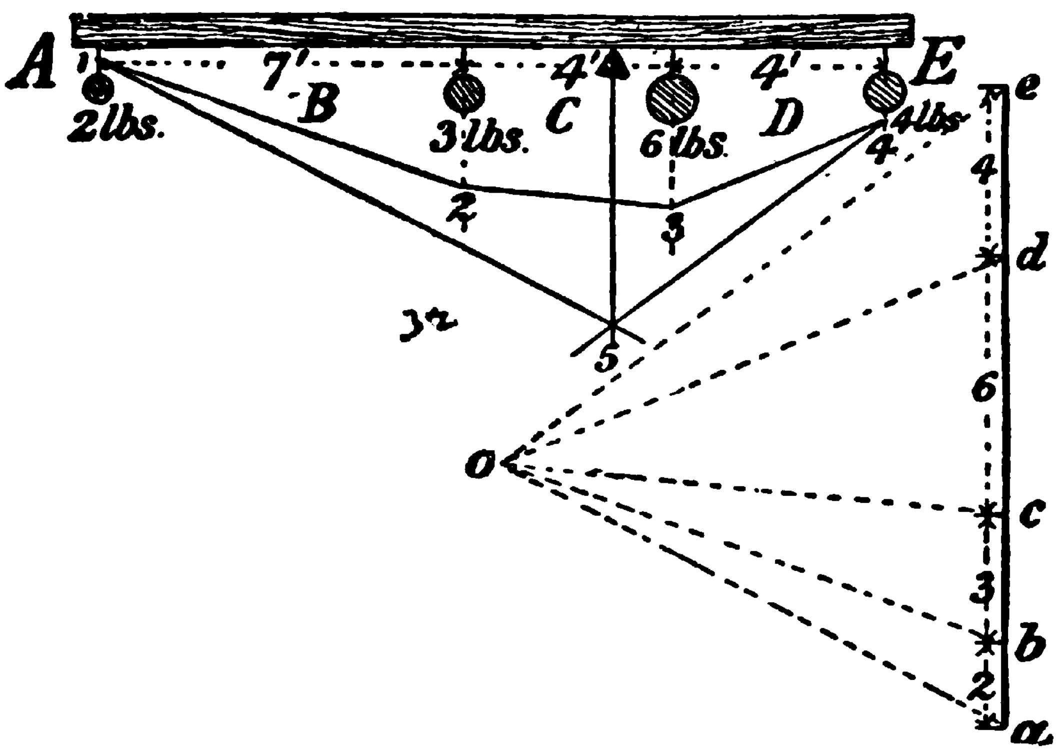


Fig. 32.

Graphically.—Draw the vertical line of loads e a, representing to scale the sum of the weights as shown (Fig. 32). Construct the polar diagram by drawing from any point O the lines e O, d O, c O, b O, and a O.

To draw the funicular polygon, draw vertical lines under each weight, and—starting anywhere in the line of the first weight as at 1—draw in the space B a line 1 2 parallel to O b; in the space C draw 2 3 parallel to O c; in the space D draw 3 4 parallel to O d. Through 4 draw 4 5 parallel to e O and through 1 draw 1 5 parallel to O a. The vertical line drawn through the point 5, where these two lines meet, gives the position of the point of support.

Although the application of the lever as a tool or machine is an everyday occurrence with the workman in such appliances as the turning bar of the bench-vice and sash-cramp, the screw-driver, brace, pincers, claw-

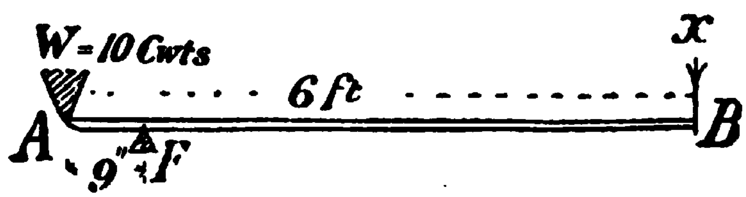


Fig. 33.

hammer, grindstone, treadle lathe, mortising-machine, etc., the detailed consideration of each of these cannot be entered into for want of space. The following examples involving the use of the crowbar will suffice further to illustrate the principles involved.

Example 1. What force must be exerted at one end of a crowbar 6 ft. long, to raise a weight of 10 cwts. at the other end: the bar resting on a fulcrum 9″ from the weight. (Neglect the weight of the crowbar.)

Let A B (Fig. 33) be the bar 6 ft. long, and F the fulcrum at 9″ from A. Consider moments in inch-lbs. about F. Let x be the required force.

Moment of 10 cwts. about $F=9\times10\times112$ inch-lbs.

Moment of x about $F=BF\times x=(72-9)\times x=63\times x$ inch-lbs.

$$\therefore x=\frac{9\times10\times112}{63}=160 \text{ lbs.}$$

Example 2. A man weighing 140 lbs. is using a crowbar 5 ft. long. What must be the position of the fulcrum to enable him to balance a weight of 1260 lbs. at the other end?

Let A B be the length of the bar; F the position of the fulcrum; and x the length of the long arm in inches; then (60—x) is the length of the short arm in inches.

Taking moments about F,

$$140x=1260(60-x);$$
$$140x=75600-1260x;$$
$$140x+1260x=75600;$$
$$1400x=75600;$$
$$\therefore x=\frac{75600}{1400}=54 \text{ in.}=4'\ 6''=\text{length of long arm.}$$

Fig. 34.

Example 3. A lever 7 ft. long is used as shown in Fig. 34. If a force of 200 lbs. is applied at P in the direction of the arrow, what weight, placed at a point 12″ from the fulcrum, can be raised?

Taking moments in ft.-lbs. about F,

$$W\times1=200\times7;$$
$$\therefore\ W=1400 \text{ lbs.}$$

Loaded Beams. The determination of the proportion of the total weight carried by each support of a loaded beam—in other words, the upward reaction of each support which is necessary to maintain equilibrium—affords a good practical example of the theory of parallel forces.

Example 1. A beam rests upon supports placed 8 feet apart. A weight of 12 lbs. is placed on the beam at a distance of 2 ft. from the right-hand support. What proportion of the weight is carried by each of the supports, the weight of the beam being neglected?

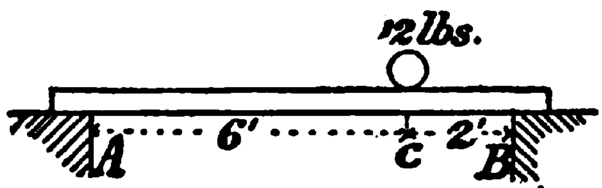


Fig. 35.

Arithmetically.—In this case (Fig. 35) the downward force (weight) of 12 lbs. must be balanced by upward forces (reactions) at the points of support, respectively equal to the pressure at these points, and together equal to 12 lbs.; and the moments of the upward forces about the point c must be equal.

∴ Reaction at A×Ac=Reaction at B×Bc,

i.e. Reaction at A : Reaction at B :: Bc : Ac,

or Reaction at A : Sum of reactions at A and B :: Bc : (Bc+Ac),

i.e. Reaction at A : 12 lbs. :: Bc : AB.

This may be expressed in general terms as follows:

Pressure on one end caused by any load : that load :: Distance of that load from other end : Length between supports.

therefore Pressure on A : 12 lbs. :: cB : AB;

$$\therefore \text{ Pressure on A} = \frac{12 \times cB}{AB} = \frac{12 \times 2}{8} = 3 \text{ lbs.}$$

$$\text{Similarly, Pressure on B} = \frac{12 \times Ac}{AB} = \frac{12 \times 6}{6} = 9 \text{ lbs.}$$

Graphically.—Fig. 36 shows the beam and supports with the load in position. The polar diagram is drawn as follows: Draw a vertical line a b, representing the

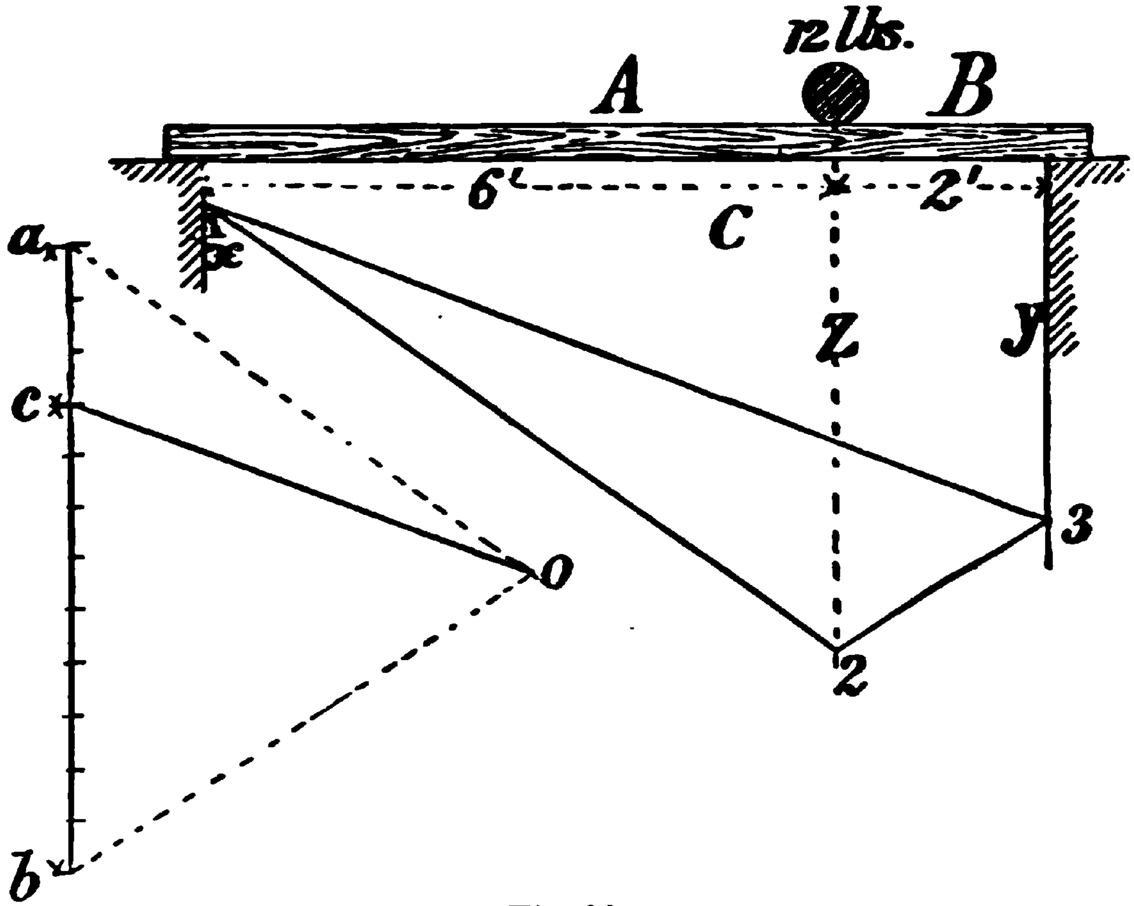


Fig. 36.

weight (12 lbs.) to a suitable scale. From any point O, which may be at any convenient distance from a b, draw the triangle O a b. Draw as in the figure a vertical line directly under the load, and one under each point of support, as Z, x, y. Letter the load A B, and the space between the supports C. These letters can now be used to denote the reaction at each point of support—i. e., the upward force required to maintain equilibrium—which is equal and opposite to the pressure exerted on each support by the load. Anywhere in x, as from 1, draw, in the space A, a line 1 2, parallel to a O; from 2, in the space B, draw 2 3 parallel to b O. Join 1 3, and through the pole O, draw O c parallel to 3 1. Then a c (on the vertical line of loads a b) represents to scale the pressure on the left-hand support, and c b to the same scale represents the pressure on the right-hand support.

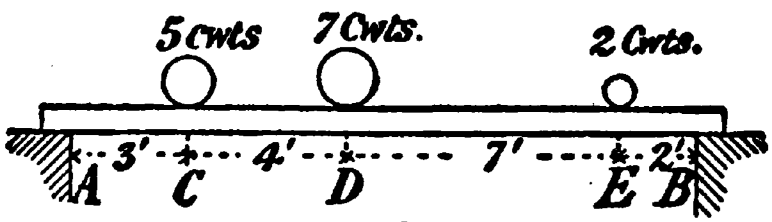


Fig. 37.

As the reaction at each end is equal in magnitude and opposite in direction to the pressure, a c gives the amount of the reaction A C, and b c gives the amount of the reaction B C; and the sum of the reactions—both acting upwards—is equal to the total weight (12 lbs.).

Example 2. A beam is loaded as shown in Fig. 37. Determine the reaction at each end, that is, the upward force required at each point of support to maintain equilibrium.

Arithmetically:

$$\text{Reaction at A due to weight at C} : \text{wt. at C} :: \text{CB} : \text{AB};$$

$$\therefore \text{Reaction at A due to weight at C} = \frac{\text{wt. at C} \times \text{CB}}{\text{AB}} = \frac{5 \times 13}{16}.$$

Similarly,

$$\text{Reaction at A due to weight at D} = \frac{\text{wt. at D} \times \text{DB}}{\text{AB}} = \frac{7 \times 9}{16}.$$

Also

$$\text{Reaction at A due to weight at E} = \frac{\text{wt. at E} \times \text{EB}}{\text{AB}} = \frac{2 \times 2}{16}.$$

The total reaction at A is equal to the sum of the partial reactions as shown above; or it may be obtained directly thus:

Total reaction at A

$$= \frac{(\text{wt. at C} \times \text{CB}) + (\text{wt. at D} \times \text{DB}) + (\text{wt. at E} \times \text{EB})}{\text{AB}}$$

$$= \frac{(5 \times 13) + (7 \times 9) + (2 \times 2)}{16} = \frac{132}{16} = 8\tfrac{1}{4} \text{ cwts.}$$

Similarly,

Total reaction at B

$$= \frac{(\text{wt. at C} \times \text{CA}) + (\text{wt. at D} \times \text{DA}) + (\text{wt. at E} \times \text{EA})}{\text{AB}}$$

$$= \frac{(5 \times 3) + (7 \times 7) + (2 \times 14)}{16} = \frac{92}{16} = 5\tfrac{3}{4} \text{ cwts.}$$

Graphically.—Construct the vertical line of loads, representing to scale the sum of the weights as shown in Fig. 38. Fix the pole O, and draw the dotted lines O a, O b, O c, O d. Letter the loads and draw a dotted vertical line directly under each load and under each support as shown. From any point in the first line, draw in the space A, a line 1 2 paralel to a O; from 2 draw 2 3 parallel to b O; from 3 draw 3 4 parallel to c O; and in the space D, from 4 draw 4 5 parallel to d O. Join 1 to 5, thus completing the funicular polygon. By drawing a line parallel to 5 1—the closing line of this polygon—through pole O, and meeting the vertical line of loads at e, it is found that e a equals the reaction E A, and e d equals the reaction E D; they are together equal to the sum of the weights in the beam.

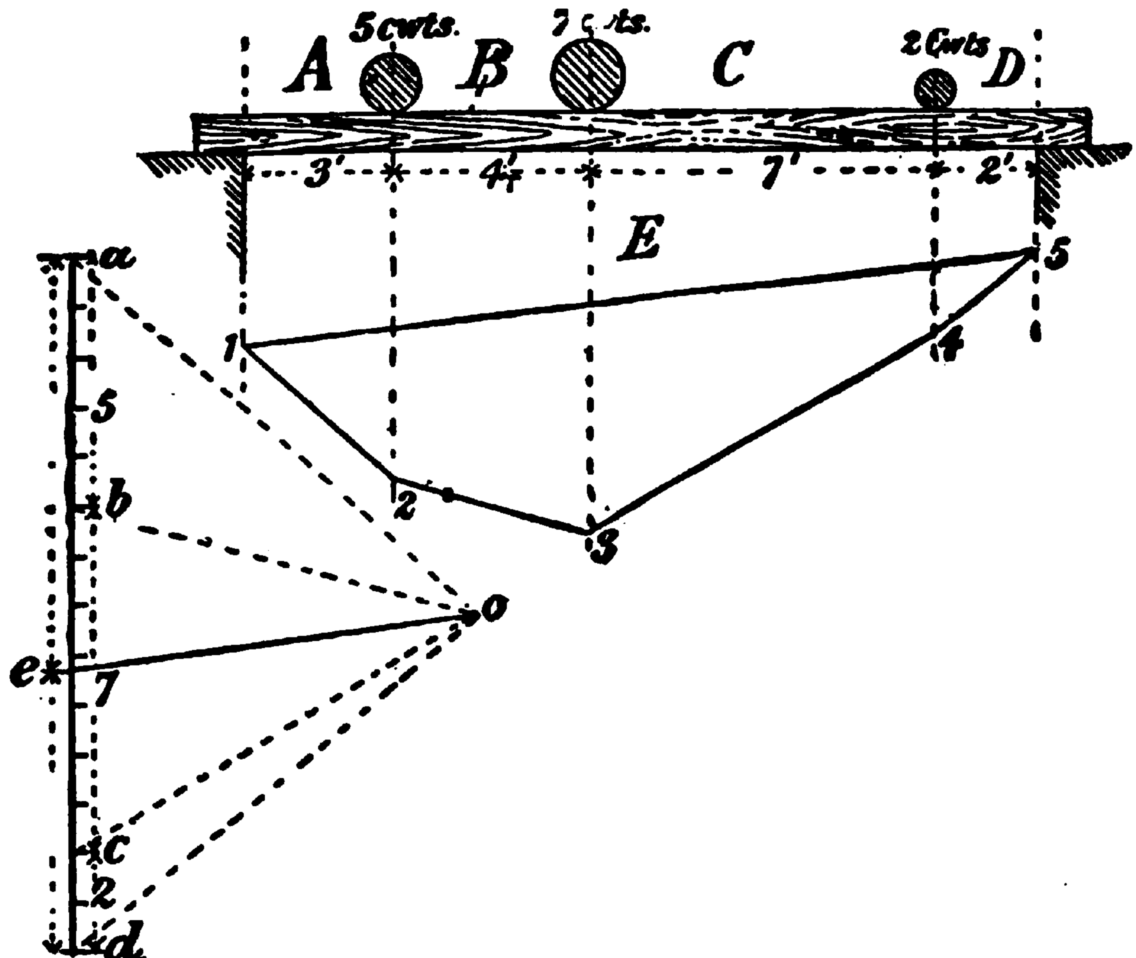


Fig. 38.

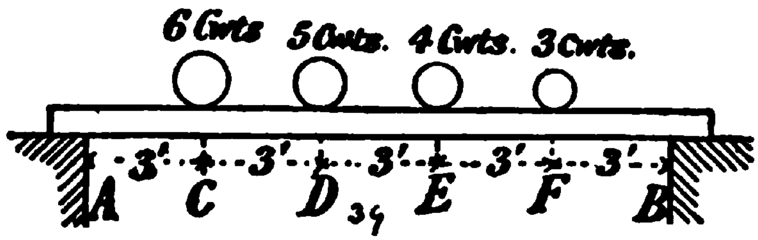


Fig. 39.

Example 3. A beam weighing 6 cwts. is loaded as shown in Fig. 39. Determine the reaction at each end necessary to produce equilibrium.

• When the weight of a uniform beam is to be considered, it may be taken as acting half-way between the supports, and thus adding half its weight to each sup-

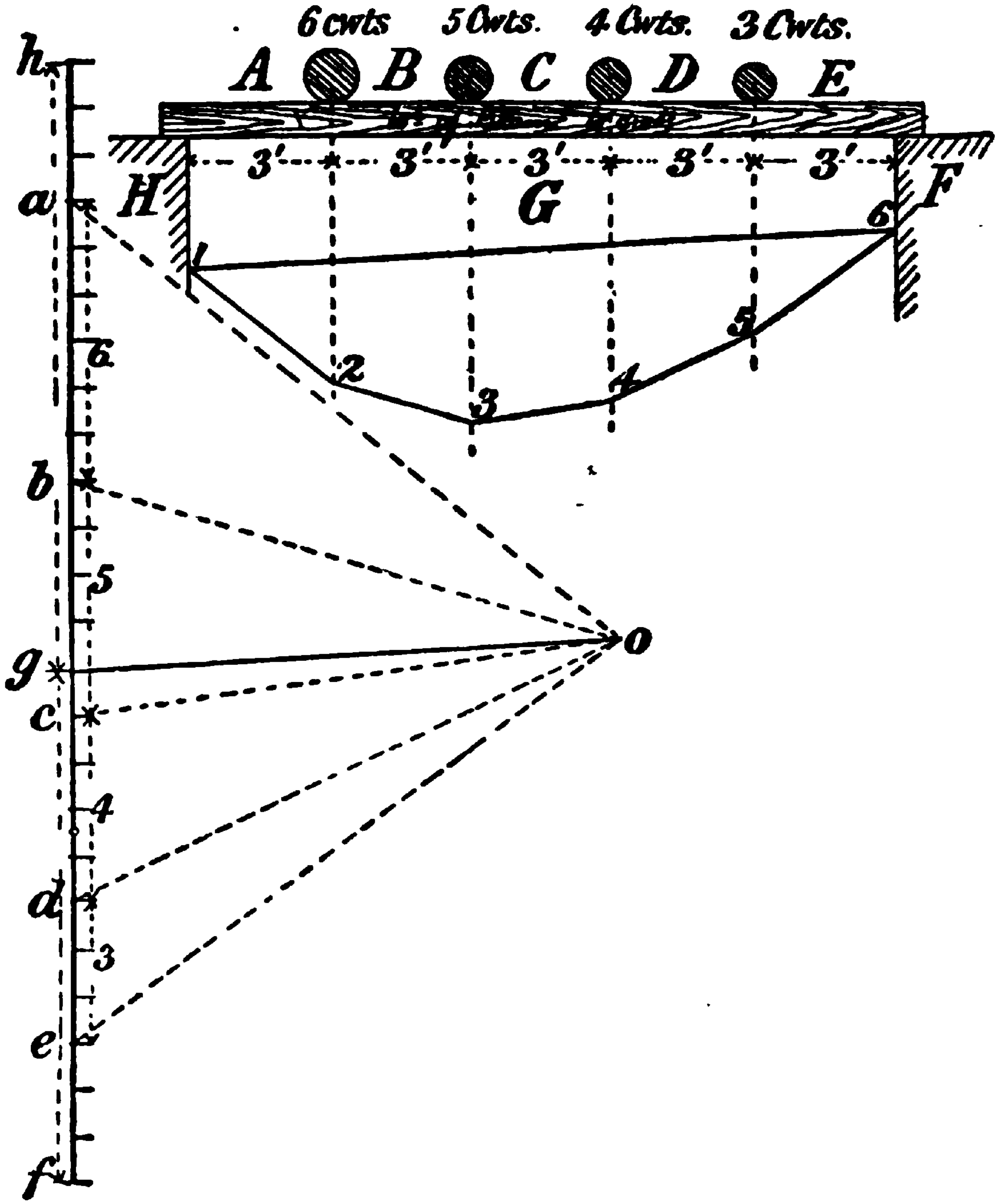


Fig. 40.

port. With this difference the method used is as in the previous example. Fig. 40 shows the graphical solution.

Stress Diagrams for Roof Trusses.—Figs. 41 to 50 show an application of the foregoing graphic methods to the determination of stresses in roof trusses. Fig. 41 is a line diagram of a king-post truss loaded in the usual way. It must be noticed that the lettering is arranged so that every member is indicated by a letter on each side. It is first necessary to determine the amount of weight carried by each point of support. This example is simplified by the symmetrical loading, as one half the weight is carried by each point of support. When this is not the case, the proportion of the weight carried by each support must be determined first, by a consideration of parallel forces, as in earlier examples.

It is usual when determining the stresses of such a truss, to draw the stress diagram, shown in Fig. 46. This diagram is a combination of Figs. 42 to 45, which are only drawn as separate figures to assist in understanding the question more clearly.

Fig. 42 is the polygon of forces for the joint (1) at the foot of the principal rafter on the left. Four forces act at the point: A B downwards, B N the principal rafter, N G the tie beam, and the upward force. A G—the reaction at the point of support. Of these four forces the amounts of two, A B and A G, are known; it is required to determine the nature and amounts of the stress of B N and of N G when acting at the angles given.

Commencing Fig. 42 by drawing a b equal to A B, and a g equal to A G, the upward force. As these two forces are in the same straight line, and in opposite directions, their resultant is the line b g. From b

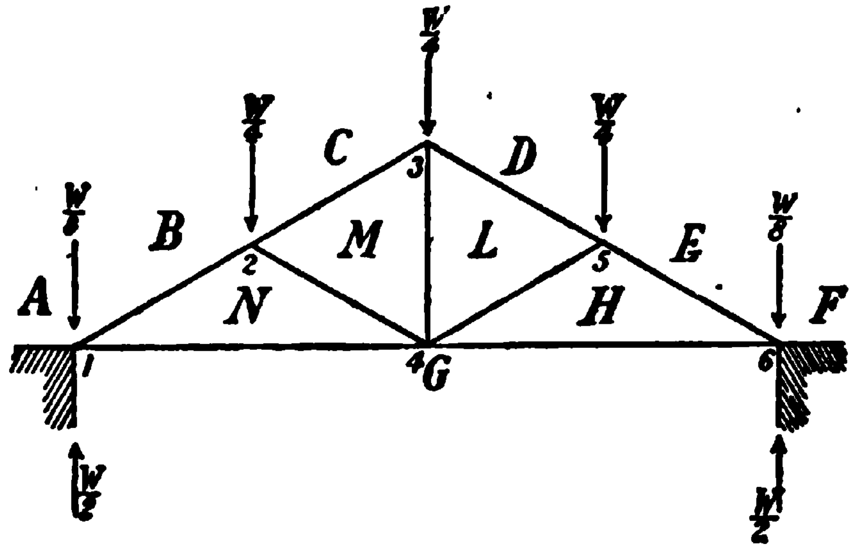


Fig. 41.

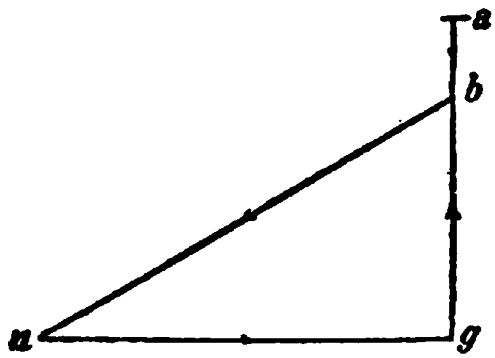


Fig. 42.

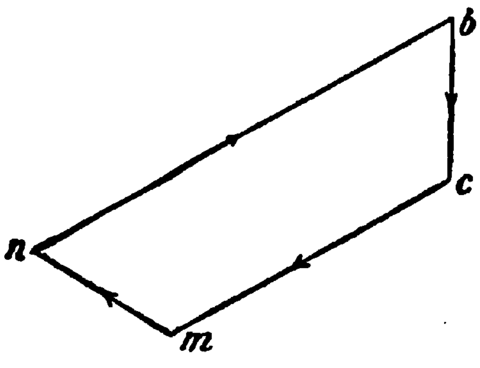


Fig. 43.

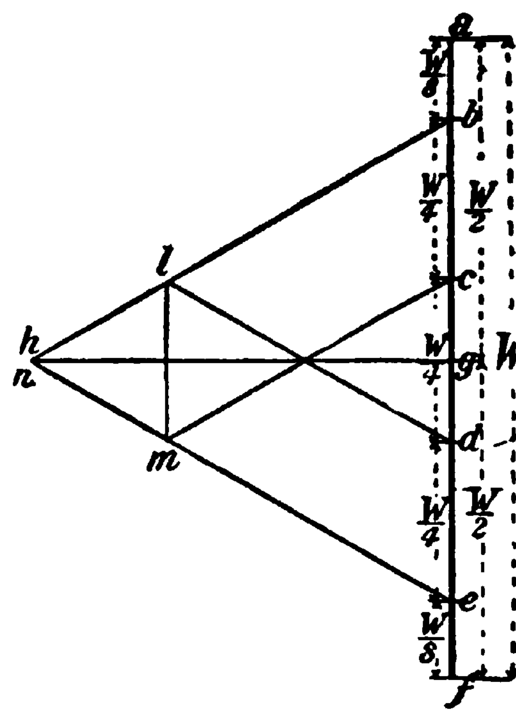


Fig. 46.

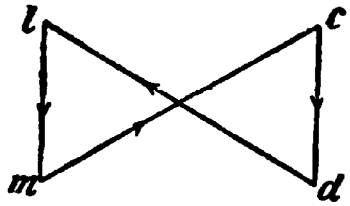


Fig. 44.

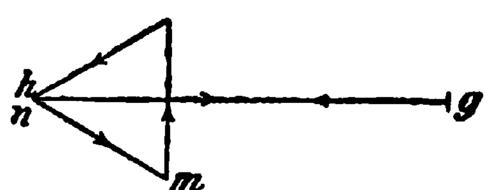


Fig. 45.

draw b n parallel to B N; and from g draw g n parallel to G N until b n and g n meet. Then b n g is the polygon (a triangle in this case) of forces acting at the point; and b n and n g represent the amount of stress in the principal rafter and tie-beam respectively.

The direction of the stress is found by taking the forces in order: thus, g b acts upwards; b n acts towards the joint, therefore this member is in compression; n g acts from the joint, which indicates that the tie-beam is in tension.

At joint (2), four forces act, namely, B C, B N, C M, and N M. The two known forces are B C acting downwards and B N towards the joint. For the magnitude of the stress on B N has already been found, and its direction of action at joint (2) is the opposite to its direction at joint (1). Fig. 43 shows the application of the polygon of forces to this joint. In it, b c and b n are drawn equal and parallel to B C and B N respectively; and, by drawing n m parallel to N M and c m parallel to C M, the stress diagram is obtained. This shows that the stress in C M, the upper part of the principal rafter, is much less than in B N, the lower part. By tracing the polygon, it is found that b c acts towards the joint, c m towards the joint (therefore C M is in compression), m n towards the joint (compression) and n b towards the joint (compression as in the previous figure).

At joint (3) there are four forces, i. e. C M, C D, D L, M L, acting as shown. Of these four forces the two C M and C D are known. Since the amount and nature of the stress in any member must be the same at any intermediate point between the joints, the stress

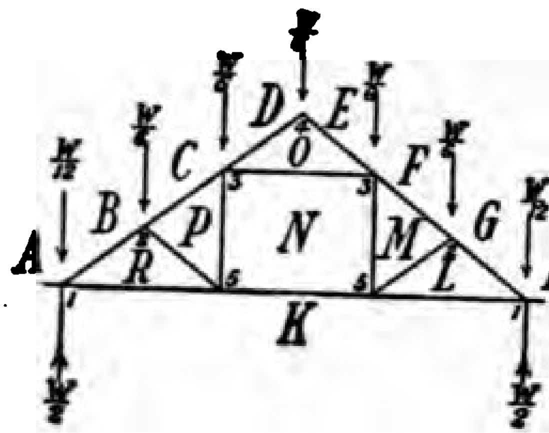


Fig. 47.

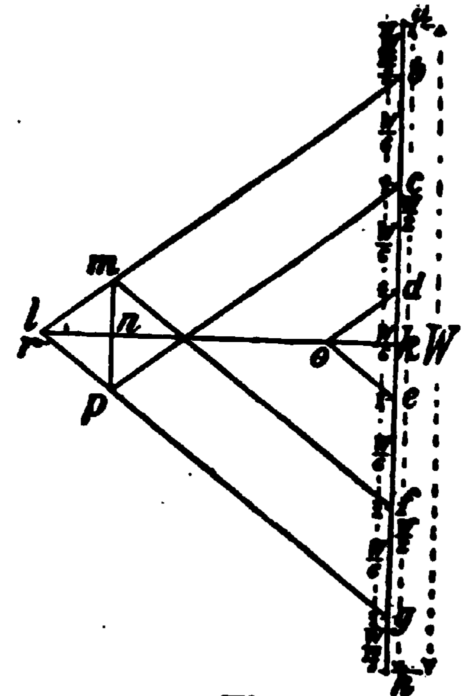


Fig. 48.

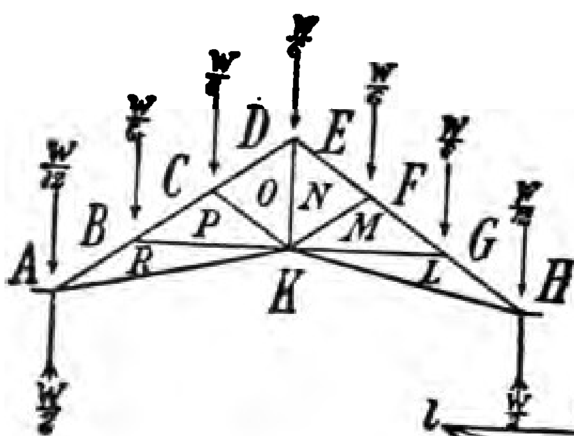


Fig. 49.

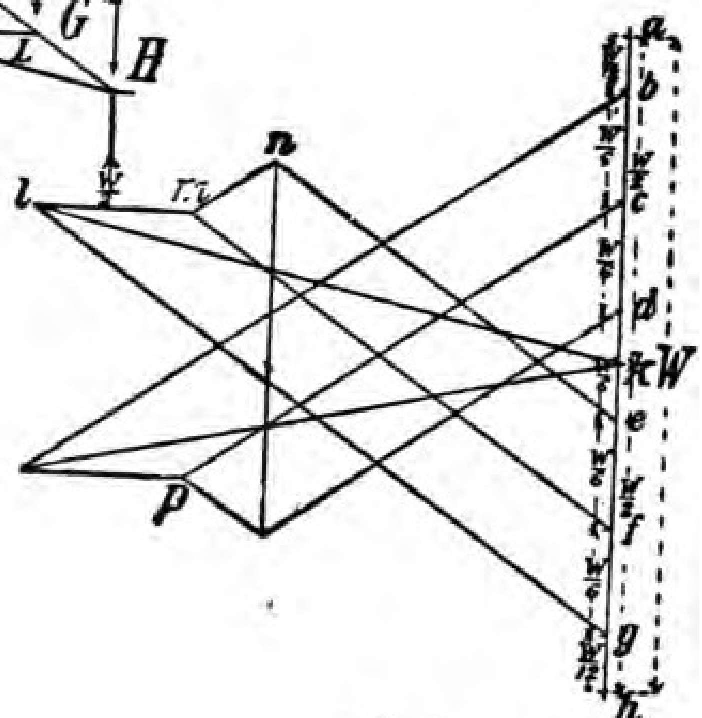


Fig. 50.

in C M acting upon joint (3) must be as determined by the diagram for joint (2). Fig. 44 is the stress diagram; c d is drawn parallel and equal to C D; c m parallel and equal to C M; m l and d l are drawn parallel to M L and L D respectively until they meet. Taking these forces in order, c d is towards the joint, D L towards the joint (compression), L M is from the joint (tension), and M C towards the joint (compression).

The tension stress in L M is caused by the struts M N and L H, which transfer part of the loads B C and D E respectively to the foot of the king-post. If no struts existed in this truss there would be no stress in M L.

Joint (4) has five forces acting, each one of which has already been determined, since the stress diagrams for one side of the truss are in this example applicable to each side. For example, the diagrams showing the stresses in the joints (1) and (2) are applicable to (6) and (5) respectively. An examination of Fig. 45 will show that g n is parallel and equal to G N; n m is parallel and equal to N M; l m is parallel and equal to L M; and l h being drawn parallel to L H meets m n in n, whilst h g is equal to n g. The diagram therefore shows the stress in each of the five members.

In Fig. 46, which is the complete stress diagram for the members of the truss, the lettering is identical with that in each of the separate Figs. 42 to 45, and will be easily understood from them.

Fig. 47 is a line diagram of a queen-post roof truss, and Fig. 48 is the stress diagram of this truss. Similarly, Figs. 49 and 50 are respectively the line diagram of and the stress diagram for, a composite roof truss, sometimes named a German truss. The detailed explanation already given will enable the figures to be understood.

STRENGTH OF WOODEN BEAMS.

For the purpose of calculating the carrying capacity of wooden beams, it is necessary to notice the nature of the stresses to which they are subjected, as well as the manner in which they are loaded, and the arrangement of the load.

Stress and Strain. When a weight, or other force, acts upon a beam, it tends to change the shape and size of the beam. The force is technically called a **stress,** while the change in shape or size is called a **strain.** When a beam or girder, supported at both

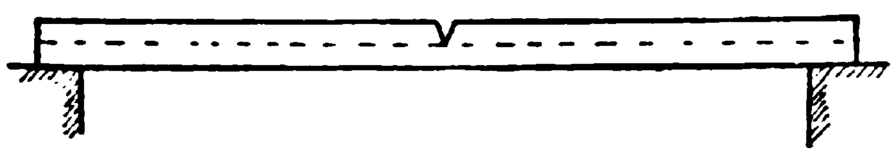


Fig. 51.

ends, is loaded, the upper part tends to shorten. The lower fibres, on the other hand, are in a state of tension, as they tend to stretch. The force acting on the upper fibres of such a beam is therefore a **compression stress**; that on the lower fibres is a **tension stress.**

The existence of these stresses may be made very apparent either by making a saw-cut across, or by actually cutting out a wedge-shaped piece from, the middle of a beam of wood for half its depth, as shown in Fig. 51. On resting the beam on two supports with the cut edge uppermost, and then loading it, it will be seen that the saw-cut closes. This shows that the fibres on the upper side are in a state of compression. If the same beam is now turned over so that the saw-cut is on the lower side, and again loaded, the tendency is for the cut to open, thus showing that the fibres on the lower side are in a state of tension.

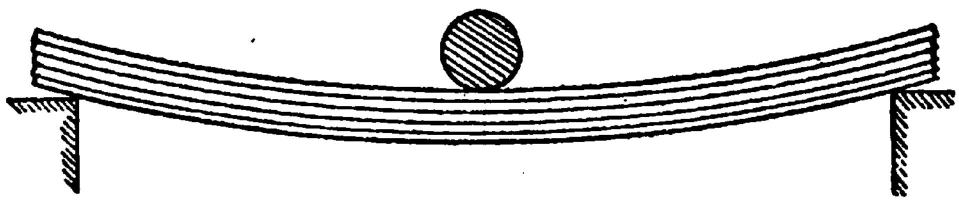


Fig. 52.

Shearing Stresses. A shearing stress is one which gives the fibres of the wood a tendency to slide over one another. A shearing stress may be either in the direction of the fibres or at right angles to them. To illustrate a shearing stress in the direction of the fibres of a beam, imagine the beam cut into a number of

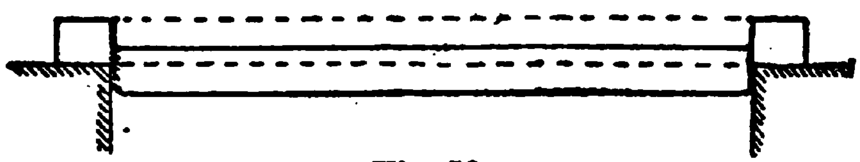


Fig. 53.

boards; place these on the top of each other in the position of a beam resting upon supports at each end, and place a load in the middle. The result will be that the beam will bend as shown in Fig. 52, and the boards will slide over each other. A shearing stress across the fibres of a loaded beam can be illustrated by taking a bar of soap, or some such soft material, resting it upon supports, and loading it. The result will be as shown in Fig. 53.

Methods of Arranging Beams. The nature and amount of stress in the fibres of a loaded beam depend upon the way the beam is supported and on the arrangement of the load. Thus a **cantilever** is a beam with one end only secured upon a support, the other end overhanging. The load upon a cantilever may be a concentrated load at the outer end, as in Fig 57, or the load may be anywhere between the outer end and

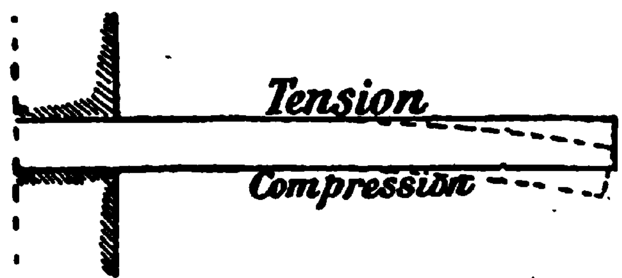


Fig. 54.

the supported end; a number of loads of varying weights may be distributed over the length; the load may be a uniformly distributed one extending over the length of the beam, or it may be a combination of a concentrated load and a distributed load. A cantilever loaded in any of the ways just described has the fibres in the upper edge in a state of tension, those in the lower half being in compression (Fig. 54).

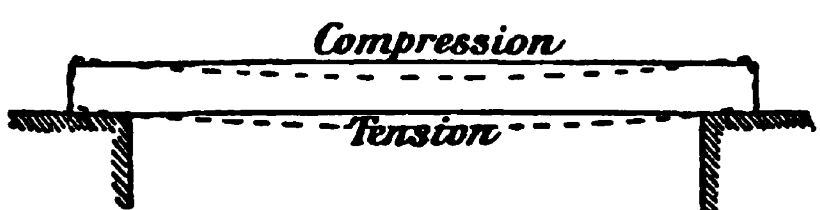


Fig. 55.

A beam supported at both ends may be loaded in any of the ways described for the cantilever, with the result that the stresses will be as shown in Fig. 55; i. e. the upper part will be in compression, and the lower half in a state of tension. The stresses in the various parts of a loaded beam which has the **ends fixed** differ from those of the beam which simply rests upon supports. They are illustrated in Fig. 56, which shows that for a distance of about one-fourth from each end the beam takes the form of a cantilever, and has the fibres in the upper half in a state of tension and the lower fibres in compression. The remainder of the beam has the upper fibres in compression and the lower part in tension. The neutral axis of all these beams is in the centre of the depth. If a long beam has intermediate supports as in Fig. 56, it may be regarded as being "fixed" at the points of intermediate support.

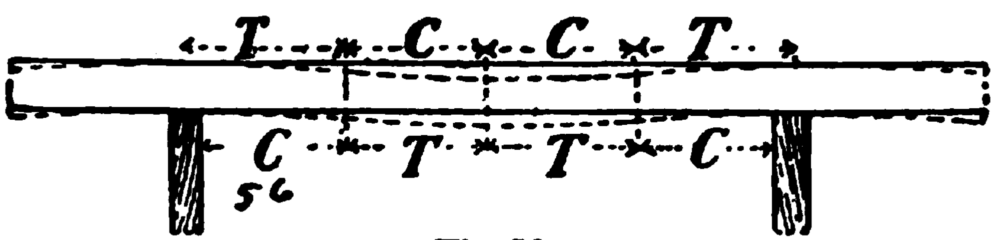


Fig. 56.

Bending Moments. For the purpose of making comparisons of the relative strengths of loaded beams, a further consideration of the "moment of a force" is necessary. Since the tendency to bending, to which a given beam is subject at any point, depends upon the moments of the stresses about that point, it is obvious that the relative strengths of beams may be measured in terms of moments. **The bending moment at any given section is the algebraic sum of all the external forces acting on one side of the section.** Since it is at the point where the greatest bending moment occurs that the beam is subjected to the greatest stress, it follows that it is of some importance to be able to determine the bending moment of beams loaded under different conditions. The bending moment—like other moments—must always be expressed in terms of a length and a force.

Example 1. A cantilever carries a load of 6 tons at its outer end, which is 5 ft. from the supporting wall. Determine the maximum bending moment, and also the bending moment at 2 ft. from the wall.

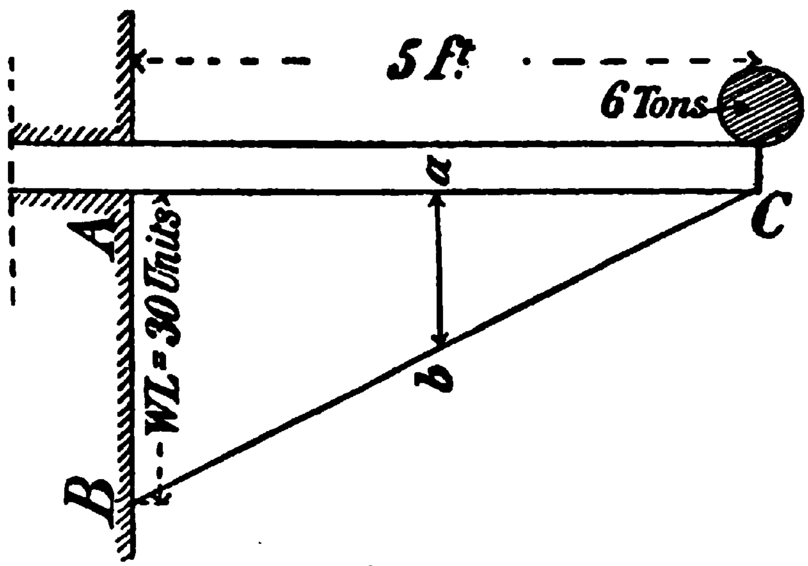


Fig. 57.

The greatest tendency to bending will be at the point of support, i. e. at a distance of 5 ft. from the load.

∴ Maximum bending moment=5×6=30 ft.-tons.

Bending moment at 2 ft. from the wall (i. e. 3 ft. from the load) =3×6=18 ft.-tons.

The bending moment at any distance from the load may be determined graphically by drawing, as in Fig. 57, a vertical line A B 30 units long (respecting the maximum bending moment) under the point of support A (i. e. the point where the bending moment is a maximum), and joining B C. Then the bending moment at the point a will be represented by the length of a b drawn parallel to A B.

Example 2. A cantilever projects 4 ft. and carries a uniformly distributed load of 8 cwts. along the upper edge. Determine the maximum bending moment, and also draw a diagram from which the bending moment at any section along the length of the cantilever may be determined.

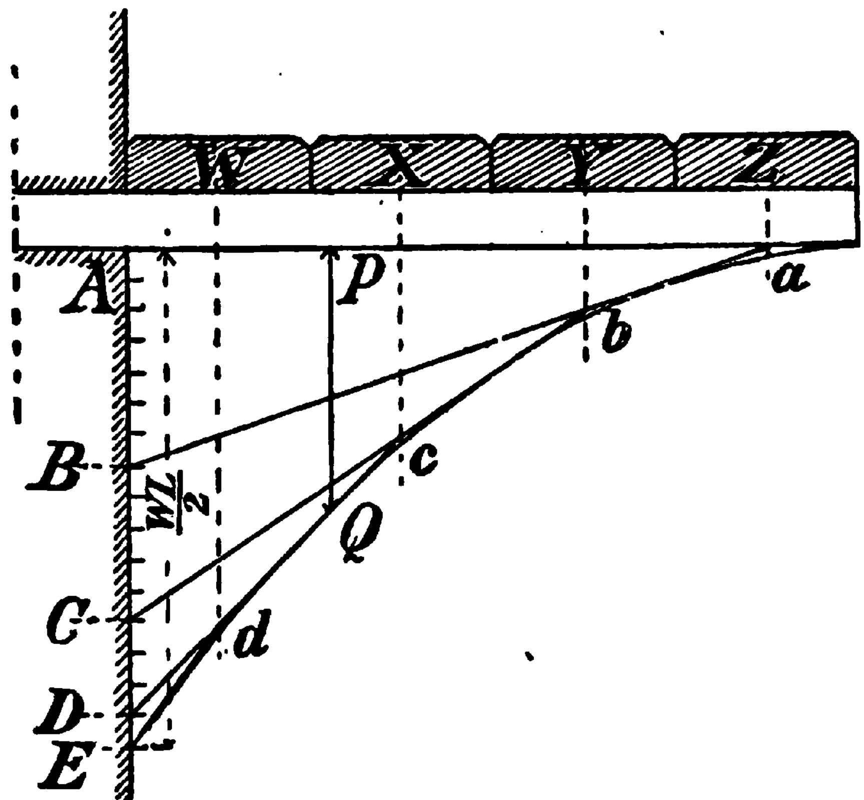


Fig. 58.

A load arranged as shown in Fig. 58 is equivalent to a concentrated load of 8 cwts. acting in the middle of the length, i. e. 2 ft. from the point of support. The maximum bending moment will therefore be

$$8 \times 2 = 16 \text{ ft.-cwts.}$$

Fig. 58 is the diagram from which the bending moment at any section may be determined. The load is supposed divided into 4 equal parts, and the bending moment due to each part is drawn to scale on the vertical line A E. The weight of Z acts at 3′ 6″ from A, and the maximum bending moment due to Z=2×3.5= 7 ft.-cwts. Draw A B 7 units long. Similarly, the maximum bending moment due to Y=2×2.5=5 ft.-cwts., and is represented by B C; maximum bending moment due to X=2×1.5=3 ft.-cwts., represented by C D; maximum bending moment due to W=2×0.5=1 ft.-cwt., represented by D E. The maximum bending mo-

Fig. 59.

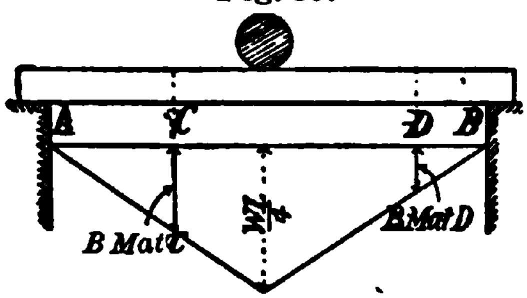
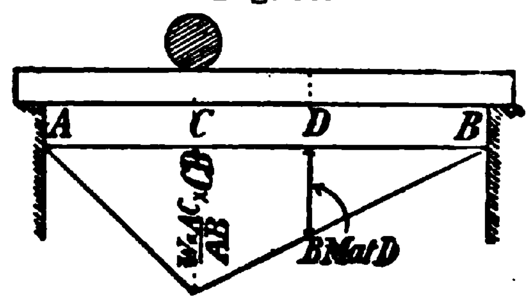


Fig. 60.



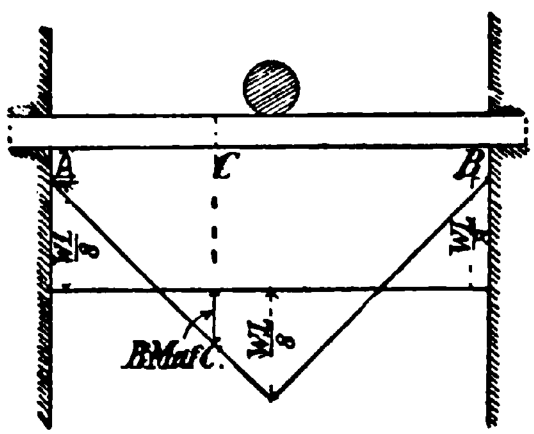


Fig. 62.

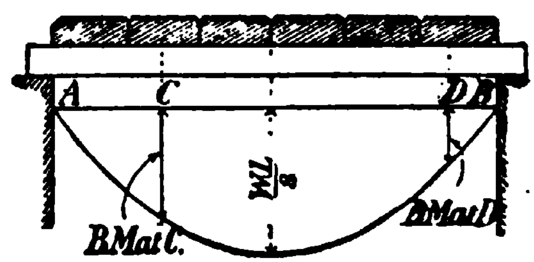


Fig. 63.

ment due to the total load is therefore 7+5+3+1=16 ft.-cwts., and is represented by A E. Draw a vertical line through the centre of each part of the load, and complete the triangles A a B, B b C, C c D, D d E. Draw an even curve touching the lines E d, D c, C b, B a. This curve is a parabola. The bending moment at any section P is represented by the length of the vertical line P Q cutting the parabola at Q.

The following formulæ are used for determining the relative bending moments (and therefore the relative strengths required) of beams loaded in various ways. In each case L=the length of the beam and W=the weight of the load.

	Maximum Bending Moment.	Relative Strength.
Cantilever fixed at one end and loaded at the other end (Fig. 57),	WL	1
Cantilever fixed at one end and loaded with uniformly distributed load,	$\frac{WL}{2}$	2
Beam supported at both ends and loaded with a central load (Fig. 59),	$\frac{WL}{4}$	4
Beam supported at both ends and loaded with a uniformaly distributed load (Fig. 65),	$\frac{WL}{8}$	8
Beam fixed at both ends and loaded with a central load (Fig. 62),	$\frac{WL}{8}$	8
Beam fixed at both ends and loaded with a uniformily distributed load,	$\frac{WL}{12}$	12

Figs. 60 to 63 show beams loaded in various ways, and serve to illustrate the method of determining graphically the bending moment at any section of the beam.

It will be noticed that the maximum bending moment of a beam supported at each end is in each case in the line of the load, and with a central or an evenly distributed load is at the middle of the length of the beam.

Calculation of the Transverse Strength of Wooden Beams. Other things being equal, the strength of a rectangular wooden beam is directly proportional to the breadth in inches multiplied by the square of the depth in inches, and inversely proportional to the length in feet. Of course the nature of the material is also an important factor, since timber, even of the same kind, varies in strength to a considerable extent. Each beam therefore has what is called a natural **constant,** which must be considered in the calculation of its carrying capacity. To obtain this constant, it is usual to take a bar of similar wood, 1 inch square in section, and long enough to allow of its being placed on supports 1 foot apart. The constant is the weight of the central load, which is just sufficient to break the bar. The constant may be expressed in lbs., cwts., tons, etc, and the carrying capacity will always be in the same terms. The following constants (in cwts.) may be adopted for the purposes of calculation: oak, ash and pitch pine, 5; red deal, red pine and beech, 4; white and yellow pine, 3.

Another important consideration is the ratio which the breaking load of a beam bears to the "safe" load. This ratio is called the **factor of safety,** and its value depends upon whether the load is a live—a constantly moving—load, or a dead (i. e., a stationary) load. The factor of safety for a dead load is usually taken at 5, which means that the safe load upon a beam must not exceed one-fifth of the breaking load; the factor of safety for a live load is often taken at 10.

For beams supported at both ends the formula $W = \frac{bd^2c}{L}$ may be used for the purposes of calculation when:

W=breaking weight or maximum carrying capacity of a centrally loaded beam, expressed in the same terms as the constant,
b=breadth of the beam in inches,
d=depth of the beam in inches,
L=length of the beam in feet,
c=the constant, found by experiment as described above, and expressed in terms of lbs. or cwts.

To illustrate the above formula, take two pieces of the same kind of wood say 7 ft. long, 6 in. wide and 2 in. thick. Place one of these pieces flat, and the other on one edge, the distance between the supports in each case being 6 ft. As the constant is the same in both (say 5 cwts.), the carrying capacity of each will be expressed by the formula $W=\frac{bd^2c.}{L}$;

for the flat beam, $$W = \frac{6 \times 2 \times 2 \times 5}{6} = 20,$$

for the one on edge, $$W = \frac{2 \times 6 \times 6 \times 5}{6} = 60;$$

and the relative strengths will be as 20:60 or as 1:3.

When it is necessary to find other terms than W, the equation $W = \frac{bd^2c}{L}$ may be expressed as follows:

$$L=\frac{bd^2c}{W}; \quad b = \frac{WL}{d^2c}; \quad d^2=\frac{WL}{bc}; \quad d = \sqrt{\frac{WL}{bc}}; \quad c = \frac{WL}{bd^2}$$

The value of W for a **distributed load** is twice that for a concentrated load, i. e., $W=\frac{2bd^2c}{L}$. When the ends are fixed the carrying capacity is increased by about one-half.

For safe central loads the formula $W=\frac{bd^2c}{LF}$ is used: F being the factor of safety.

Example 1. Find the maximum carrying capacity of a centrally loaded wooden beam of pitch pine, 13 ft. long (12. ft. between the supports), 10 in. wide, and 6 in. thick, (1) when placed on edge; (2) when placed flat. Assume a constant of 5 cwts.

Applying the formula

$$W=\frac{bd^2c}{L},\ (1)\ \ W=\frac{6\times 10\times 10\times 5}{12}=250 \text{ cwts. when on edge.}$$

$$(2)\ \ W=\frac{10\times 6\times 6\times 5}{12}=150 \text{ cwts. when placed flat.}$$

Example 2. What would be the maximum safe load to which the beam in Ex. 1 may be subjected (1) as a central load; (2) as a uniformly distributed load?

Formula for safe central load using a factor of safety of 5, is

$$\text{Safe central load}=\frac{bd^2c}{LF}$$

$$=\frac{6\times 10\times 10\times 5}{12\times 5}=50 \text{ cwts. for beam on edge,}$$

$$\text{or}\quad =\frac{10\times 6\times 6\times 5}{12\times 5}=30 \text{ cwts. for beam placed flat.}$$

Formula for safe uniformly distributed load, again using factor of safety of 5, is

$$\text{Safe distributed load}=\frac{2bd^2c}{LF}$$

$$=\frac{2\times 6\times 10\times 10\times 5}{12\times 5}=100 \text{ cwts. for beam on edge,}$$

$$\text{or}\quad =\frac{2\times 10\times 6\times 6\times 5}{12\times 5}=60 \text{ cwts. for beam placed flat.}$$

Example 3. Find the breadth of a beam of oak resting upon supports 18 feet apart, the beam being 12 in. deep, to carry safely a uniformly distributed load of 5 tons. Constant 5 cwts.

$$\text{Safe distributed load, } W = \frac{2bd^2c}{LF};$$

$$\therefore\ b = \frac{WLF}{2d^2c} = \frac{(5 \times 20) \times 18 \times 5}{2 \times 12 \times 12 \times 5} = \frac{25}{4} = 6\tfrac{1}{4}\text{ inches.}$$

Example 4. A beam of red or yellow pine 20 ft. long (between supports), and 10 in. broad has to carry safely (1) as a central load, (2) a distributed load of 4 tons. What must be the minimum depth of the beam in each case? (Constant 4 cwts.)

$$\text{With a central load } d^2 = \frac{WLF}{bc};$$

$$\therefore\ d = \sqrt{\frac{WLF}{bc}} = \sqrt{\frac{80 \times 20 \times 5}{10 \times 4}} = \sqrt{200} = 14.14 \text{ inches.}$$

With a distributed load

$$d = \sqrt{\frac{WLF}{2bc}} = \sqrt{\frac{80 \times 20 \times 5}{2 \times 10 \times 4}} = \sqrt{100} = 10 \text{ inches.}$$

Example 5.—What size of beam is required to carry safely a central load of 35 cwts. over a 10 ft. span; the depth and breadth of the beam being in the proportion of 7 : 5? (Constant 5 cwts.)

$$b = \tfrac{5}{7}d;$$

$$d^2b = \frac{WLF}{c}$$

$$\text{i.e. } d^2 . \frac{5}{7}d = \frac{WLF}{c}$$

$$\frac{5d^3}{7}=\frac{WLF}{c}$$

$$d^3=\frac{7WLF}{5c}=\frac{7\times 35\times 10\times 5}{5\times 5}=400.$$

$$d=\sqrt[3]{490}=7.88''=\text{nearly } 8''.$$

$$\therefore\ b=\frac{5\times 7.88}{7}=5.6 \text{ inches.}$$

The strength of flitched girders may be calculated by considering the wooden beam and iron flitch separately. The thickness of the flitch is usually about 1-12 that of the wooden beam. The constant for wrought iron is 25 cwts.

Deflection. In arranging beams it is necessary to consider not merely the strength of the beam, but also its liability or otherwise to be bent out of shape—or deflected—by the load placed upon it; since a beam which is overloaded and bent to a large extent has the fibres strained and therefore permanently weakened. The resistance which a beam offers to deflection is called its **stiffness.** It should be noticed that the "strongest" beam is not necessarily the "stiffest," nor the stiffest beam the strongest.

It is of importance to be able to determine the cross-sections of the strongest and the stiffest beams respectively which can be cut from a given log.

Suppose the log is of circular cross-section.

(a) To find the cross-section of the strongest beam.—Draw a diameter AB (Fig. 64) and divide it into 3 equal parts at 1 and 2. From 1 and 2 draw perpendiculars to AB cutting the circumference at C and D respectively. Join ACBD. The rectangle ACBD is the section required.

(b) To find the cross-section of the stiffest beam.—Divide the diameter AB (Fig. 65) into 4 equal parts at 1, 2, 3, and draw 1C and 3D perpendicular to AB, and cutting the circumference in C and D respectively. The rectangle ACBD is the section required.

Since the strength of a beam is proportioned to $\frac{bd^2}{L}$, and the value of this fraction increases as d increases when bd (i. e., the sectional area) remains

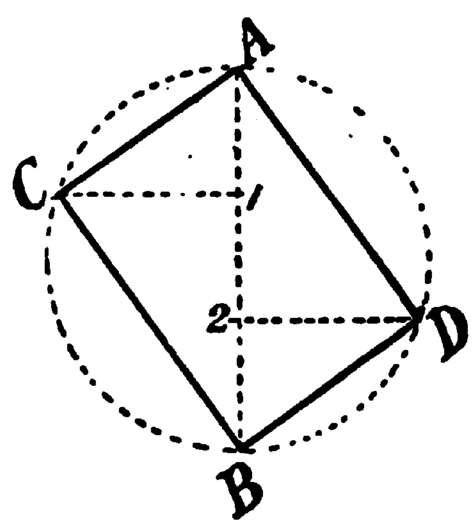


Fig. 64.

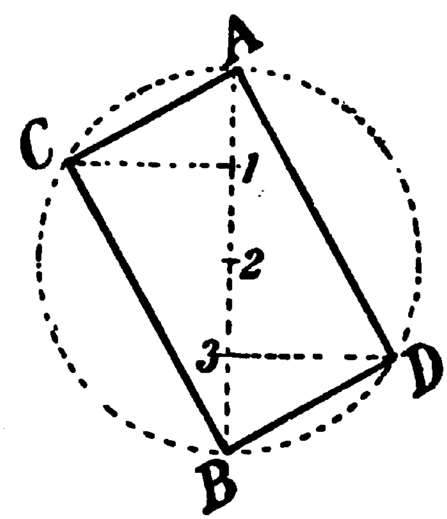


Fig. 65.

constant, the strongest beam of any given sectional area would be that of greatest depth if the tendency to buckling could be avoided. In the case of floor joists the ratio of depth to breadth is often as 3:1 or even 4:1, and the tendency to buckling is overcome by strutting. The strongest beam is that which has the depth to the breadth as 7:5.

Pulleys. It is necessary to consider one or two simple arrangements by which pulleys are used for hoisting purposes. In the following examples the friction will for the sake of simplicity be neglected, although in practice it must be taken into account. Fig. 66 illustrates the simplest application of the pulley. It is

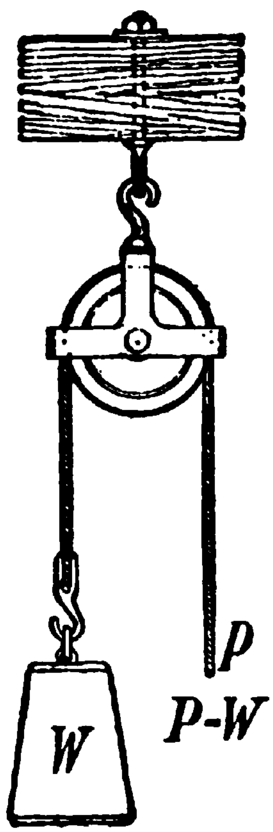


Fig. 66.

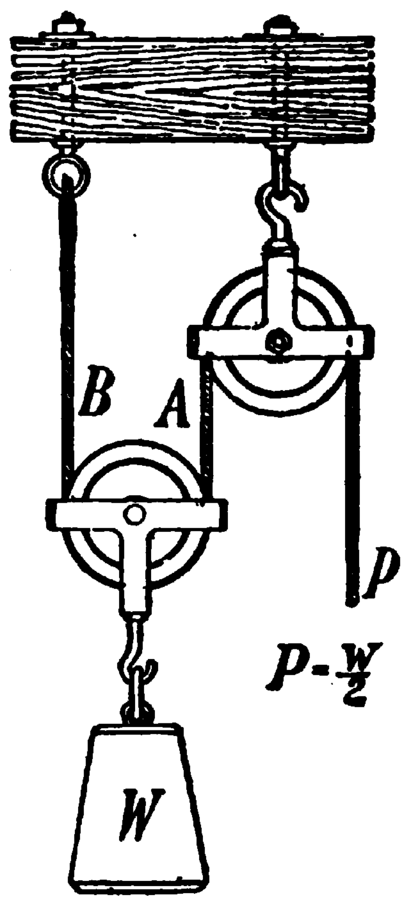


Fig. 67.

plain that when the forces acting on the pulley are in equilibrium they are equal, and the only advantage gained is in the change of direction of the force required to balance W. Therefore in this example P=W.

In Fig. 67 the force balancing P is the tension of the cord A, which is equal to that of B. The sum of these two equal tensions is plainly equal to the weight W. Therefore $P=\frac{W}{2}$, and the mechanical advantage is 2.

Figs. 68 and 69 are illustrations of a two- and three-sheaved pulley block respectively. By arranging pulleys side by side in this manner, and using a combination of two similar blocks as in Fig. 70 a mechanical advantage equal to the number of pulleys around which the rope passes is obtained. In other words, the

Fig. 68.

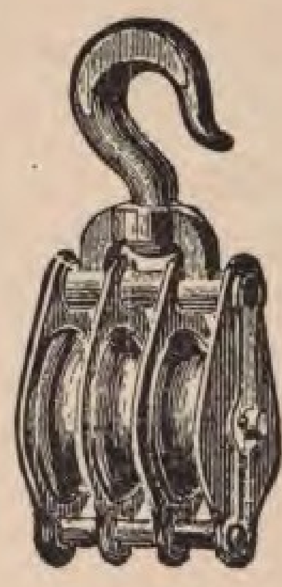


Fig. 69.

power required is equal to the weight raised divided by the number of pulleys around which the rope passes. Thus with 3 pulleys in each block there will be six cords, and the power required to balance a weight of 18 cwts. will be $18 \div 6 = 3$ cwts., plus the force required to overcome friction.

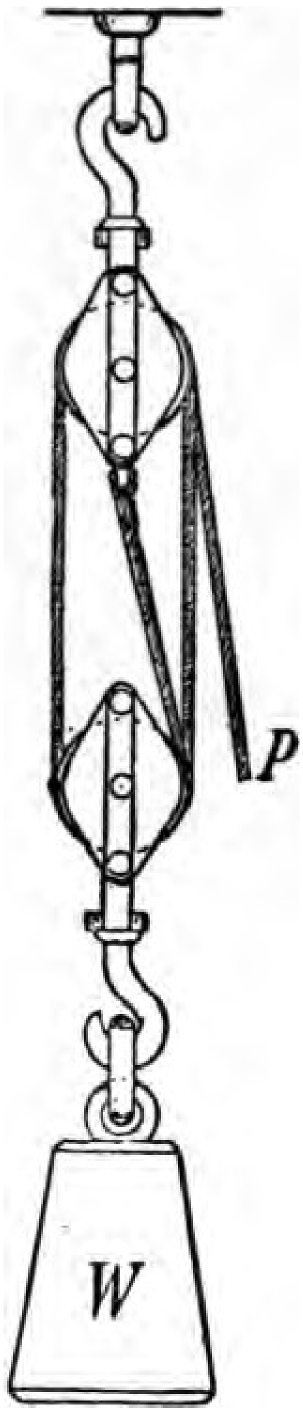


Fig. 70.

Specific Gravity. The specific gravity, or relative density, of a body is the ratio of the weight of that body to the weight of an equal volume of water. Thus a block of wood weighing 40 lbs. per cubic foot has a specific gravity of $\frac{40}{62.5}=0.64$ (since a cubic foot of water weighs 62.5 lbs.).

When a body floats in water, and is therefore in equilibrium, the weight of the body is balanced by an equal upward reaction, the weight of the water displaced being equal to the total weight of the floating body.

Example. A block of wood 9″x9″ x9″, floats in water with its upper surface 2.5″ above the surface of the water. Find its specific gravity.

$\frac{6\frac{1}{2}}{9}=\frac{13}{18}$ of the block is submerged.

By definition, the specific gravity of the wood is the ratio of the weight of any portion of the block to the weight of an equal volume of water. Consider the part of the block below the surface of the water.

$$\text{Specific gravity} = \frac{\text{Weight of submerged part of wood}}{\text{Weight of displaced water}}$$

$$= \frac{\text{Weight of submerged part of wood}}{\text{Weight of whole block}}$$

$$= \frac{\text{Volume of submerged part of wood}}{\text{Volume of whole block}}$$

$$= \frac{13}{18} = 0.72.$$

In addition to the foregoing I have thought it essential to add the following special notes which are the results of actual experiments, and which will, I am sure, appeal to the American workmen, especially those who are located in the West, where timber is yet often employed in heavy structures.

STRENGTH OF TIMBER.

Proposition I. The strengths of the different pieces of timber, each of the same length and thickness, are in proportion to the square of the depth; but if the thickness and depth are both to be considered, then the strength will be in proportion to the square of the depth, multiplied into the thickness; and if all the three dimensions are taken jointly, then the weights that will break each will be in proportion to the square of the depth multiplied into the thickness, and divided by the length; this is proved by the doctrine of mechanics. Hence a true rule will appear for proportioning the strength of timbers to one another.

Rule. Multiply the square of the depth of each piece of timber into the thickness; and each product being divided by the respective lengths, will give the proportional strength of each.

Example. Suppose three pieces of timber, of the following dimensions:

The first, 6 inches deep, 3 inches thick, and 12 feet long.

The second, 5 inches deep, 4 inches thick, and 8 feet long.

The third, 9 inches deep, 8 inches thick, and 15 feet long. The comparative weight that will break each piece is required.

Ans. 9, 12½, and 43 1-5.

Therefore the weights that will break each are nearly in proportion to the numbers 9, 12, and 43, leaving out the fractions, in which you will observe, that the number 43 is almost 5 times the number 9; therefore the third piece of timber will almost bear 5 times as much weight as the first, and the second piece nearly once and a third the weight of the first piece; because the number 12 is once and a third greater than the number 9.

The timber is supposed to be everywhere of the same texture, otherwise these calculations cannot hold true.

Proposition II. Give the length, breadth, and depth of a piece of timber; to find the depth of another piece whose length and breadth are given, so that it shall bear the same weight as the first piece, or any number of times more.

Rule. Multiply the square of the depth of the first piece into its breadth, and divide that product by its length: multiply the quotient by the number of times as you would have the other piece to carry more weight than the first, and multiply that by the length of the last piece, and divide it by its width; out of this last quotient extract the square root, which is the depth required.

Example I. Suppose a piece of timber 12 feet long, 6 inches deep, 4 inches thick; another piece 20 feet long, 5 inches thick; required its depth, so that it shall bear twice the weight of the first piece.

Ans. 9.8 inches, nearly.

Example II. Suppose a piece of timber 14 feet long, 8 inches deep, 3 inches thick; required the depth of another piece 18 feet long, 4 inches thick, so that the last piece shall bear five times as much weight as the first.

Ans. 17.5 inches, nearly.

Note.—As the length of both pieces of timber is divisible by the number 2, therefore half the length of each is used instead of the whole; the answer will be the same.

Proposition III. Given the length, breadth, and depth, of a piece of timber; to find the breadth of another piece whose length and depth is given, so that the last piece shall bear the same weight as the first piece, or any number of times more.

Rule. Multiply the square of the depth of the first piece into its thickness; that divided by its length, multiply the quotient by the number of times as you would have the last piece bear more than the first; that being multiplied by the length of the last piece, and divided by the square of its depth, this quotient will be the breadth required.

Example I. Given a piece of timber 12 feet long, 6 inches deep, 4 inches thick; and another piece 16 feet long, 8 inches deep; required the thickness, so that it shall bear twice as much weight as the first piece.

Ans. 6 inches.

Example II. Given a piece of timber 12 feet long, 5 inches deep, 3 inches thick; and another piece 14 feet long, 6 inches deep: required the thickness, so that the last piece may bear four times as much weight as the first piece.

Ans. [illegible] inches.

Proposition IV. If the stress does not lie in the middle of the timber, but nearer to one end than the other, the strength in the middle will be to the strength in any other part of the timber, as 1 divided by the square of half the length is to 1 divided by the rectangle of the two segments, which are parted by the weight.

Example I. Suppose a piece of timber 20 feet long, the depth and width is immaterial: suppose the stress or weight to lie five feet distant from one of its ends, consequently from the other end 15 feet, then the above portion will be $\frac{1}{10 \times 10} = \frac{1}{100} : \frac{1}{5 \times 15} = \frac{1}{75}$ as the strength at five feet from the end is to the strength at the middle, or ten feet, or as $\frac{100}{100} = 1 : \frac{100}{75} = 1\frac{1}{3}$.

Hence it appears that a piece of timber 20 feet long is one-third stronger at 5 feet distance for the bearing, than it is in the middle, which is 10 feet, when cut in the above proportion.

Example II. Suppose a piece of timber 30 feet long; let the weight be applied 4 feet distant from one end, or more properly from the place where it takes its bearing, then from the other end it will be 26 feet, and the middle is 15 feet; then, $\frac{1}{15 \times 15} = \frac{1}{225} : \frac{1}{4 \times 26} = \frac{1}{104}$ or as $\frac{225}{225} = 1 : \frac{225}{104} = 2\frac{17}{104}$ or nearly $2\frac{1}{6}$.

Hence it appears that a piece of timber 30 feet long will bear double the weight, and one-sixth more, at four feet distance from one end, than it will do in the middle, which is 15 feet distant.

Example III. Allowing that 266 pounds will break a beam 26 inches long, requireth the weight that will break the same beam when it lies at 5 inches from either end; then the distance to the other end is 21 inches; $21 \times 5 = 105$, the half of 26 inches is 3:.$13 \times 13 = 169$; therefore the strength at the middle of the piece is to the strength at 5 inches from the end, as $\frac{169}{169} : \frac{169}{105}$ or as $1 : \frac{169}{105}$ the proportion is stated thus: $1 : \frac{169}{105} :: 266 : 428+$, Ans.

From this calculation it appears, that rather more than 428 pounds will break the beam at 5 inches distance from one of its ends, if 266 pounds will break the same beam in the middle.

By similar propositions the scantlings of any timber may be computed, so that they shall sustain any given weight; for if the weight one piece will sustain be known, with its dimensions, the weight that another piece will sustain, of any given dimensions, may also be computed. The reader must observe, that although the foregoing rules are mathematically true, yet it is impossible to account for knots, cross-grained wood, &c., such pieces being not so strong as those which are straight in the grain; and if care is not taken in choosing the timber for a building, so that the grain of the timbers run nearly equal to one another, all rules which can be laid down will be baffled, and consequently all rules for just proportion will be useless in respect to its strength. It will be impossible, however, to estimate the strength of timber fit for any building, or to have any true knowledge of its proportions, without some rule; as without a rule everything must be done by mere conjecture.

Timber is much weakened by its own weight, except it stands perpendicular, which will be shown in the following problems; if a mortice is to be cut in the side of a piece of timber, it will be much less weakened when cut near the top, than it will be if cut at the bottom, provided the tenon is driven hard in to fill up the mortice.

The bending of timber will be nearly in proportion to the weight that is laid on it; no beam ought to be trusted for any long time with above one-third or one-fourth part of the weight it will absolutely carry: for experiment proves, that a far less weight will break a piece of timber when hung to it for any considerable time, than what is sufficient to break it when first applied.

Problem I. Having the length and weight of a beam that can just support a given weight, to find the length of another beam of the same scantling that shall just break with its own weight.

Let l=the length of the first beam,
L=the length of the second;
a=the weight of the first beam,
w=the additional weight that will break it.

And because the weights that will break beams of the same scantling are reciprocally as their lengths, therefore $\frac{1}{l} : \frac{1}{L} : : w+\frac{a}{2} : \frac{w+\frac{a}{2}}{L} l = W$ = the weight that will break the greater beam; because $w+\frac{a}{2}$ is the whole weight that will break the lesser beam.

But the weights of beams of the same scantling are to one another as their lengths:

Whence, $l : L : : \frac{a}{2} : \frac{La}{2l} = W$ half the weight of the greater beam.

Now the beam cannot break by its own weight, unless the weight of the beam be equal to the weight that will break it:

Wherefore, $\frac{La}{2l} = \frac{w+\frac{a}{2}}{L} l = \frac{2w+a}{2L} l \therefore L^2a = 2wl^2 + al^2$,

$\therefore a : 2w+a : : l^2 : L^2$, consequently $\sqrt{L^2} = L$ = the length of the beam that can just sustain its own weight.

Problem II. Having the weight of a beam that can just support a given weight in the middle, to find the depht of another beam similar to the former, so that it shall just support its own weight.

Let d = the depth of the first beam;
x = the depth of the second;
a = the weight of the first beam;
w = the additional weight that will break the first beam;

then will $w+\frac{a}{2}$ or $\frac{2w+a}{2}$ = the whole weight that will break the lesser beam.

And because the weights that will break similar beams are as the squares of their lengths,

$$\therefore d^2 : x^2 :: \frac{2w+a}{2} : \frac{2x^2 \times w + ax^2}{2d^2} = W$$

the weights of similar beams are as the cubes of their corresponding sides:

$$\text{Hence } d^3 : x^3 :: \frac{a}{2} : \frac{ax^3}{2d^3} = W$$

$$\therefore \frac{ax^3}{2d^3} = \frac{2x^2w + x^2a}{2d^2} \therefore ax = 2wd + ad$$

$\therefore a : a+2w :: d : x =$ the depth required.

As the weight of the lesser beam is to the weight of the lesser beam together with the additional weight, so is the depth of the lesser beam to the depth of the greater beam.

Note.—Any other corresponding sides will answer the same purpose, for they are all proportioned to one another.

Example. Suppose a beam whose weight is one pound, and its length 10 feet, to carry a weight of 399.5 pounds, required the length of a beam similar to the former, of the same matter, so that it shall break with its own weight.

here $a=1$
and $w=399.5$
then $a+2w=800=1+2\times 399.5$
$d=10$

Then by the last problem it will be $1:800:10:8000 = x$ for the length of a beam that will break by its own weight.

Problem III. The weight and length of a piece of timber being given, and additional weight that will break it, to find the length of a piece of timber similar to the former, so that this last piece of timber shall be the strongest possible:

Put l=the length of the piece given
w=half its weight,
W=the weight that will break it;
x=the length required.

Then, because the weights that will break similar pieces of timber are in proportion to the squares of their lengths,

$\therefore l^2 : x^2 : : W+w : \frac{Wx^2+wx^2}{l^2}$ = the whole weight that breaks the beam;

and because the weights of similar beams are as the cubes of their lengths, or any other corresponding sides, .:.

$$\text{then } l^3 : x^3 : : w : \frac{wx^3}{l^3} \text{ the weight of the beam;}$$

consequently $\frac{Wx^2+wx^2}{l^2}$ less $\frac{wx^3}{l^3}$ is the weight that breaks the beam = a maximum; therefore its fluxion is nothing.

$$\text{that is, } 2W x x + 2 w x x - \frac{3 w x^2 x}{l} = \text{nothing.}$$

$$2W + 2w = \frac{3wx}{l} \text{ therefore, } x = l \times \frac{2W+2w}{3w}$$

Hence it appears from the foregoing problems, that large timber is weakened in a much greater proportion than small timber, even in similar pieces, therefore a proper allowance must be made for the weight of the pieces, as I shall here show by an

Example. Suppose a beam 12 feet long, and a foot square, whose weight is three hundred pounds, to be capable of supporting 20 hundred weight, what weight will a beam 20 feet long, 15 inches deep, and 12 thick, be able to support?

```
      12 square inches              15
      12                            15
     ---                           ---
     144                            75
      12                           15
     ---                           ---
 12)1728                           225
     ---                            12
     144                          ----
                              2.0)270.0
                                  -----
                                   135
```

But the weights of both beams are as their solid contents:

```
therefore 12 inches square
          12 ..
         ---
         144
         144 inches=12 feet long
        ----
         576
        576
       144
       -----
       20736 solid contents of the 1st beam
```

```
      15 deep
      12 wide
     ----
     180
     240 length in inches
    -----
    7200
   360
   -----
   43200 solid contents of the 2d beam


 20736:43200::3
              3
        ------ cwt. lb.
20736)129600(6 .. 28=the weight of the 2d beam
      124416
      ------
        5184
         112
      ------
       10368
       5184
      5184
      ------
20736)580608(28
       41472
      ------
      165888
      165888
      ------
```

```
  144::135::21.5 by prop. 1.
          21.5
        ------
          67.5
         135
        270
        ------
    12)2902.5
        ------
   12)241.875
        -------
        20.25625
             112
        -------
           31250
          15625
         15625
        -------
        17.50000
        16
      ----
        30
        5
      ----
        8.0
```

21 cwt. 56 lbs. is the weight that will break the first beam, and 20 cwt. 17 lb. 8 oz. the weight that will break the second beam; deduct out of these half their own weight.

```
  20::17::8
   3::14::0 half
  ----------
  17...3..8
```

Now 20 cwt. is the additional weight that will break the first beam; and 17 cwt. 3 lb. 8 oz. the weight that will break the second: in which the reader will observe, that 10::3::8 has a much less proportion to 20, than 20 cwt. 17 lb. 8 oz. has to 21::46. From these examples, the reader may see that a proper allowance ought to be made for all horizontal beams; that is, half the weights of beams ought to be deducted out of the whole weight that they will carry, and that will give the weight that each piece will bear.

If several pieces of timber of the same scantling and length are applied one above another, and supported by props at each end, they will be no stronger than if they were laid side by side; or this, which is the same thing, the pieces that are applied one above another are no stronger than one single piece whose width is the width of the several pieces collected into one, and its depth the depth of one of the pieces; it is therefore useless to cut a piece of timber lengthways, and apply the pieces so cut one above another, for these pieces are not so strong as before, even if bolted.

Example. Suppose a girder 16 inches deep, 12 inches thick, the length is immaterial, and let the depth be cut lengthways in two equal pieces; then will each piece be 8 inches deep, and 12 inches thick. Now, according to the rule of proportioning timber, the square of 16 inches, that is, the depth before it was cut, is 256, and the square of 8 inches is 64; but twice 64 is only 128, therefore it appears that the two pieces applied one above another, are but half the strength of the solid piece, because 256 is double 128.

If a girder be cut lengthways in a perpendicular direction, the ends turned contrary, and then bolted together, it will be but very little stronger than before it was cut; for although the ends being turned give to the girder an equal strength throughout, yet wherever a bolt is, there it will be weaker, and it is very doubtful whether the girder will be any stronger for this process of sawing and bolting; and I say this from experience.

If there are two pieces of timber of an equal scantling, the one lying horizontal, and the other inclined, the horizontal piece being supported at the points e and f, and the inclined piece at c and d, perpendicularly over e and f, according to the principles of mechanics, these pieces will be equally strong. But, to reason a little on this matter, let it be considered, that although the inclined piece D is longer, yet the weight has less effect upon it when placed in the middle, than the weight at h has upon the horizontal piece C, the weights being the same; it is therefore reasonable to conclude, that in these positions the one will bear equal to the other.

The foregoing rules will be found of excellent use when timber is wanted to support a great weight; for, by knowing the superincumbent weight, the strength may be computed to a great degree of exactness, so that it shall be able to support the weight required. The consequence is as bad when there is too much timber, as when there is too little, for nothing is more requisite than a just proportion throughout the whole building, so that the strength of every part shall always be in proportion to the stress; for when there is more strength given to some pieces than others, it encumbers the building, and consequently the foundations are less capable of supporting the superstructure.

No judicious person, who has the care of constructing buildings, should rely on tables of scantlings, such as are commonly in books; for example, in story posts the scantlings, according to several authors, are as follows:

For 9 feet high	6 inches square
12 ————	8
15 ————	10
18 ————	12

Now, according to this table, the scantlings are increased in position to the height; but there is no propriety in this, for each of these will bear weight in proportion to the number 9, 16, 25, and 36, that is, in proportion to the square of their heights, 36 being 4 times 9; therefore the piece that is 18 feet long, will bear four times as much weight as that piece which is 9 feet long; but the 9 feet piece may have a much greater weight to carry than an 18 feet piece, suppose double: in this case it must be near 12 inches square instead of 6. The same is also to be observed in breast-summers, and in floors where they are wanted to support a great weight; but in common buildings, where there are only customary weights to support, the common tables for floors will be near enough for practice.

To conclude the subject, it may be proper to notice the following observations which several authors have judiciously made, viz.; that in all timber there is moisture, wherefore all bearing timber ought to have a moderate camber, or roundness on the upper side, for till that moisture is dried out, the timber will swag with its own weight.

But then observe, that it is best to truss girders when they are fresh sawn out, for by their drying and shrinking, the trusses become more and more tight.

That all beams or ties be cut, or in framing forced to a roundness, such as an inch in twenty feet in length, and that principal rafters also be cut or forced in framing, as before; because all joists, though ever so well framed, by the shrinking of the timber and weight of the covering will swag, sometimes so much as not only to be visible, but to offend the eye: by this precaution the truss will always appear well.

Likewise observe, that all case-bays, either in floors or roofs, do not exceed twelve feet if possible; that is, do not let your joints in floors exceed twelve feet, nor your purlines in roofs, &c., but rather let their bearing be eight, nine, or ten feet. This should be regarded in forming the plan.

Also, in bridging floors, do not place your binding or strong joists above three, four, or five feet apart, and take care that your bridging of common joists are not above twelve or fourteen inches apart, that is, between one joist and another.

Also, in fitting down tie-beams upon the wall plates, never make your cocking too large, nor yet too near the outside of the wall plate, for the grain of the wood being cut across in the tie-beam, the piece that remains upon its end will be apt to split off, but keeping it near the inside will tend to secure it.

Likewise observe, never to make double tenons for bearing uses, such as binding joists, common joists, or purlins; for, in the first place, it very much weakens whatever you frame it into, and in the second place, it is a rarity to have a draught to both tenons, that is, to draw both joists close; for the pin in passing through both tenons, if there is a draught in each, will bend so much, that unless it be as tough as wire, it must needs break in driving, and consequently do more hurt than good.

Roots will be much stronger if the purlins are notched above the principal rafters, than if they are framed into the side of the principals; for by this means, when any weight is applied in the middle of the purlin, it cannot bend, being confined by the other rafters; and if it do, the sides of the other rafters must needs bend along with it; consequently it has the strength of all the other rafters sideways added to it.

This volume will be followed by at least three more, on the subject of **Carpentry and Joining**, one volume of which will be devoted to **Framing** and **Heavy Timbered Work**. The other two or more volumes will be devoted to fine joining and cabinet work, so far as the latter art comes within the purview of the joiners' work.

Each volume will, of course, be illustrated with diagrams and working details. The matter will be taken from executed works and from every available source.

INDEX

A

An Annular Vault 128
An Ogee Dome 129
Angle Brackets for Core.................... 153
Angle Bracket for Curved and Straight Walls.... 155
Angle Bracket on a Rake.................... 157
A Frame Splayed Lining..................... 175
A Window with Splayed Soffit................ 178
Angle Sash Bars 187
Axle Pulleys 213
A Lantern Light 233
A Bay Window 237
A Cased Frame Bay Window................... 240
A Riveted Light 243
A Bull's-Eye Frame 247
A Segment Headed Frame..................... 286
A Semi-Headed Frame 287
A Solid Frame 288
Architrave Grounds 294
Architraves 295
A Variety of Panel Doors..................299-303

B

Bracketing for Cornices 153
Brackets for Re-entrant Angles.............. 154

Bracket for Curved Cornice 155
Boxing Shutters 191
Balancing Shutters 193
Bay Windows 216
Box Shutters 225
Barge Boards 230
Battered Doors 252
Brased Doors 252
Bead Flush Doors 255
Bead and Butt Doors 256
Bolection Moulded Doors 257
Bead Flush Panels 259
Bead Butt Panels 259
Bolection Mouldings 261
Bed-room Doors 263
Blind Screwing 281
Blind Nailing 281
Bending Ornaments 345

C

Cube 43
Cube Projection 44
Cube and Octagon 45
Cone, Sphere, and Cylinder 57
Cylinder, Cone and Sphere 57
Conic Sections 72
Cones from Parabolas 75
Cones and Hyperbolas 76
Cuneoids 78
Cylinder Sections 78
Cylindrical Rings 82
Cylinder and Sphere 90

Coverings of Solids.............................. 96
Covering Prisms 98
Covering of Pyramids.............................. 99
Covering of Right Cylinders.................... 103
Covering of Oblique Cylinders.................. 105
Covering Semi-Cylindric Surfaces............... 106
Cone Coverings 108
Covering a Segmental Dome...................... 117
Covering an Elliptic Dome...................... 122
Covering an Ellipsoidal Dome................... 123
Covering an Annular Vault...................... 128
Covering an Ogee Dome.......................... 129
Cutting Oblique Spouts or Timbers.............. 143
Cutting Different Pitches...................... 144
Ceilings in Wood or Stucco..................... 146
Cove Brackets 153
Cornice Bracketing 153
Corner Bracket on a Rake....................... 157
Curb Ribs for Circular Ceiling................. 158
Circular Splayed Work.......................... 176
Construction of Windows........................ 201
Casement Windows 204
Cased Pulley Styles............................ 209
Circular Head Windows.......................... 220
Corner Posts and Cross-Rails................... 229
Conservatories 236
Chamfered Doors 256
Constructive Memoranda 257
Common Doors 264

D

Diagram Showing Solid Geometry................. 10
Dodecahedron 48

Dodecahedron Solids 51
Development of Coverings.................... 97
Diagram for Oblique Cuts.................... 145
Double Semi-Circular Mitre.................. 160
Double Curvature Work....................... 166
Development of Conical Surfaces............. 177
Diminishing of Mouldings.................... 180
Double Tenons 203
Double Sunk Rails........................... 208
Double Hung Sashes 214
Double Bolection Moulded Doors.............. 257
Double Margined Doors....................... 267
Door and Window Grounds..................... 279
Door and Jambs Complete..................... 298
Deflection of Beams and Girders............. 354

E

Ellipsoids 86
Ellipsoidal Coverings 114
Elliptical Dome Coverings................... 120
Elliptical Niche 162
Enlarging of Mouldings...................... 180
Entrance Doors 263
External Doors 264
External Door Frames........................ 283

F

Finding Projection Points in a Cone......... 60
Finding Point of Projection in Sphere....... 64
Finding an Ellipsoid........................ 81
Frustrum of Cones........................... 111

Finding the Stretchout of Curves................ 131
Framed Splayed Circular Soffit.................. 175
Folding Shutters 188
Franking Sash Bars......................... 204
Fixing Window Frames 221
Finials 230
Framed and Braced Doors.................... 253
Framed Paneled Doors....................... 254
Framed Grounds 295
Forces which Balance Each Other................ 310

G

Given Points in Planes......................... 32
Generating Curves 84
Gable Mitres 140
Grounds 226
Greenhouses and Conservatories................ 236
Grounds for Base.......................... 291

H

Horizontal Traces 24
Horizontal Projection 28
Helix in Cylinder.......................... 62
Helix in a Cone........................... 63
Helical Curve on Sphere..................... 65
Hexagonal Pyramid 67
How to Cut Oblique Spouts, etc.................. 143
Hook Point 207
Hinged Skylights 227
Heavy Architraves 296

I

Intersecting Lines 18
Intersection of Curved Surfaces 86
Intersection of Cylinders 87
Intersecting a Sphere and Cylinder 90
Intersection of Cones 91
Irregular Mitres 133
Interior Shutters 188
Inclined Forces 323

J

Joints of Sashes 203

L

Lamet Curves 84
Large Panels for Ceilings 151
Long Shutters 189
Lifting Shutters 193
Linings 223
Large Sky-Lights 231
Lantern Lights 231
Laying Out a Circular Louvre 248
Louvre Boards 249
Louvre Gains 250
Loaded Beams 331

M

Mitres for Irregular Forms 133
Mitres on a Rake 135
Mitring Irregular Mouldings 138
Mitring Sprung Mouldings 185

Mitring Curved and Straight Mouldings......... 185
Mullions 215
Moulded and Square Doors.................... 255
Moulded Grounds 294
Mechanics of Carpentry...................... 305
Methods of Arranging Beams................ 344

N

Niches 159
Niche on Elliptical Plane..................... 164

O

Octagon Diagram 13
Octagon and Cube.............................. 44
Octahedron 46
Octahedron Projection 47
Oblique Planes 57
Octagonal Pyramid 69
Oblique Axes 70
Ogee Pyramids 85
Oblique Cuts 143
On Mitres 159
Octagonal Mitre 163
Outside and Inside Linings 209
Other Paneled Framing....................... 278

P

Preface 5
Part I. 9
Points, Lines and Planes...................... 16
Planes and Projections........................ 22
Perspective Lines and Traces................. 26

Projection of Cylinder.......................... 58
Pyramid 67
Prisms ....
Pyramids Developed ....
Practical Solutions 131
Projection of Solids........................... 141
Paneled Ceilings, Wood or Stucco............... 146
Parting Strips 210
Pocket Piece 212
Position to Cut Beads.......................... 245
Planted Moulded Doors.......................... 258
Projections of Doors 263
Plain Jamb Linings 289
Plinth Blocks 297
Panel Doors in Various Styles.................. 300
Polygon of Forces.............................. 319
Parallel Forces 324
Pulleys 356

R

Right Lines 36
Regular Polyhedrons 96
Raking Moulding Mitres......................... 136
Raking Bracket for Corner...................... 157
Ribs for Circular Arch and Ceiling............. 158
Ribs for Octagonal Niche....................... 163
Regular Octagonal Niche 165
Ribs for Irregular Octagonal Niche............. 166
Radial Sash Bars............................... 173
Raking Mouldings 183
Raking Sash Bars............................... 187
Rolling Shutters 188

Raised Panel Door.............................. 256
Ra..ed and Flat Door.......................... 256
ed, Sunk and Moulded Door............... 256
.ed, Sunk and Moulded........................ 257
Reception Room Doors 263
Resistant of Two or More Forces................ 306
Reciprocal Diagrams 332
Revolving Doors 278

S

Solid Geometry 9
Straight-Sided Solids 36
Sphere and Tetrahedron....................... 55
Sphere, Cone and Cylinder..................... 57
Sections of Solids............................. 66
Sections of Cones.............................. 76
Sections of Cones from Parabolas.............. 75
Sections of Cones from Hyperbolas............. 76
Sections of Cuneoids.......................... 78
Sections of a Cylinder......................... 78
Solids of Revolution........................... 83
Sphere and Cylinder 90
Semi-Cylindric Coverings 106
Spheres and Ellipsoids........................ 113
Spherical Niche 159
Semi-Circular Niche 160
Segmental Niche 161
Semi-Elliptical Niche 163
Segment Heads 170
Staving for Circular Heads.................... 170
Sash Bars for Circular Windows................ 173
Splayed Work Circulars........................ 176

Sprung Mouldings 185
Shutters 188
Sliding Shutters 193
Spring Shutters 197
Size and Position of Windows........................... 201
Sashes 201
Sashes Opening Inward........................... 205
Sashes Opening Outward........................... 208
Sash and Frame Window........................... 209
Sashes and Window Frames........................... 209
Sashes of Various Kinds........................... 212
Single Hung Sashes........................... 214
Sliding Sashes 215
Shop Windows 221
Sliding Shutters 226
Sash Frame 228
Side-Lights 233
Square and Sunk Doors........................... 254
Stop Chamfered Doors........................... 257
Solid Moulded Doors........................... 258
Superior Internal Doors........................... 264
Square Shoulder Rail........................... 271
Superior Doors 279
Secret Fastening 281
Solid Door Frames........................... 283
Semi-Circular Frame 288
Skeleton Frame 289
Set of Linings........................... 289
Stress Diagrams for Trusses........................... 337
Strength of Wooden Beams........................... 342
Stiffners of Beams........................... 342
Specific Gravity 358
Strength of Timber........................... 359

T

To Determine Length of Oblique Lines........... 16
To Determine Various Angles.................... 21
Traces of Intersecting Planes.................... 25
Tetrahedron 37
Tetrahedron .Projection 40
To Find Inclination of Two Faces............... 49
To Describe a Helix.............................. 64
The Cone 71
To Describe the Section of a Sphere............. 80
To Describe an Ellipsoid......................... 81
To Intersect Two Unequal Cylinders............. 88
To Find Covering of Frustrum of Cone.......... 112
To Find the Covering of an Elliptic Dome....... 122
To Find Angle Bracket for Cornice.............. 154
To Obtain Soffit Mould.......................... 166
To Form a Head.................................. 168
To Obtain a Developed Face Mould.............. 172
To Find Mould for a "Cot Bar"................. 173
To Find Mould for Radial Bars.................. 173
To Obtain Face Moulds.......................... 176
To Apply the Moulds............................. 177
The Development of Conical Surfaces........... 177
To Diminish a Moulding.......................... 181
Throating 205
To Hang the Sash................................ 245
To Find Position to Cut Beads.................. 245
The Construction of Doors...................... 251
To Set Out Gunstock Stiles..................... 270
To Set Out Rail for Gunstock Stile............. 271
The Nature of Force............................. 306
The Parallelogram of Forces.................... 308

Triangle of Forces.............................. 312
Transverse Strength of Wooden Beams.......... 349

V

Vertical and Horizontal Projections.............. 17
Vertical Projection 31
Vertical Projection of Dodecahedron............. 53
Veneering Soffits 170
Venetian Windows 215
Vertical Sashes 215

W

Windows Generally 201
Window Frames 204
Window and Sash-Frames.......................... 210
Windows with Curved Heads....................... 219
Windows for Shops................................ 221
Wedges 221
Window Linings 223
Window Shutters 225
Window Grounds 279

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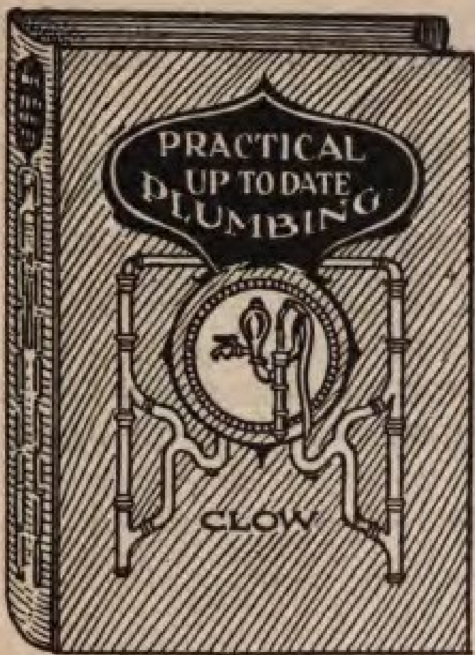
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